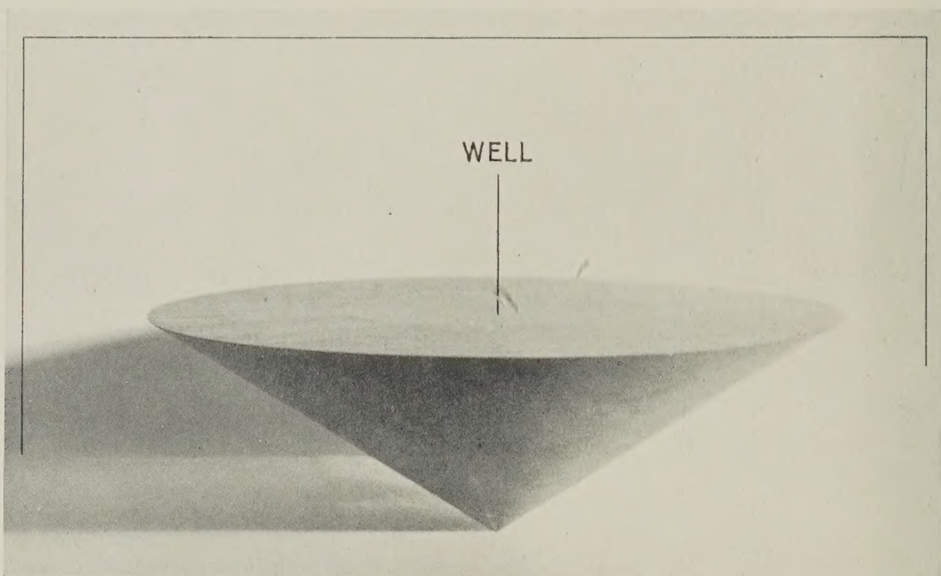
# ANALYTICAL PRINCIPLES *of* THE PRODUCTION *of* OIL, GAS, AND WATER FROM WELLS

STANLEY C. HEROLD

# PRODUCTION OF OIL, GAS, AND WATER




*"The isolated oil, gas, or water well in Capillary Control produces one-third of the fluid underlying its drainage area, such fluid being mobile at the given constant back pressure which is exerted against production. The quantitative distribution of this produced fluid, irrespective of the thickness of the formation which serves as a reservoir, may be represented by a space diagram in the form of a right circular cone with a base of radius* R."

(See Section 174)

# ANALYTICAL PRINCIPLES *of* THE PRODUCTION *of* OIL, GAS, AND WATER FROM WELLS

A TREATISE BASED UPON A SYSTEM OF FLUID MECHANICS PARTICULARLY ADAPTED TO THE STUDY OF THE PERFORMANCE OF NATURAL RESERVOIRS

BY

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WITH A FOREWORD BY CYRUS F. TOLMAN
AND A FINAL SUMMARY BY ERNEST K. PARKS

STANFORD UNIVERSITY PRESS
STANFORD UNIVERSITY, CALIFORNIA
1928

PRINTED AND BOUND IN THE UNITED STATES OF AMERICA
BY STANFORD UNIVERSITY PRESS

To

CORNELIA M. FARLEY
LEANDER M. HOSKINS
HALCOTT C. MORENO

*Studying with you I saw how sound concepts and conclusions may conform to the rigor of exact mathematical thinking, and I thereafter had the inspiration to do this book with confidence*

# FOREWORD

With pleasure and profit I have followed Mr. Herold's work on fluid delivery from oil and gas wells from the time he brought his field data to Stanford University in 1923 for detailed analysis and investigation to the completion of this treatise, approximately four years later.

Mr. Herold's treatment is largely mathematical, because the data available to the petroleum engineer are in mathematical form, chiefly as production curves, and because mathematical methods are not only an aid to clear thinking and concise statement, but also are essential to the solution of many of the problems investigated.

Although the mathematics used in his treatise is simple, nevertheless the concise mathematical statement of a problem in the form of an equation or a curve is oftentimes terrifying to one who does not readily translate mathematical expressions into the spoken word about physical events.

For this reason, and also because an understanding of the three "controls" which govern fluid production from oil and gas wells is essential to a conception of the behavior of these wells and to a satisfactory manipulation of them, the attempt is made in this foreword to illustrate these controls by familiar examples, such as the flow of water from tanks, without any reference to equations or curves.

For two of the controls the task is easy. A water tank delivering fluid with head (or depth of water) maintained at a constant level illustrates his Hydraulic Control, and a water tank with lowering head during fluid delivery illustrates his Volumetric Control.

The mathematics of these two types has long been familiar to students of hydraulics, and it must apply to well production if the head is due to the hydraulic pressure of a column of liquid. If replenishment of the water in the formation is so rapid that the production from wells does not affect the height of the water column, the laws controlling fluid delivery from a tank of constant head will govern the production of oil and gas in Hydraulic Control. If the production of fluid from the wells is more rapid than replenishment of water in the column, then the laws controlling fluid delivery from a tank with decreasing head will govern production in Volumetric Control.

Mr. Herold first investigated production curves of oil and gas wells in the Mid-Continent and Eastern fields. These curves showed uniformly a significant difference in the behavior of the wells from that called for when hydraulic pressure governs fluid delivery. His mathematical analyses of these curves proved that hydraulic pressure here does not force the oil to the well, and suggested that the energy was supplied by the expanding gas dissolved in the oil; also that the energy thus derived is able to force oil to the well from a definite and limited distance out from the well against a resistance of an origin unknown at the time.

While studying the literature of physics in search for the physical explana-

tion of his mathematical analyses, he encountered Jamin's classic work on capillarity, in which appears a discussion regarding capillary resistance due to a chain of bubbles in a capillary tube, and the proof that the resistance is directly proportional to the number of bubbles in the tube.

These experiments gave him the basis for a concept of the physical conditions governing fluid delivery from wells having production curves such as those first studied. Unfortunately there is no familiar water tank to illustrate the behavior of capillary tubes.

The Jamin capillary tube may be taken to represent a line of the innumerable, interconnected, capillary interstices that drain into the well. The tube filled with liquid and bubbles represents the physical condition of the fluids within the formation, when the relief of pressure accompanying the escape of the fluids from the open well is sufficient to cause the gas that is dissolved in the oil to flash into bubbles.

Thus a body of foam is formed encircling the well and increasing in diameter as the pressure decreases in the formation, and sooner or later the Jamin resistance becomes greater than the hydraulic pressure in the oil and gas pool. Thereafter the pressure that moves the fluids is internal, and not external to the pool. The force of the expanding gas can push a line of bubbles of definite length in the capillary interstices which discharge toward the well. The length of this line of bubbles is the radius of the circular area of production around the well. No production beyond this area is possible from the well as long as capillary resistance exceeds hydraulic pressure. Mr. Herold aptly calls this control Capillary Control. His early studies were limited to wells of this control, but subsequently he found that wells in certain fields, notably those in Mexico, obeyed the laws of Hydraulic Control, while other fields, exemplified by those in California, obeyed the laws of Volumetric Control.

The measurements that should be made to furnish the data necessary for the solution of the equations he discusses are indicated throughout his work. He shows that the control under which the well is producing can be easily determined, and the importance of this determination lies in the fact that by it conditions existing within the productive formation can be definitely known.

In conclusion it may be stated that the problems of mutual interference of wells, spacing of wells, recovery of oil and gas, drainage of a pool by a well or group of wells, and other important problems that have confronted the industry, the government, and the judiciary, can only be solved by the application of the theory expounded in this treatise.

This treatise is the first attempt to state the fundamental laws governing fluid delivery from oil and gas wells, and it will prove to be the foundation for further advances of knowledge.

CYRUS F. TOLMAN

STANFORD UNIVERSITY, CALIFORNIA
June, 1928

# PREFACE

It was ten years ago, to be exact, when I first became seriously interested in the characteristic performance of oil and gas wells. As I look back now, it seems only natural that I should have begun the study of this performance from the standpoint of the facts presented by the empirical data of field observations. These data, I believed, were alone to be taken as the criteria of performance. Physical laws would undoubtedly explain the behavior of the wells, and the data, it seemed, would surely be competent to indicate these laws. I made little, if any, progress by this procedure, for this empirical method of study afforded me no way of making a discrimination between right and wrong conclusions. Laws which appeared to express the quantitative relations between, say, pressure, rate of production, and time, held satisfactorily so long as I applied them to wells in the same field. Clearly they did not admit of general application. An alternative method of attack remained, however, and this was to take up the subject seemingly from the reversed direction, namely, from the standpoint of the established laws in physics. Empirical data would continue to be useful, for they would indicate the various paths to be traversed in a detailed investigation, they would prevent erroneous digressions from these paths by serving as a means of final judgment concerning the competency and the comprehensiveness of any deductions to be made in accordance with fundamental principles.

This alternative method has in fact required of me four full years of study. But I shall not say that this length of time was due to inherent difficulties of the problem before me; rather must I admit that my education in geology had to be supplemented by one in adequate branches of physics and mathematics. I found it necessary to construct a convenient system of fluid mechanics, one that is sufficiently flexible, and not too idealistic in nature, in its application to natural reservoirs and their wells, where perfection in physical conditions, according to our usual standards of dynamics, is quite unknown. The foundation for such a system was already at hand. Theoretical mechanics and hydromechanics, wherein principles are based upon a variety of experiments including those associated with falling bodies and fluids at rest and in motion, had established certain basic laws upon which the system could be constructed. Furthermore, they had established reliable methods of construction. These principles, these methods, and the laws of physics concerned with the properties of liquids, gases, and vapors have indeed led me to the subject-matter which forms the present treatise on the analytical principles of the production of oil, gas, and water from wells.

In so far as we accept these laws as expressions which faithfully represent Nature's behavior upon specified circumstances, my basic assumptions—if thus they may be called—are incontestable. The physicist knows we are not to observe exact fulfillment of these laws; he tells us that we are to be satisfied with closest approximations. The philosopher alone is permitted to question

their quality of fitness. I invent no new laws of physics, neither do I discover new properties of fluids. The investigations pursued by physicists in the years between 1460 and 1860 appear to furnish sufficient material for the present.

From these basic laws I make my deductions which relate specifically to the performance of reservoirs. Most of these deductions are immediately acceptable to the petroleum engineer; a few of them I fear are for the present contrary to the traditions of his profession. For these I shall deny responsibility, unless it may be proved that I have erred. I do not apologize for administering a rather large dose of elementary mathematical principles. The mathematics I use was invented for the explicit purpose of describing the behavior of Nature, and I concede the fact that many of my deductions might forever remain obscure, were it not for the efficacy of simple mathematical operations. Granting this to be the case, I do not ask for the acceptance of these deductions without due observations either in the laboratory or in the field. We are to remember that purely mathematical reasoning never yields physical results; we are to concede nothing except upon verification by deliberate observations. I am confident that the results will fully justify the present means of studying the delivery of fluid from reservoirs.

In saying that these mathematical operations are simple I do not mean to infer that their proper interpretations will always be found so. When we understand the relation between mathematical operations and physical events in all its phases, we can appreciate the difficulties to be encountered in the interpretation of either one in terms of the other. It is clear that facility and confidence are to be acquired only by diligent practice and application.

The treatise at large is confined to analytical theory. I do not dwell upon hypotheses, for I take these to be unnecessary in the light of our established laws of physics and mathematics. We are to investigate natural events which take place within productive reservoirs. These, of course, are found to be both independent of and dependent upon the manner in which the operator equips and regulates his wells. No attempt to include details of practical application shall be made, inasmuch as these are to be considered as matters of technology, rather than matters of science. I take the province of science to be that of indicating the way to technology. Thus in science I may say that all wells are alike, in that they are merely the orifices of natural reservoirs in performance, while in technology each well has characteristics which differentiate it from all other wells.

Specifically, the treatise is divided into four parts. The first includes a brief review of the laws of physics and the properties of matter, as these appear related to the problem of production. Here I have also found it convenient to adopt new terminology, in order to avoid possible ambiguity with respect to those points which must be defined in a precise manner, and in order to describe in a word or phrase an entire process or combination of propositions which appear to be new. Conservatism and practicability have guided me in the choice of these terms which we are to add to our glossary

pertaining to the behavior of natural reservoirs and wells. I trust that no confusion may arise by conflict with their previously established uses. The succeeding three parts deal with the three great classes of reservoirs. To one of these classes each and every oil well, gas well, and water well must belong. These I designate as Hydraulic, Volumetric, and Capillary Controls. They are defined by their mathematical features as displayed by their curves of production. The controls are analyzed by means of selected artificial reservoirs which serve as types. The treatment is a progressive one throughout, where discriminations that are made in the first control aid in the understanding of the second; and, in turn, where discriminations that are made in the second aid in the understanding of the third, the latter control being, perhaps, the most difficult to comprehend, and yet the most precise in its performance in the field.

Mr. Ernest K. Parks gives us a final summary entitled, "Analytical Principles of Production: Their Value to the Oil and Gas Industry." In this he emphasizes the practical application of the theory herein set forth to the problems of oil and gas field development, operation, and rejuvenation.

As appendixes I have included tables and supplementary articles which may prove to be of interest to those wishing to proceed with their studies of the theory.

Parts of the text have appeared from time to time in issues of the *Oil and Gas Journal* of Tulsa, Oklahoma. In the present writing special attention has been given to the arrangement of the material so that it may serve the needs of students in petroleum engineering. Where it is to be used as a text in a regular curriculum, I do not hesitate to suggest two courses in preparation for the one it is to serve. They are the following:

I. A lecture and laboratory course on the properties of liquids, gases, and vapors, with special reference to those found in natural reservoirs. This course would include such considerations as molecular state, expansion and compression, phenomena accompanying heat changes, the phenomena of adhesion and cohesion, viscosity, surface tension, capillary attraction and resistance, the behavior of mixtures composed of any two or all three of the classes of fluids, and so on.

II. A lecture course on the foundations of mathematical and physical sciences, including a review of the methods of mathematical analysis and the philosophical bases of mathematical deduction. Of particular importance is the relation between the two sciences.

I furthermore suggest that laboratory experimentation should accompany the study of the present text. In pursuing these courses the student should, from the beginning, hold in mind the words of Michael Faraday, who said: "The philosopher should be a man willing to listen to every suggestion, but determined to judge for himself. He should not be biased by appearances; have no favorite hypothesis; be of no school; and in doctrine have no master. He should not be a respecter of persons, but of things. Truth should be his

primary object. If to these qualities be added industry, he may indeed hope to walk within the veil of the temple of nature."

In preparing this book I have been dependent upon the assistance and encouragement rendered by several members of the faculty of Stanford University. I have most frequently consulted, in the Department of Geology, Cyrus F. Tolman, Eliot Blackwelder, James Perrin Smith, and Austin Flint Rogers; in the Departments of Physics and Chemistry, Stewart W. Young, Frederick J. Rogers, Perley A. Ross, Joseph G. Brown, and George S. Parks; in the Department of Mathematics, Leander M. Hoskins, Halcott C. Moreno, William A. Manning, Robert E. Allardice, and Hans F. Blichfeldt; in the Departments of Engineering, Charles N. Cross and Frederick G. Tickell; and in the Department of Philosophy, Harold C. Brown. To Cyrus F. Tolman I am particularly indebted for the close attention he has given the work from the beginning of my theoretical pursuits. His constructive criticisms have invariably led me to the study of problems, the practical importance of which I had previously not appreciated. From Benjamin F. Hake and Ernest K. Parks I have received suggestions and comments which have enlarged the scope of my work and improved the technique or rigor of the proofs it contains. The latter has given my manuscript a most thorough and critical examination. Theodore K. Sawyer, Thomas H. Acres, and Clifford S. Wilson have contributed essential data of experimentation in the laboratory.

With respect to data from the field I confess that I have been served most generously by many oil and gas companies through their administrators, geologists, and engineers. Lastly, with respect to sources of information, I would say that I have drawn freely from current textbooks on mathematics, physics, and physical chemistry, and also from original records of particular laboratory experiments that were performed by early investigators. Many of these records are to be found in the library at Stanford University. I might add that it has been necessary to repeat some of these experiments with modifications that permit a closer approach to the conditions which with reason we believe to exist within natural reservoirs of oil and gas.

Stanley C. Herold

Stanford University, California
June, 1928

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*Profound study of Nature is the most fertile source of mathematical discoveries. Not only has this study, in offering a determinate object to investigate, the advantage of excluding vague questions and calculations without issue; it is besides a sure method of forming analysis itself, and of discovering the elements which it concerns us to know, and which natural science ought always to preserve. These are the fundamental elements which are reproduced in all natural effects.*

*"The analytical equations, unknown to the ancient geometers, which Descartes was first to introduce into the study of curves and surfaces, are not restricted to the properties of figures, and to those properties which are the object of rational mechanics; they extend to all general phenomena. There cannot be a language more universal and more simple, more free from errors and from obscurities, that is to say more worthy to express the invariable relation of natural things.*

*"Considered from this point of view, mathematical analysis is as extensive as Nature itself; it defines all perceptible relations, measures times, spaces, forces, temperatures. This difficult science is formed slowly, but it preserves every principle which it has once acquired; it grows and strengthens itself incessantly in the midst of the many variations and errors of the human mind.*

*"Its chief attribute is clearness; it has no marks to express confused notions. It brings together phenomena the most diverse, and discovers the hidden analogies which unite them. If matter escapes us, as that of air and light, by its extreme tenuity, if bodies are placed far from us in the immensity of space, if man wishes to know the aspect of the heavens at successive epochs separated by a great number of centuries, if the actions of gravity and of heat are exerted in the interior of the earth at depths which will always be inaccessible, mathematical analysis can yet lay hold of the laws of these phenomena. It makes them present and measurable, and seems to be a faculty of the human mind destined to supplement the shortness of life and the imperfection of the senses; and what is still more remarkable, it follows the same course in the study of all phenomena; it interprets them by the same language, as if to attest the unity and simplicity of the plan of the universe, and to make still more evident that unchangeable order which presides over all natural causes."*

—Joseph Fourier

# Part I. Elementary Principles

CHAPTER I

# Nature and Scope of the Treatise

"Is Nature governed by caprice, or does harmony rule there? That is the question. It is when it discloses to us this harmony that science is beautiful and so worthy to be cultivated. But whence can come to us this revelation, if not from accord of a theory with experiment?"—HENRI POINCARÉ

1. *The empirical method of study.*—With respect to the physical sciences we are to admit that all our knowledge of them is founded upon experience or experiment. All this knowledge, then, possesses an empirical foundation. We might conceive ideas of physical events and conditions independently of actual operations either in the field or in the laboratory, but these ideas are worthless without the proper verification. The verification itself is dependent upon empirical observations and empirical data. While this, as we say, is conceded to be true of physical sciences in general, a more restricted conception of the empirical is to be desired in formulating our ideas concerning the nature and scope of the present treatise.

Merely as a matter of convenience we can say that, in so far as the study of the performance of reservoirs and wells is concerned, all speculative laws relating to their behavior during the process of production, as based upon field observations, are of an empirical nature, while any laws derived from the well-established principles of fluid mechanics are of an analytical nature. To justify ourselves in so classifying these laws we can cite the uncertainty of the former methods of derivation, and the universal acceptance of the principles involved in the latter. The classification is an arbitrary one, to be sure, but it is an expedient one.

From the point of view of the physicist the status of the principles of fluid mechanics is such as to render unnecessary their further verification by experiments in the laboratory, unless these are performed solely for the purposes of instruction.

We are then to make a clear distinction between the data of observations in the field and the data of observations in the laboratory. Our field apparatus—if such we may call it—consists of two parts, the natural reservoir and the well proper, the first as provided by Nature herself, and the second as equipped with any appurtenances by the operator. The reservoir may be said to be inaccessible to us, except in so far as we may claim to be in contact with

it by means of the well.[1] The operator has no jurisdiction in things pertaining to the reservoir; he can only influence the motion of the fluid within it by manipulating things at the well. No part of the reservoir is visible; and any observations on its behavior are to be made at the well. We readily see how different this apparatus is from one which we may construct and set up in the laboratory.

What if we find the natural reservoir and the well, as separate entities, to perform in accord with separate sets of physical laws which, aside from general physical properties of fluids, have few if any items in common? Is it not clear that our data of observations at the well might involve quantities which are dependent upon a complex physical situation? These appear to me to be unquestionably true circumstances. We have for many years attempted to formulate physical laws, and thereby diagnose physical events and conditions within the reservoir by means of empirical data obtained at the well. Unknown factors have been deterrent factors in the determination of these laws, events, and conditions, for their nature and magnitude, in their complexity, have remained obscure, and therefore indeterminate. Empirical data, as we have agreed to construe the term, have been and will continue to be incompetent for the purposes of clarifying our ideas of performance. And it is unfortunate, I would say, that our experiences with these data in the past have too universally established the belief that any mathematical study of reservoir and well performance will avail us nothing.

2. *The analytical method of study.*—In contrast to the empirical method there is the analytical method, the method of the present treatise. By analytical I mean mathematical. Mathematical analysis permits the diagnosis of events and conditions within the reservoir with accuracy. The diagnosis is of value; indeed its value will become evident as we proceed with the investigation we are to make. While we shall encounter features already known to us, we shall find others of which certainty replaces doubt, and still others of which certainty replaces ignorance, for in these we are to recognize features heretofore supposedly beyond our reach and even some beyond our suspicion. By analysis we determine the general principles in accordance with which a reservoir and its orifice perform. Here we call upon empirical data to act as our guide, for without experience we would undoubtedly wander from the appropriate paths. Analysis will not permit us to determine the specific values of quantities associated with production from a given natural reservoir or from its well in advance of the proper observations at the well. It will guide us in the manner of making these observations, and it will reveal to us the accuracy of the resulting data. Furthermore, it will guide us in the subsequent manipulation of these data in their application to the specific case at hand. Thus empirical data are consigned to their proper place by the methods

[1] Here we ignore access to the outcrop of the productive formation, if there is one.

of analysis. Their rôle is admittedly an important one, although a subordinate one in the present method. That we are justified in assuming this attitude toward empirical data is a matter which does not require a specific argument, for the results will speak for themselves.

In pursuing the analytical method it is obvious that we must resort to the use of mathematics. The operations involve the simplest, the most elementary principles in the domains of algebra, co-ordinate or analytical geometry, and differential and integral calculus. The present treatment of our subject does not call for principles of a complex nature. Undoubtedly there will come a time when complex principles are necessary, but then the treatment will have passed beyond the scope of the present investigation.

In virtue of laboratory experiments originally performed by well-known physicists we have our so-called laws of physics. These are capable of expression in simple forms of equations. While by means of mathematical manipulations these equations may, and frequently do, become somewhat complex in form, it seems improper to say that their meaning becomes complex because of this fact.

It is by means of these mathematical methods that we construct a comprehensive system of fluid mechanics, sufficiently flexible to suit actual conditions within the various classes of natural reservoirs we are to meet. The system is convenient because of its simplicity, and the ease with which we pass from a consideration of the ideal to a consideration of the actual renders it a particularly advantageous one. It complies with the requirements of all systems of mechanics; namely, it is permissible, consistent, and categorical. If we will accept our fundamental principles of physics and mathematics as correct ones, we may thereafter classify our mathematical deductions and their physical interpretations as correct or incorrect ones. The final test for the latter lies within the domain of field observations and their empirical data.

The analytical method permits us to deal with the unknown factors of production in an efficient manner. These are no longer deterrent factors. We shall continue to regard them as "unknown," but in this we shall construe the term in a purely mathematical sense—unknown, but definite and determinable. Ordinarily we shall eliminate them from our equations by the simple expedient of dividing them by themselves. Let us say that we have such a term in a particular investigation, and that we know this term to possess a definite numerical value, although we do not know this value. We can call it $K$ until its value may be determined. Then if we bring about the operation whereby $K$ is divided by $K$, it is obvious that this division results in unity, since any quantity, determined or undetermined, when divided by itself, is equal to one. Consequently the term itself disappears from the equation; the remaining quantities appear in the form of ratios. The process will be fully illustrated in the text ahead; it is for us a most important mathematical expediency, for it is primarily that which permits us to analyze performance in no uncertain terms.

[ § 2

It is well to note further with respect to these unknown quantities $K$ that we can at any time actually determine their numerical values by substitutions into the proper equations. We are in no way restricted to the "carrying of ratios" in our analysis, and therefore we are not restricted in our procedure from the theory of performance to the application of its principles in the field. This mathematical process will also be illustrated within the text.

The analytical method is based upon idealized conditions in Nature. These conditions are clearly defined in a manner to conform as closely as possible to actual physical conditions within the natural reservoir. Idealized experiments in the laboratory and idealized data at large permit us to determine the desired general principles of production. These principles are themselves independent of ideal and actual conditions; from the ideal we learn them, and to the actual we apply them. In this respect I would say that the situation here does not at all differ from the situations in theoretical mechanics and hydromechanics, wherein their general principles are determined by means of idealized conditions, and thereafter applied to actual conditions. We may regard the present work as one closely paralleling the investigations of the older sciences.

Following the general principles we have the specific ones. These are determined by removing, one at a time, the stipulated ideal conditions from our problem. In this we clearly approach actual conditions in the field, and consequently we are better able to appreciate the performance of individual wells. Again we are here following closely the procedure of investigations in theoretical mechanics and hydromechanics.

3. *Properties of matter and laws of physics.*—The classified properties of solids, liquids, gases, and vapors are our closest approximations in accordance with observed conditions in Nature. We are particularly interested in solids in so far as they are associated with natural reservoirs in the form of porous and permeable strata which lie between impermeable ones. As to fluids we are interested in their molecular state, their expansion and compression, the phenomena accompanying heat changes, the phenomena of adhesion and cohesion, their viscosity, surface tension, capillary attraction and resistance, the behavior of mixtures composed of any two or all of the three classes of them, and so on. In this treatise it is sufficient that we thoroughly understand the nature of these properties. Their numerical values appear only in the form of comparative data; this is to say, the numerical values of the properties of oil, gas, and water are immaterial, except in the form of proportional values for like properties of the three fluids.

Science makes a distinction between the physical properties of liquids, gases, and vapors. It makes no distinction between them as to the fundamental principles of mechanics, for here all are to be treated alike. We are to study reservoirs which produce liquid alone, gas alone, and liquid and gas in combination, in accordance with the same principles.

The laws of physics so necessary in our investigation of production are based upon the experiments of such men as Leonardo da Vinci, Galileo, Guericke, Torricelli, Pascal, Boyle, Huygens, Newton, the Bernoullis, Avogadro, Dalton, Gay-Lussac, Henry, Joule, Thomson, Jamin, and so on. By no means is this list complete. These investigators were active in the years between 1460 and 1860. The earlier ones began their experimentation where Archimedes had left off some 1,600 years before. The most important of laws for us are those of Boyle and Henry, and those which pertain to theoretical mechanics and hydromechanics in their subdivisions of kinematics and dynamics, the last of these being again subdivided into statics and kinetics. Our task is to apply these laws, and the others ranking next in importance, to the performance of natural reservoirs.

Certain investigations concerning production can be based upon the laws of thermodynamics. We shall indeed encounter phenomena of production to which these laws are applicable, but it will be sufficient for us at the present time to consider merely the place and manner of application, and to make note of the differences in the methods of studying energy in thermodynamics and mechanics. Thereafter we shall confine our attention solely to mechanics in recognition of the fact that of the two sciences it is the first to be thoroughly mastered.

4. *Practical application of theory.*—Detailed discussion of the practical application of general principles is beyond the scope of the present treatise. The study of theory and the study of practice constitute two tasks, even though these are admitted to be mutually closely related. I believe that the understanding of theory will prepare us for application. Most of the points of advantage and disadvantage become so evident that we need not have further directions to guide us in our operations. As we proceed with the development of principles we are not to ignore the practical significance of features which have a favorable or an unfavorable bearing upon the recovery of oil and gas from productive formations. Neither are we to ignore the fact that the interpretation of these features as favorable or unfavorable requires us to consider two points of view, namely, that of the individual among a group of operators in the same field, and that of the group as a whole. Where these points of view differ, they do so because of the presence or absence of competition. Upon them depends our national welfare in oil and gas. Unwise over-production and unwise measures for conservation follow readily in the path of ignorance respecting the general principles which govern these features, inasmuch as their proper interpretation from any viewpoint becomes impossible.

We are to consider the present investigation as one in science, and not as one in technology. Each, it seems to me, has its place in our oil and gas field operations, and neither one can replace the other. They can only supplement and reinforce each other. I take theory and practice as the substance of

science and technology, respectively. I prefer to consider technology to be primarily dependent upon science, while science is primarily dependent upon laboratory experiments and mathematical operations. From what has been said before it is evident that technology and application are concerned with the empirical data of field observations.

The treatise calls for the measurement of quantities that are not universally observed in practice. It is possible that these measurements may be made only at the risk of interfering with the performance of the well. Where we would avoid this risk we might substitute indirect methods of measurements for direct ones. One such indirect measurement is illustrated in the case of determining the final closed-in pressure at the well without the complete cessation of flow.[2] Again, the treatise infers the measurement of quantities that can be observed only with the greatest difficulty, if at all. We are to remember that the actual measurement of these quantities will not be required for the purposes of practical application. Science frequently utilizes quantities in the abstract, and the advantage to be gained by them lies in understanding them rather than in applying their concrete values. Where we know how Nature performs in regions beyond our sight and touch, we are better able to influence her actions to our benefit.

What is legally practicable or impracticable, and what is economically possible or impossible, can be of no concern to Nature; consequently, if our compliance with natural features proves to be hindered by legal statutes, regulations, and decisions, by property rights, contracts, or other agreements, or by the vagaries of supply and demand, no allowance for these conditions shall be made in this text. We are to imagine a hand free from all these restraints, and hold to our purpose of eventually being able to understand Nature in a way that will permit us to obtain the greatest recovery of oil and gas from known pools in the most effective, and therefore the most profitable, manner.

**5.** *The classification of reservoirs.*—An analysis of the performance of natural reservoirs can be measurably simplified by a generalization of the problem. This is accomplished principally by including the study of artificial reservoirs. But it will not suffice to confine our classification of all possible reservoirs into the natural and the artificial, for both of these possess other characteristics that are capable of classification in ways that facilitate our study.

Now there are, we may suppose, an indefinite number of schemes in accordance with each of which we may classify all reservoirs. Those which are of interest to us are few in number, since the classifications which they permit are sufficiently varied in their scope to accommodate our requirements. Following is a list of these classifications, including the one which has already been mentioned:

[2] See § 119.

*a*) Those which are natural, and those which are artificial;

*b*) Those which are hollow, and those which are filled with a porous medium;

*c*) Those which are either apparently or actually below their orifices, and those which are above them;

*d*) Those which are open to the atmosphere at some place other than possibly at the orifice, and those which are closed at all places other than possibly at the orifice;[3]

*e*) Those which produce liquid alone, those which produce gas alone, and those which produce liquid and gas in combination; and

*f*) Those which have distinct mathematical systems of curves of production without regard to the preceding classifications.

It is easy to see that these classifications are of importance in a complete analysis of production. They indicate likenesses and unlikenesses between reservoirs, and they therefore can guide us in our laboratory experiments and field observations. They permit us to carry on our investigations largely by the method of analogy.

The last classification forms the basis of our text. All reservoirs, whether natural or artificial, are divided into three classes, or, as we shall say, into three "controls." These are the following:

I. *Reservoirs in Hydraulic Control,* where pressure and rate of production remain constant throughout finite time, typified by the simple gas holder with a floating top, and the solution tank with the pressure head on the orifice maintained at a constant value by a sufficient inflow of liquid.

II. *Reservoirs in Volumetric Control,* where pressure and rate of production decline and approach zero in finite time, typified by the rigidly constructed gas tank, and the solution tank, both with the pressure head lowering in accord with production from the orifice.

III. *Reservoirs in Capillary Control,* where pressure and rate of production also decline and approach zero in finite time, typified by the Jamin capillary tube, a tube of capillary bore, filled with alternating globules of liquid and bubbles of gas.[4] The pressure head here likewise lowers in accord with production from the orifice.

If natural reservoirs in these three controls possess a porous medium, such as a consolidated sandstone, and if they contain both liquid and gas, we may expect alternating globules of liquid and bubbles of gas to exist throughout this medium. Now in the first two controls we are to find that the hydrostatic pressure within the reservoirs is able to overpower these globules and bubbles, and consequently the pressure and rate of production curves bear the same

[3] It is necessary to say "other than possibly at the orifice" because the orifice may open either directly to the atmosphere or to a flow-line. The orifice of this flow-line need not be taken as the orifice of the reservoir.

[4] These tubes are fully described in § 149.

relation to each other as they would in the absence of such globules and bubbles. In the last control, however, the hydrostatic pressure within the reservoirs is unable to overpower these globules and bubbles. *These globules and bubbles dictate new relations for the pressure and rate of production curves.*[5]

No reservoir, either natural or artificial, can be in more than one control at the same time. As between Hydraulic Control and Volumetric Control the pressure head during production either remains constant or declines. Obviously it cannot do both at one time. There can be no compromise between these situations. And as between Capillary Control on the one hand and Hydraulic and Volumetric controls on the other, where globules and bubbles are present in all, the action of these globules and bubbles either overpowers the hydrostatic pressure within the reservoir, or it does not do so. Again there can be no compromise between these situations.

However, it is possible, and it frequently happens, that reservoirs are converted from one control to another at some instant in the course of their lives as producers. This is a different matter, for the controls clearly continue to possess their individualities in spite of the conversion, and the reservoirs behave strictly in accord with one control at a time.

We shall study the three controls in order that we may understand the behavior of reservoirs in accord with them. It will be convenient to attack the problem first with reference to artificial reservoirs. These are to be divided into two sub-classes: namely, those wherein flow may be said to be "tubular," and those wherein flow may be said to be "radial." The differences between these systems are due solely to the design of the container and the position of the orifice. In analysis the first is the simpler. When we have studied the behavior of reservoirs belonging to this subclass, we shall be prepared to take up those which belong to the second, and these are obviously more closely analogous to our natural reservoirs in the field.

**6.** *Primary functions of performance.*—In the early history of production many expected that oil and gas wells should continue indefinitely at the same rate of production in the way of the average water well. There were in fact such wells which produced in this manner for years, and we still find them in certain regions today. But more and more wells do we find where there is an appreciable decline in the rate of production from month to month, or from year to year. While it has been a simple matter to distinguish between those wells which do not decline and those which do, it has not been easy for us to make note of two paths for this decline where it exists. That there are two such paths we now know from the brief description given to Volumetric and Capillary controls above.

[5] The three sets of pressure and rate of production curves are shown in Fig. 34, § 58. (Pressure is denoted by $P$, and rate of production by $Ve$.)

What can these different paths mean to us? This may be answered with these two statements:

First. Our forecasting of the future performance of wells, whether by numerical or graphical methods, will be more accurate because of our knowledge of the characteristics of the appropriate paths. In this we shall know how to profit by the use of a few accurate data in the place of a mass of haphazard data.

Second. The recognition of either path will permit us to tabulate at once a series of physical events and conditions which take place or exist within the reservoir supplying the particular well. These have a most important bearing upon recovery. They are extremely different in Volumetric and Capillary controls. With a few exceptions they are the same in Hydraulic and Volumetric controls.

Pressure and rate of production are, we shall say, two functions of performance. As such we shall find that we may treat them in the way of mathematical functions; that is, as mathematical quantities which bear a definite relation to each other, a relation which can be expressed in the form of a mathematical equation. Pressure and rate of production are but two of several functions of performance. Cumulative production and time are two others. Following is a complete list of the functions that we shall use in our analysis with a few remarks concerning each one:

*a*) *Pressure.* This is to be measured by closing the well. No other pressure than one obtained in this manner can be used in forecasting performance or in identifying physical events within the reservoir.

*b*) *Velocity.* This is simply another name for rate of production. It suggests the fact that it represents the rate at which the volume of oil and gas within the reservoir is leaving it by production.

*c*) *Volume.* This is either the total amount produced to date or the total amount to be produced from date. As the former it is another name for cumulative production.

*d*) *Acceleration.* This is the change in the rate of production.

*e*) *Energy.* In virtue of this we obtain our oil and gas. There is an interesting relation between this function and the amount of gas accompanying oil within the reservoir.

*f*) *Power.* This is the rate at which energy is leaving the reservoir. It is also the rate at which gas is produced where both oil and gas issue from the well.

*g*) *Time.* This is to be reckoned either as time elapsed or as time remaining. Together these constitute the total life of the reservoir as a producer.

These are our "primary functions" of performance. We shall determine their mechanical relations by interpreting their mathematical relations,[6] for, as

[6] These mathematical relations appear in the form of equations. See Appendix A and Appendix B.

stated above, they are to be treated both as reservoir functions and as mathematical functions.

Observed values of any one of the first six functions in the list can be plotted as ordinates (vertically) with the observed values of the seventh, time, as abscissas (horizontally), and the curve representing the variation of the function can then be drawn. Of such curves as these we are familiar in practice with the curves between pressure and time, velocity and time, and volume and time. Ordinarily we would plot the functions in their measured units, but if we should instead plot them in percentage values, horizontally as well as vertically, we would have what we are to call the "relative curves." For Volumetric Control these curves are shown in Figure 92, section 109, and for Capillary Control they are shown in Figure 158, section 159. They are always the same, regardless of the size of the reservoir, the size and condition of the orifice, and the time at which values are taken as 100 per cent. For Hydraulic Control such curves are impossible, inasmuch as pressure, velocity, and power are the only functions capable of being expressed in percentage values.[7]

Observed values of any one of the first six functions can be plotted as ordinates with the observed values of any other of the same six as abscissas.[8] Of such as these we are familiar with the curves between pressure and volume, and velocity and pressure. The latter curve is worthy of mention here, for if we construct this curve on a logarithmic plat[9] in place of the ordinary Cartesian plat, the "curve" is a straight line with a slope of one to two for Volumetric Control, and another straight line with a slope of three to two for Capillary Control.[10] The latter slope is quite the usual one in the gas fields of the Mid-Continent and Eastern regions of this country. We may take this situation as an illustration of many which are to be interpreted in the present treatise.

Thus far in discussing the behavior of the primary functions ideal performance of the reservoirs and wells has been inferred. We shall first study ideal performance and thereafter make allowance for actual performance as we find it in the field. This is to be done by considering what we are to call "theoretic performance," where definitely known changes in the conditions of production shall be made and the corresponding changes in the behavior of the reservoir functions shall be noted. In this manner we will appreciate the significance of performance as we know it in the field, and no doubt we will be able to take advantage of feasible changes that prove to be beneficial.

[7] See Fig. 45, § 67, where a complete set of primary function curves for Hydraulic Control is illustrated.

[8] Curves for all possible combinations can be so constructed for Volumetric and Capillary controls, but some of the curves for Hydraulic Control reduce to single points, because there is no variation between certain functions; for example, between velocity and pressure.

[9] For a description of this plat see §§ 120, 121, etc.

[10] See Fig. 136, § 145.

7. *Secondary functions of performance.*—To generalize a statement made above we may say that the recognition of either one of three sets of paths that are traversed by the primary functions of performance in the course of production will permit us to tabulate at once a series of physical events and conditions which take place or exist within the natural reservoir supplying the particular wells. As the paths of the primary functions are defined by the controls, so are physical events and conditions within the reservoir. We are to call these events and conditions the "secondary functions" of performance. They include fluids produced, source of energy, conservation desired, drainage radius and drainage area, water encroachment upon the pool, storage and reserves, recovery by natural flow, possible conversion of control, restoration of gas pressure, and the forced drive, as the most important of these functions.[11] In the study of these we of course encounter other features, such as pressure gradients within the reservoir at the time the well is closed and at the time it is open and flowing, the lineal or radial motion of fluid within the reservoir, and the effects of viscosity, surface tension, and capillarity within the reservoir. The proportional production of gas to oil is a problem to be interpreted in terms of both primary and secondary functions.

We are to investigate all these secondary functions in this treatise; we shall learn why they are as they are, and how they are to be interpreted in terms of performance and recovery. Again, in regard to these functions, we are to establish general principles on the basis of ideal performance, note the effects of changes in accordance with theoretic performance, and thereby come to understand actual performance as we know it in the field.

Here, then, we have seen a brief survey of the nature and scope of the treatise. We shall study those features which may guide us in application, but we shall not dwell at length upon the method or significance of application, inasmuch as these details become self-evident on understanding. Finally, it seems to me that the proper conception of the principles established herein will lead us eventually to the proper conception of our resources in oil and gas. By these principles we shall be able to ascertain quantities of oil and gas in place as effectively as we do with such solid commodities as coal or copper. Our road is a long one; here it has thirty-three chapters with one hundred and ninety-two sections, and this is not the end.

[11] See frontispiece, and also Appendix C. So long as we remain unaware of three controls, confusion of ideas concerning these secondary functions will prevail, and observed differences in the various fields of the world will remain unexplained. The tabulated statements in the appendix here referred to can be expected to receive amplification in the text, and any obscurity now noted in them is due to their brevity.

CHAPTER II

# Fundamental Data

"Nature with its myriad phenomena assumes a unified aspect only in the rarest cases; in the majority of instances it exhibits a thoroughly composite character . . . . ; it is accordingly one of the duties of science to conceive phenomena as made up of sets of partial phenomena, and at first to study these partial phenomena in their purity. Not until we know to what extent each circumstance shares in the phenomenon as an entirety do we acquire a command over the whole . . . ."—P. VOLKMANN

8. *Laws of fluid delivery.*—The production of fluids—liquids, gases, and vapors, either separately or jointly—takes place from all reservoirs because of a disturbance of equilibrium. We provide the reservoir with an orifice, and leave it open. When there is a pressure exerted by this fluid at one side of the orifice greater than the pressure exerted by any fluid at the other side, the equilibrium which previously existed is interfered with; the reservoir immediately attempts to equalize the pressure on the two sides by passing a volume of fluid in the proper direction. It does this for the purpose of establishing a new equilibrium, and thus it is that with natural reservoir formations containing oil, gas, or water, under pressure, we obtain this fluid with comparatively little effort on our part.

But if the open orifice does not upset an equilibrium, we obtain no fluid. The pressures on the two sides are equal before the orifice is provided. And if, as sometimes happens, the pressure is greater on the exterior side of the orifice, our reservoir undergoes a process of filling. We might say that the reservoir produces negatively.

Again, if it should be possible that the pressure on the interior of the orifice is maintained at the same value by a sufficient inflow of fluid from another source, the new condition of equilibrium will never be reached. Fluid will be produced until the end of time, assuming a perpetual source for inflow, with the reservoir fruitlessly attempting to establish a state which is impossible. We can say that this reservoir is, mathematically, one of infinite size; it will never become emptied of its contents.

When the pressure is not maintained on the interior of the orifice, but is allowed to lower in accord with flow from the reservoir, a state of equilibrium will certainly be approached, and will actually be reached in any case where there is no replenishment of fluid by inflow. While approaching equilibrium the pressure must gradually pass through a succession of values, each smaller

than the one preceding. The rate of production also passes through a succession of declining values. Such reservoirs, so long as they are approaching equilibrium, appear to be, mathematically, of finite size, for they are in the process of becoming empty within finite time.

Now, remembering our classification of all reservoirs into three controls, we may say that Hydraulic Control is infinite, whereas Volumetric and Capillary controls are finite. In all controls the six functions of performance, as listed in section 6, are passing through a succession of values, while time, the seventh one, lapses in the course of production. In virtue of the fact that they are of perfect order in ideal performance, these successions of values, be they of constant or variable nature, define the laws of fluid delivery from reservoirs. These laws may be expressed mathematically by statements in words, by equations, or by curves drawn on paper that is provided with co-ordinates.

In summary, we may say that *the laws of fluid delivery are the laws of the re-establishment, or the attempted re-establishment, of equilibrium within the reservoir.*

9. *Primary functions of performance.*—The seven reservoir functions are, I have said, mathematically dependent upon one another. A particular phase of each of them is to be defined, and together these arbitrarily selected representatives of the functions shall constitute the primary functions of performance. As previously stated, the rate of production will be designated as velocity, when we are dealing with the analysis of performance. There can appear to be no objection to this, provided we have the privilege of retaining the older, well-established term for expressed interpretations.

As stated in section 6, observed values of any one of the first six functions there listed can be plotted as ordinates with observed values of time as abscissas. The six possible curves constitute the *fundamental primary function curves.* They show graphically the relations between (*a*) Pressure and Time, (*b*) Volume and Time, (*c*) Velocity and Time, (*d*) Acceleration and Time, (*e*) Energy and Time, and (*f*) Power and Time.

Again, as stated in the same article, observed values of any one of the first six functions can be plotted as ordinates with observed values of any other of the same six as abscissas. All possible curves so obtained constitute the *derived primary function curves,* because their equations may be obtained easily from those of the fundamental curves by the simple expedient of algebraically eliminating the function time between them. There are fifteen of these curves showing the relations between (*g*) Pressure and Volume, (*h*) Velocity and Pressure, (*i*) Acceleration and Pressure, (*j*) Pressure and Energy, (*k*) Pressure and Power, (*l*) Velocity and Volume, (*m*) Acceleration and Volume, (*n*) Volume and Energy, (*o*) Power and Volume, (*p*) Acceleration and Velocity, (*q*) Velocity and Energy, (*r*) Velocity and Power, (*s*) Acceleration and Energy, (*t*) Acceleration and Power, and (*u*) Power and Energy.

With any pair of functions it is immaterial which is taken as the abscissa and which as the ordinate. Incidental considerations, convenience of uniformity, or custom, may induce us to adopt a particular arrangement of the two.

All these derived relations are of importance in the mathematical analysis of fluid delivery, although admittedly some are of greater importance than others.

Heretofore our engineering work has been practically confined to the consideration of the following relations:

*a*) Pressure and Time, in gas field practice;

*b*) Volume and Time, as cumulative production in both oil and gas field practice;

*c*) Velocity and Time, as rate of production in both oil and gas field practice;

*g*) Pressure and Volume, in gas field practice; and

*h*) Velocity and Pressure, in gas field practice.

While stress is laid upon these particular relations, on account of their economic as well as their analytic importance, irrespective of the nature of the fluid, others among the complete list must be given special attention. Those which are not specifically dealt with in detail are left for subsequent investigators to consider, if ever their practice demands them.

The particular phase of the functions to be selected and defined will be taken up within the discussions upon the separate functions now following.

**10.** *Pressure.*—Instead of the general term "force," as treated in theoretical mechanics, we are concerned with pressure as its special representative in fluid mechanics. By pressure we actually mean intensity of pressure, for a fluid exerts its force upon the walls of a container, or upon particles suspended within it, at so much per unit of area; for example, pounds per square inch, grams or kilograms per square centimeter, atmospheres of approximately 14.7 pounds per square inch, inches or centimeters of mercury, feet of water, and so on. It is clear that force for us is purely an intensive factor, whereas it is generally an extensive one in the study of the behavior of solids.[1]

The present system of fluid mechanics is in no way dependent upon the particular unit of pressure intensity chosen, for our calculations are to be based upon ratios. Ratios are, obviously, abstract numbers; they are nondimensional, and therefore they are the same regardless of the unit employed. Nevertheless, for convenience, and for uniformity, we shall continually refer to pounds-per-square-inch intensity of pressure, or, more briefly, pounds-per-square-inch pressure. In using the latter expression the completed one, which alone is exact, must ever be borne in mind.

[1] For example, a force, or weight, of 100 pounds is extensive in its nature, whereas a pressure of 100 pounds per square inch is intensive. We cannot know the value of the force in the latter case until the number of square inches is specified.

Pressure is, we might say, the master function of performance. It is the most easily, and therefore the most accurately, observed evidence of energy possessed by the fluid within the reservoir. But is this statement not somewhat incomplete and ambiguous? Let us see. A closed orifice confines the *reservoir system* to that space ordinarily recognized as the fluid reservoir. By such closing an interior adjustment establishes an equilibrium within, and all other space in the universe is then external to the reservoir. The opening of the orifice places two portions of space into communication; the system is enlarged, because some part of space previously external to the system now becomes part of a new internal system. But suppose the new portion of space possesses a pressure equal to that on the inside of the closed orifice. Now clearly the fluid within the reservoir possesses no energy which may be utilized in the production of fluid, although it cannot be said that it does not possess energy. Evidently it is not pressure, but a difference between two pressures, which is of interest to us—the difference between the pressure obtaining at the orifice when it is closed, with the fluid within at rest, and the pressure external to the orifice under the same conditions. *Thus we should prefer to say that the difference between these two pressures is the master function of performance. It is the most easily, and therefore the most accurately, observed evidence of energy which is possessed by the fluid within the reservoir and which is capable of being expended by the reservoir in the process of production.*

It is true, of course, that the pressure in the new portion of the reservoir system may be lowered by us, and, whereas the reservoir was previously incapable of producing fluid, it may now do so. In this case we need only repeat the statement as applied to the newly established conditions.

**11.** *The potential phase of pressure.*—Measurements must ordinarily be made from some point taken as a zero or base; otherwise a number, used to express such a measurement, has little significance to us. In many cases custom or science has determined for us certain convenient points to use in this way. For example, pressure is often expressed in pounds per square inch above the pressure of the atmosphere, although it can also be expressed in pounds per square inch above absolute zero. The number in the one case differs from that in the other; yet both are equally correct in expressing identically the same amount of pressure. We might say that one of the numbers expresses pressure in one phase, while the other expresses it in another phase. As an example, we may state that the pressure is, say, 20 pounds in *atmospheric phase,* or 34.7 pounds in *absolute phase*. The equivalence of the two is, of course, dependent upon the correctness of 14.7 pounds absolute for the pressure of the atmosphere. If this is not correct, the proper amount may be used instead.

The atmospheric and absolute phases have both been of great service to us, the one in the physics laboratory, and the other in engineering practice,

where pressures are ordinarily read by gauges whose indicators point to zero when the mechanism is open to the atmosphere. Inasmuch as the delivery of fluid from a reservoir is dependent upon a difference of pressures, it is clear that the zero for measuring this difference is determined by the amount of pressure exerted at the exterior of the orifice. For example, let us suppose that the pressure exerted on the interior of a closed orifice is 100 pounds, while that exerted on the exterior of the same orifice closed is 20 pounds; now the latter determines a zero such that we measure an amount of 80 pounds. The 80 pounds, we will say, is measured in *potential phase*. Three zeros are here evident: namely, *atmospheric zero, absolute zero,* and one which may be called a *potential zero*. Thus we have in this example 114.7 — 34.7 in absolute phase, 100 — 20 in atmospheric phase, or simply 80 in potential phase.

Only in case production takes place from the orifice directly into the atmosphere are the atmospheric and potential phases identical; and only in case production takes place into a perfect vacuum are the absolute and potential phases identical.

Let us suppose that two reservoirs are identical in all respects except in the fact that the internal and external pressures are, respectively, 125 pounds and 45 pounds in one system, while they are 100 pounds and 20 pounds in the other. The two potential pressures are evidently equal at 80 pounds. The two reservoirs are more alike than might be imagined at first thought, for both will produce the same amount of fluid, and both will produce at the same rate at corresponding instants during their lives.

It is important to note the fact that we need not hold in mind continually the position of the potential zero, either on the scale of atmospheric readings or on the scale of absolute readings. In the example above, 80 is our important number; it is immaterial where it begins and where it ends on any scale of measurement. Furthermore, if it should be altered to 100 or to 60 by an act of ours, it is immaterial whether this alteration occurs at the bottom or at the top of the scale of measurement.

The potential phase of pressure, or, briefly, the potential pressure, is the first of our primary functions of performance. To determine its value in any natural reservoir we may proceed in a manner not differing from that in the case of artificial reservoirs: Close the orifice, and note the highest point reached by the pressure gauge. Either measure or compute the pressure bearing upon the exterior of the closed orifice. The difference is the value of the potential pressure which we seek. It is often advisable to express the two pressures in absolute phase, before obtaining the difference. This method will obviate confusion in the case of a reservoir of the closed type, or in the case where the exterior pressure is less than that of the atmosphere.

12. *Volume.*—Volume is quantity of fluid. It may be expressed in many different units: for example, cubic feet, cubic meters, gallons, liters, barrels, metric tons, and so on. Volume with us shall always be a *mass-volume,* in

that it shall be independent of density, whether the fluid is a liquid or a gas. In regard to liquids, which are practically incompressible, no comment seems necessary concerning its mass-volume; but with gas, which we recognize to be very sensitive to the pressure bearing upon it, we need say only that it shall be expressed in cubic feet, measured at, or calculated as of, atmospheric pressure. Thus it does not matter what its actual density is under various conditions of the given problem. For both fluids measurements are always assumed to be made at one temperature which, if desired, may be adopted as a standard. It is not necessary to specify a particular temperature in our analysis of reservoir performance.

Corresponding to the three phases of pressure there are three phases of volume. By *absolute volume* we are to mean the entire contents of a reservoir. The portion of this volume that can be produced against the pressure of the atmosphere is the *atmospheric volume,* and the portion that can be produced in virtue of the potential pressure is the *potential volume.* For example, a rigidly constructed gas tank measures, let us say, exactly one cubic foot inside, and it holds gas under a pressure of 14.7 pounds per square inch, as shown by an attached gauge. The absolute volume of gas is two cubic feet of free gas. If the tank is permitted to produce into the atmosphere, the potential pressure is equal to the gauge pressure, and the corresponding potential volume of gas is one cubic foot of free gas. In this case the potential and atmospheric phases of pressure are identical; consequently the same two phases of volume are likewise identical. When the same tank is caused to produce against a line pressure, say, one of 7.35 pounds above the pressure of the atmosphere, the potential volume is one-half a cubic foot of free gas, while the atmospheric volume remains unchanged.[2]

The rigidly constructed gas tank is a reservoir of the closed type, while the solution tank is one of the open type, according to the classification of section 5, paragraph (*d*). We scarcely need concern ourselves with the absolute volume of the latter, for it includes not only all liquid within the tank, but also the air above the tank, since this air is a part of the fluid system involved in production. It will be practical, however, to consider the *absolute liquid volume* of such a tank. This will obviously mean the entire liquid contents of the reservoir, and of this either the whole or a part is potential volume, according to the conditions of production.

Given the size of a solution tank, the height of the liquid within it, and the position of the orifice, it is an easy task to compute the volume of liquid to be produced from it. This amount of liquid is none other than that which is here termed the potential volume. We must compute the volume of gas to be delivered from a rigidly constructed tank in a similar manner, given the size of the tank, the pressure bearing upon the orifice when closed, and the

[2] The atmospheric phase of volume, as apart from potential volume, is ordinarily of little interest to us.

pressure against which the production is to take place. In both types of tanks it is the potential volume in which we are interested, and this potential volume corresponds to the potential pressure in each case. When we calculate the volume of gas to be produced from a reservoir, why should we use the value of the absolute pressure to determine a volume to issue from the reservoir against a back pressure that is equal to or greater than the pressure of the atmosphere? The answer is that we should not do so. It is the potential pressure which specifies a definite potential volume.

The potential volume is that volume of fluid which is registered by the production records of a reservoir.[3] The curve between potential volume and time, for a reservoir in finite control, is the inverted cumulative production curve, for the former pertains to the amount of fluid within the reservoir yet to be produced at a specified instant, whereas the latter pertains to that which has been withdrawn from the reservoir from the beginning to the same instant. For a reservoir in infinite control the two curves are identical, because of the manner in which we are to treat its record of production.

13. *Velocity.*—Velocity is the rate of displacement. This definition is general; therefore it must be made more specific in each particular case. In theoretical mechanics the problems concerning velocity usually refer to the lineal motion of particles or objects along a straight or curved path. But velocity may also refer to change of length or distance, change of area, and change of volume. It is clear that the notion of a rate of displacement in position is somewhat different from that of a rate of displacement in quantity, whether this quantity be measured in one, two, or three dimensions.

The quantity of interest to us in reservoir performance is, as we have noted, volume of fluid. Obviously the volume within a reservoir during the process of production is being displaced; it is passing from the inside to the outside of the container. Velocity for us, then, shall be the rate of displacement of the volume of fluid within the reservoir at any instant we may specify. It may be expressed in such units as cubic feet per day, cubic meters per day, liters or gallons per minute, barrels per day. In fact, any unit of mass-volume per any unit of time signifies a velocity. Our velocity is thus a *mass-velocity,* as distinguished from lineal velocity in particular.

If the volume of fluid within a reservoir at any instant may be defined as a function of time, or, in other words, if the relation between volume and time may be expressed by an algebraic equation—and this we shall do in our study of each of the three controls—then the first derivative of the expression, with respect to time, is an equation showing the relation between the velocity and time. I believe this relation between volume and velocity, like

[3] By our definition of potential volume these records are necessarily complete. Adsorbed fluid, or fluid isolated within the formation either by textural barriers or structural features, plays no part in production.

others which are to be mentioned shortly, will be better appreciated when we consider the separate controls. The principle will be recognized as one in differential calculus, one which has familiar applications in theoretical mechanics.

When we think of a well producing at the rate of a million cubic feet of gas per day, or of one producing at the rate of a hundred barrels of oil per day, it seems that we do not usually have in mind the mathematical relation between the rate and the volume of fluid in the reservoir at the instant. For the purpose of forcing us to think continually of velocity in its relation to volume, I have proposed to deal with the term *velocity* instead of using the established expression, *rate of production,* in all purely analytical work.

Velocity and volume are not clearly distinguished in our literature on the performance of wells, or on the metering or flow of liquids and gases. This fact appears to have led to confusion in many cases, and certainly to errors in some. Suppose an engineer should say, "We do not plot the rate of production from our wells, but actual volume as shown by our sales records. For instance, our records show the sale of 5,250,000 cubic feet of gas in January, 5,000,000 in February, 4,775,000 in March, etc. These actual quantities we plot." Does he plot volume of gas? No; the wells are simply producing at the rate of 5,250,000 cubic feet per month at one time, 5,000,000 cubic feet per month at another time, and so on.

Velocity, or a rate of production, is not actual fluid. It is a number which, when multiplied by time, gives a result actually signifying volume. For example, "a thousand barrels of oil per day" is not oil. If this number is multiplied by $\frac{1}{24}$ day, by 1 day, or by 30 days, the result obtained certainly is oil; but not otherwise. Any curve, the ordinates of which, when fully expressed in words, indicate a volume of fluid per unit of time, is actually a velocity curve. Such a curve must be integrated in order to show the corresponding volume curve. For the example above, we must plot 5,250,000 cubic feet for January, 10,250,000 for February, 15,025,000 for March, and so on. Clearly this curve is actually cumulative production. By adding the quantities in this manner the potential volume curve becomes inverted, as stated in the preceding section.

*Potential velocity* is the rate of displacement of potential volume. It is the rate of flow under the actual conditions demanded in the field. The open-flow rate of a gas well is a good example of a displacement which is not in accord with the definition of potential velocity, for the normal production of a gas well takes place against a line pressure. Only in the case where the production continually takes place into the atmosphere is the open-flow rate a potential velocity. Now we may see that the engineer who plotted the gas sales mentioned above might know that he was doing something different from common practice. He plotted potential velocity, whereas it has been the custom to plot open-flow curves for gas wells.

14. *Acceleration.*—Acceleration is the rate of change of velocity. Our most common experience with this function is in connection with falling bodies and projectiles at the surface of the earth.[4] This is obviously a lineal acceleration, related as it is to lineal velocity. In this treatise we shall use a *mass-acceleration,* corresponding to volume and velocity as already defined.

If an oil well produced, say, exactly 500 barrels of oil per day at some time, and exactly 440 barrels per day thirty days later, then the rate of production declined either at the exact rate or at the average rate of two barrels for each day of production. In other words, the acceleration was two barrels per day *per day* of operation. The units employed in expressing acceleration are basically those of velocity, as cubic feet per day per day, cubic meters per day per day, liters or gallons per minute per minute, barrels per day per day, and so on. In some cases there may be reason for stating the acceleration in two units of time; for example, barrels per day per month. Ordinarily the unit of time is given in duplicate.

If the relation between velocity and time may be expressed by an algebraic equation, then the first derivative of the expression, with respect to time, is an equation showing the relation between acceleration and time. Where there is no change in velocity, the value of acceleration is zero; but where velocity declines, either in the process of producing or in that of filling, the value of acceleration is positive and real.[5] In this case the acceleration may be constant, or it may itself decline in value.

*Potential acceleration* is the rate of change in potential velocity. When this acceleration is either zero or of a constant value, a change in the back pressure maintained against production has no effect upon its value; consequently the potential acceleration is equal to the acceleration of open flow in these reservoirs.[6] But when acceleration declines, such a change in the back pressure does affect the value of acceleration. Thus the potential acceleration and the acceleration of open flow are not equal at a given instant in the life of these reservoirs.[7]

[4] Acceleration of bodies near the earth's surface is always positive in value, and directed downward, toward the center of the earth. As we know, the velocity of a rising body decreases, and that of a falling body increases, because of this uniformly directed acceleration. Plus and minus signs may well be used to designate directions of motion, or velocity, while acceleration retains only the plus sign.

[5] By "real" I mean "sensible"; not to be regarded as negligible, even if its value, as expressed in the customary units, is exceedingly small. For the present there is apparently no reason for using the term "deceleration" for a decreasing velocity or rate of production. The situation is perfectly analogous to that of a rising body under the influence of the earth's attraction. When we study production under the conditions of "constant power" in the finite controls in §§ 118 and 164, we shall meet a true negative acceleration, or what we might properly term a "deceleration."

[6] This is the situation in Hydraulic and Volumetric controls.

[7] This, on the other hand, is the situation in Capillary Control.

**15.** *Energy.*—We have already seen, in connection with pressure, that under the proper circumstances the fluid in a reservoir possesses energy, or capacity to perform work against pressure that opposes production at the orifice. This pressure, known as back pressure, is of two kinds. These are to be discussed in section 21.

Mechanical energy possessed by a fluid within a reservoir may be due to either one of two sources. The first is in virtue of a hydrostatic pressure head, the weight of a column of liquid bearing upon the orifice; and the second is in virtue of a gas pressure head, the gas being in a state of compression. These distinct sources call for a classification of all reservoirs into the open type and closed type, respectively. Among natural reservoirs of oil, gas, and water, both types exist. With wells which produce on account of a hydrostatic head, any gas in the reservoir possesses no energy intrinsically its own. That energy which it appears to have is due solely to the pressure exerted upon it by the column of liquid. Again, with wells which produce on account of a gas pressure head, the liquid in the reservoir possesses no energy intrinsically its own. That energy which it appears to have is due solely to the pressure exerted upon it by the gas. In this way we are able to make a distinction between different natural reservoirs.[8] *No natural reservoir owes its production to a simultaneous action of the two pressure heads.* A given natural reservoir is either of the open type or of the closed type; it cannot be of both types simultaneously. It follows then that the energy of such a given natural reservoir is due either to a hydrostatic pressure head or to a gas pressure head, and it cannot be due to both simultaneously.

Energy is the product of pressure and volume. The units employed in the two functions determine the unit for energy. The energy within a reservoir may be expressed in foot-pounds, in gram-centimeters, and so on.[9] Rarely, if at all, do we need to refer to energy in its specific units, inasmuch as our interest in this function is purely analytical. By our method of carrying ratios the actual units vanish from the expressions, and until we set up the ratios for a particular problem, we can use the general term, "units of energy."

[8] We shall find that natural reservoirs in Hydraulic and Volumetric controls have energy due to a hydrostatic pressure head, whereas those in Capillary Control have energy due to a gas pressure head.

[9] To express energy in its customary units it is essential that the units for pressure and volume be consistent. For gas we have

$$\frac{\text{Pounds}}{\text{Square Feet}} \times \text{Cubic Feet} = \text{Foot-pounds}$$

Here the pressure is expressed in terms of square feet instead of square inches. For oil we have to express barrels in terms of cubic feet. By carrying ratios these conversions become unnecessary.

It is of interest to compare the present conception of energy in terms of its components with the conception of energy in theoretical mechanics. There energy is defined as the product of force and distance.

The *absolute energy* possessed by fluid in a reservoir may be arbitrarily limited to, and therefore defined as, the product of the absolute pressure bearing on the closed orifice and the absolute volume of fluid within the reservoir.

*Potential energy,* the fifth of our primary functions of performance, is defined as the product of the potential pressure and the potential volume.[10]

The absolute energy of a reservoir can be considered as *available energy* in the sense that it can be withdrawn from the reservoir by causing production to take place into a perfect vacuum. Potential energy can be considered as *effective energy* under any conditions of production; that is, for production into a perfect vacuum, against the pressure of the atmosphere, against a line pressure (gas), or against the weight of a column of liquid (oil or water). In any case, the difference between the absolute energy and potential energy is *non-effective energy*.[11] In general we are to consider only these *mechanical energies* in processes of production.

16. *Power.*—Power is the rate of displacement of energy. It is the rate of the performance of work. Power is, in fact, a velocity of a special nature, in that it deals with the removal of energy from within the reservoir, just as velocity deals with the removal of volume from within the same reservoir. There is an important difference between the two, however, in so far as volume, after the removal, is at our command as a commodity, while energy at the same time has become degraded, or dissipated. It is not destroyed; it is simply reduced to a condition of unavailability. While the work performed by the reservoir results in the delivery of fluid, what actually becomes of the energy in this performance? Following is a list of items concerning this energy in its dissipated forms:

*a*) The fluid may acquire a position at a higher elevation. If it does so, its newly acquired *energy of position* cannot be utilized in aiding the performance of the reservoir.[12]

*b*) The fluid may be allowed to retain a pressure greater than that exist-

[10] This definition of potential energy is to be taken as a purely qualitative one. In calculating actual amounts of potential energy in given reservoirs it will be necessary to include constants in the equations between pressure and volume on the one hand, and energy on the other. Each control has its unique constant, as follows:

| | |
|---|---|
| Hydraulic Control .................................... | 1 |
| Volumetric Control .................................... | ½ |
| Capillary Control .................................... | ⅔ |

These are to be fully discussed in the proper places. As to the qualitative definition we shall have frequent occasion to refer to it without a consideration of quantitative features.

[11] The difference is clearly of zero value when production takes place into a perfect vacuum.

[12] Energy of position is of particular consequence in the case of liquids.

ing at the point of consumption.[13] This retained *pressure-energy* cannot be utilized in aiding the performance of the reservoir.

*c*) The fluid acquires a velocity of motion. This *energy of motion* cannot be utilized in aiding the performance of the reservoir.

*d*) The fluid acquires heat on account of friction along the line of flow. If this does not take place interior to and exterior to the orifice, it in any case takes place at the orifice. This *heat-energy* cannot be utilized in aiding the performance of the reservoir, in so far as it has left the reservoir with the fluid.

It is not necessary that the first two dissipations of energy accompany all processes of production. In production from natural reservoirs they should be reduced to a minimum, compatible with conditions in economics and technology. The last two dissipations are unavoidable. They necessarily accompany the process of production.

If the relation between energy and time may be expressed by an algebraic equation—and it may, provided pressure and volume can be so expressed—then the first derivative of the expression, with respect to time, is an equation showing the relation between power and time. Or, power is the product of pressure and velocity.

*Potential power* is the rate of displacement of potential energy. It is equal to the product of potential pressure and potential velocity.

**17.** *Time and life.*—The reservoir's task of establishing, or attempting to establish, equilibrium requires time. Time, as a function of performance, is of the same mathematical importance as the six preceding functions.

So far as we know, time is infinite—it had no beginning and it will have no ending. But, as a function of performance, *time has a beginning which dates from the instant of opening the orifice, or from any other subsequent instant in the life of the reservoir which we may choose as a starting-point of reckoning performance.* Whether it has an ending or not depends upon the control of the reservoir.

Acknowledging the fact that engineering practice deals with minutes and hours in problems concerning fluid motion in artificial reservoirs, and with days, weeks, months, and years in problems concerning the performance of natural reservoirs, we will choose to confine ourselves to hours and to days for the respective problems. Our analysis is not dependent upon a particular dimension of time; the two mentioned are referred to only for convenience and uniformity. Others would serve equally well.

Time may be measured in two equally logical directions: that is, either as time elapsed or as time remaining. The scale of measure for the first extends either from the past to the present, from the present to the future, or from

[13] The situation is illustrated by a gas well delivering gas into a line wherein a pressure greater than that of the atmosphere exists. The point of consumption may be assumed to be located at the end of this line, where the gas is liberated at atmospheric pressure.

the past to the future; while that for the second extends either from the future to the present, from the present to the past, or from the future to the past.

Time elapsed, heretofore so universally used in oil and gas field practice, is primarily only of statistical or economic importance, whereas time remaining is of analytical importance, because of the fact that it is the true function of reservoir performance.[14]

*Potential time* is expressed in units of time remaining. It specifies the duration of a period of time under consideration, and this period may be either the whole or a part of the time required for the reservoir to reach equilibrium.

For a given reservoir the total time elapsed, plus the total time remaining, equals the total life of the reservoir. At any given instant, time remaining indicates life, as of the instant.

The time required by the reservoir to deliver its potential volume constitutes its *potential life,* measured, now obviously, in units of time remaining. It is of infinite duration in Hydraulic Control, but of finite duration in Volumetric and Capillary controls.

Time enters our analytical problems in two ways: (*a*) as a function of performance, in accordance with the preceding discussion, and (*b*) by the definition of the functions of velocity, acceleration, and power.

**18.** *The functions in retrospect.*—Each of the seven functions appears to be necessary in a proper analysis of reservoir performance. Taken together they seem to be sufficient for our present purposes, although future studies may include the consideration of others. More than the seven are possible; for example, inasmuch as power is a velocity for energy, so might we add a function with a new name, signifying the rate of change of power, or differently expressed, the acceleration of energy.

While the expression "rate of production" is retained for interpretations, as stated, it will be observed that I am omitting such expressions as, "ultimate production," "ultimate recovery," and "discharge." Probably the first two are synonymous, and intended to signify "absolute volume," as defined above. The third appears to have two current meanings, equivalent to volume and to velocity, as these are here defined. Some authors speak of "rate of discharge," and follow their discussion with an equation which obviously represents volume, and not velocity.

Having considered a "potential" function in connection with each of the seven in their generality, we are now prepared to define the "potential phase" in the following terms: *The potential phase is that phase which possesses the capacity of fulfilling the laws of fluid delivery in accord with the laws*

[14] There is one exception to this statement to be noted in § 137, where we consider the delivery of liquid from the cylindrical tank with its axis horizontal.

*of fluid mechanics.* If, and when, any function in this phase reaches a zero value, all others—except acceleration in Volumetric Control—reach a zero value at the same instant. As a group the potential functions constitute the primary functions of performance.

The term "potential" is synonymous with "effective" and "analytical." Indeed the former of the two has been used in the past, notably in the expression, "effective pressure," to convey the same idea as potential pressure. The present term defines a definite phase in no ambiguous manner; it calls our attention forcibly to this phase, and it can cause no confusion with previously established uses for the same word. Our expression "potential energy" is in perfect agreement with the definition of the physicist's identical expression.

In analytical equations the potential functions will appear in the symbols, *P, Vo, Ve, Ac, E, Po,* and *T* or *L.* The corresponding general functions will ordinarily appear in other convenient and obvious symbols.

CHAPTER III

# Fundamental Data *(Continued)*

"As is known, scientific physics dates its existence from the discovery of the differential calculus. Only when it was learned how to follow continuously the course of natural events, attempts to construct by means of abstract conceptions the connection between phenomena met with success. To do this two things are necessary: First, simple fundamental concepts with which to construct; second, some method by which to deduce, from the simple fundamental laws of the construction which relate to instants of time and points in space, laws for finite intervals and distances, which alone are accessible to observation."—GEORG RIEMANN.

**19.** *The potential reservoir.*—In the ordinary sense a reservoir is either an especially constructed hollow container, or an extensive porous and permeable geological formation, capable of holding and producing fluid. For us, any space whatever, regardless of size, so capable of holding and producing fluid shall constitute a reservoir. This space may be:

*a*) Coextensive with the physical container, as we see it or picture it by inference; for example, in the rigidly constructed gas tank.

*b*) In excess of this container, as for examples, in the gas holder and solution tanks, in which the atmospheric column overlying, and coextensive with the lateral dimensions of the container, is included.

*c*) Any fractional part of this container, be it a large, small, or exceedingly minute part. We shall meet with this conception in connection with reservoirs of Capillary Control, where we are to consider the individual pore space as an integral reservoir.

The notion conveyed by the term reservoir assumes either the actual existence of an open orifice, or the possibility of one being provided. Furthermore, a reservoir shall not be considered "empty," unless a perfect vacuum exists within it. When no flow takes place, we shall say that the reservoir is in equilibrium under the existing conditions of production; that is, under the existing back pressure at the orifice, and so forth.

In accordance with the three items above, the space occupied by the potential volume of fluid, at any given particular instant during the process of production, shall constitute the *potential reservoir,* as of the given instant. Compliance with this definition in no way depends upon the various methods of classifying reservoirs in general (section 5), including that of being hollow or filled with a porous medium. The spatial dimensions of the reservoir

extend to the limiting position of the potential volume in all cases. In addition, the size of the potential reservoir is not dependent upon the nature of the fluid with respect to its economic value. This is to say, the potential reservoir may include both desirable and undesirable fluids; for example, an oil pool does not define the limits of the potential reservoir—nor for that matter, those of the reservoir in the more generalized sense.

In brief, *the potential reservoir is precisely that analytical unit which either produces or is capable of producing in accord with the laws of fluid delivery.* Its size is determined by conditions interior and exterior to the orifice.

**20.** *The orifice.*—In section 10, I stated that the opening of the orifice places two portions of space into communication; the system is enlarged, because some part of space previously external to the system now becomes part of a new internal system. I chose then to consider the two portions of space, internal and external to the orifice, together as constituting the internal system, inasmuch as both are certainly involved in the process of production from the reservoir. We shall have no further need of referring to this internal system, so inclusive of space, and rather than adopt new terminology, *let us hereafter divide the potential reservoir into the internal and external systems at the orifice.* Thus we divide the reservoir into that part which controls the functions of performance and that where back pressure is applied. The orifice thus has great importance in our system of fluid mechanics.

Any opening through which flow can take place constitutes an orifice. It may consist of either a single hole or a number of holes relatively close together. An oil, gas, or water well taken in its entirety constitutes an orifice according to the definition, but it becomes necessary for us to specify some portion of the well as the specific orifice; otherwise the separation of the reservoir into the internal and external systems is not sufficiently precise for the purposes of our analysis. I suggest that we arbitrarily designate the specific orifice to be at the bottom of the well. Here the fluid leaves the productive porous medium, and enters a hollow chamber that generally contains a portion of the casing which has been perforated. And here the territory over which we certainly have some influence is separated from that over which Nature alone is in command.

**21.** *The back pressures.*—There are, in fact, two distinct kinds of back pressure exerted against the orifice during production. Let us see how these differ.

It is a well-known fact that usually during an investigation in physics or mechanics, or during some portion of it, some quantities which enter into the equations or calculations preserve their values unchanged. These quantities are called *constants.* Their name does not infer that they can never change their values, but rather that so long as conditions imposed upon the experi-

ment are unaltered, the value of each constant is unchanged. Consequently, it is clear how the "life" of a constant at a given value may extend over the entire experiment, or over only some portion of it. One of the back pressures in reservoir performance is perfectly analogous to the mathematical constant. A gas well produces against atmospheric pressure and a line pressure; an oil well produces against atmospheric pressure and the weight of a column of liquid in the hole. Obviously the line pressure and the column of liquid may be partially or entirely removed, and further, by a vacuum pump a small or large part of the atmospheric pressure itself may be taken off the orifice. Because of its mathematical behavior we shall call this type of back pressure the *constant back pressure.*

Again, during an investigation in physics or mechanics some quantities which enter into the equations or calculations alter their values continuously and uniformly. These quantities are called *variables.* Now the second back pressure which we have to consider behaves exactly in this manner. It is the back pressure exerted against production by virtue of friction at the orifice and in any flow-line exterior to it during flow from the reservoir. This we shall call *frictional back pressure,* or better, the *external friction head,* to distinguish it from an *internal friction head* within the reservoir.

The value of the external friction head is greatest when the rate of production is greatest, and is zero when equilibrium is established. Furthermore, its value may be changed at any time during production, either by altering the size of the orifice or by making any alterations in the size, length, and incidental features of the flow-line itself. But these alterations affect only certain constants which we shall always find associated with the variable back pressure.

A study of the performance of all reservoirs is greatly facilitated by making this definite distinction between the two back pressures.

22. *The ideal natural reservoir.*—We are to adopt arbitrarily any convenient specifications for an *ideal reservoir* in each of the three controls. In selecting these specifications it will be better to allow mechanical features to predominate over the geological, and to comply with the latter only in so far as convenience may not suffer thereby. Later applications to particular reservoirs in the field may call for necessary slight changes to suit local geological structure.

Certainly some of the specifications shall apply to all reservoirs, regardless of control. These pertain to internal conditions, as follows:

*a*) The formation must be porous, permeable, and of homogeneous texture in the vertical and all horizontal directions.

*b*) It must be of uniform thickness, and extend over an area for the present undefined.

*c*) It must lie either as an inclined mathematical plane, or as a cylindrical surface (of any convenient and reasonable vertical section) with an axis perfectly horizontal.

*d*) It must contain one perfect gas, one perfect liquid, or one of each of these.

*e*) The formation must lie between two impermeable strata,[1] and it shall be optional whether all formations are perfect conductors or perfect non-conductors of heat. The former option shall be held in preference.[2]

For the *ideal performance* of a well in this reservoir we need add that:

*f*) The constant back pressure and the external friction head are acknowledged to be real during the process of production. These undergo no alterations as a result of any act originating with Nature or with us.

*g*) The maximum pressure attained at the orifice when closed behaves only in a manner which complies with the laws for the control; it is either constant or it declines during the life of the reservoir, depending upon the control. No erratic act originating with Nature or with us shall cause its alteration.

*h*) The internal friction head is likewise acknowledged to be real during the process of production. It shall not be altered by any act originating with us.[3]

Subsequent to the study of ideal performance we shall deliberately make alterations in the conditions of production, in order to investigate what we may designate as the *theoretic performance* of a well in the above-mentioned reservoir. Three cases in this performance are defined as follows:

CASE 1. The external and internal friction heads will be altered by the manipulation of the orifice; that is, by partially closing or opening more fully any contrivance which regulates the rate of flow.

CASE 2. The maximum pressure attained at the orifice by its closing will be assumed altered erratically either by an act of Nature's or by an act of ours.

CASE 3. The constant back pressure will be altered by an act of ours. We shall consider the individual alteration and the rapid succession of erratic alterations.

These alterations are to be studied both in connection with their effects upon the particular well, and in connection with any possible effects upon an adjoining well. Furthermore, the cases are to be studied singly and in combination.

[1] These strata are impermeable to the particular fluid under the particular maximum pressure existing within the reservoir.

[2] Where we assume all formations to be perfect conductors of heat, we are to stipulate further a transfer of heat into the reservoir formation during the process of producing, and out to the adjoining formations in a process of filling, the latter being exemplified by storage of gas, restoration of gas pressure, and forced drive by air or gas. In either of the two processes it is essential that we regard the mass of the adjoining formations sufficiently great in comparison with that of the productive formation so that the temperature of the former is not appreciably affected by the transfer of heat.

[3] We can alter the internal friction head only by altering the rate of production at the orifice.

23. *Decline and the potential reservoir.*—An infinite life signifies non-decline, and non-decline signifies an infinite life. On the other hand, a finite life signifies decline, and decline signifies a finite life. In the two events the notions are inseparable.

Decline is a perfectly normal phenomenon in reservoirs of Volumetric and Capillary controls. It is continuously mathematical during life only in ideal performance, and interruptedly so, for periods of time which are short of life, under alterations in accord with the three cases in theoretic performance. These alterations by themselves cause the functions of performance to increase or decrease their values. While these new values are independent of natural decline, they are mathematically related to its laws.[4]

Respecting the size of the potential reservoir during natural decline we may choose between the following two notions:

*a*) That each instant determines a new potential reservoir, each smaller than its predecessor; or

*b*) That there is but one potential reservoir, and this is variable in size, the variation being toward zero.

It is preferable, I believe, to hold the latter view, inasmuch as the variable status in size is perfectly natural.

But in regard to the same reservoir in theoretic performance, let us consider that each act in accord with one or more of the three cases sets up a new potential reservoir, one of which may or may not have the same size as its predecessor, depending upon the individual, or combined, alterations.

*The laws for natural decline are independent of the nature of the fluid contained within the reservoir.* For example, the laws are independent of the fact that a pool of oil and gas may be completely surrounded by water. Any well within the pool having water within its drainage radius may or may not have its functions decline to zero before such water reaches the well.

24. *Physical state of the reservoir interior and secondary functions of performance.*—The interior of a natural reservoir is filled with a porous and permeable material. This material may be quite different in separate fields, or in separate formations in the same field. Frequently it is, to our advantage, a more or less coarse, lightly cemented sandstone which can hold and permit to pass large quantities of fluid. At times it is found to be too "tight" to serve well in its capacity of producing fluid. Again, the productive formation is not always a sandstone. It may sometimes be a cavernous limestone, the texture of which very probably does not possess the high degree of homogeneity displayed by most sandstone strata, but it is a formation which can serve none the less as an excellent reservoir. Fractured shales and cherty formations may likewise serve as productive formations.

[4] This will become evident when we study theoretic performance in Volumetric and Capillary controls, specifically as this performance is illustrated by Figs. 121 and 174.

Regardless of the nature of the formation we may speak of its being a porous and permeable medium—the act of production being sufficient evidence of this—for the limiting dimensions, and the shape of pores never have been, and likely never will be specifically defined.

The type reservoirs for the three controls are hollow, although one of them, the capillary tube, approaches the texture of a porous material. The tanks illustrate the performance of natural reservoirs in spite of their different interior state. We can say that reservoirs, either artificial or natural, filled with a porous medium, containing and producing liquid alone or gas alone, do not have their laws of delivery disturbed in any way by virtue of the fact that the medium is present.[5] But if the reservoir contains liquid and gas, then the presence of the porous medium may or may not affect the laws of delivery. We can say that in some such reservoirs, in consideration of the texture of the particular medium present, and in consideration of the value of surface tension possessed by the particular liquid in contact with bubbles of free gas, the pressure exerted by the fluids upon the orifice is sufficient to overpower capillary action within the reservoir, while in other such reservoirs it is not sufficient to do so.[6] It would not be proper to conclude from this statement that the pressure in the former class of reservoirs is necessarily greater than the pressure in the latter class, for the reason that the texture and the surface tension determine the particular intensity of pressure necessary to overpower the capillary action in each porous reservoir.

When I speak of the laws of fluid delivery as being or not being affected by the presence of the porous medium, I refer specifically to the fact that the analytical equations expressing the laws of decline for the various functions are or are not altered, respectively. Of course, the presence of the medium does affect the duration of time in the process of production. But corresponding to any porous reservoir, wherein pressure is sufficiently intense to overpower capillarity, we may construct, or imagine constructed, a hollow reservoir of the proper size, and having the proper orifice, which is capable of producing at the same rate continuously as the first one, and therefore one which will produce the same volume and have the same life as the first one. This is possible only because the laws as expressed analytically are not affected by the presence of the porous material. A reservoir in which the action of capillarity is not overcome by the pressure cannot thus be duplicated by a hollow one.

All the events and conditions within the natural reservoir, the so-called secondary functions of performance, as stated in section 7, depend upon the

[5] In making this statement we rely upon experiments with artificial reservoirs in the laboratory. We find the same laws of delivery from tanks before and after they are filled with sand.

[6] Thus we may have either Hydraulic or Volumetric Control on the one hand, and Capillary Control on the other.

physical state of the reservoir interior. To analyze this state we must consider not only the properties of porous solids, but also the properties of fluids, for all these properties play their rôle simultaneously in the process of production.

The forces which are concerned with the properties of matter are those which act between neighboring parts of the same substance, and which are called *forces of cohesion,* and those which act between portions of substances of different kinds, which are called *forces of adhesion.* These forces are quite insensible between two portions of matter separated by any distance we can measure directly. It is only when the distance becomes exceedingly small (approximately of the order of a thousandth of a millimeter), that these forces become perceptible. At distances more nearly approaching contact (say of the order of a twenty-thousandth of a millimeter) the forces are known to be of tremendously great intensity. We meet with the effects of these forces in absorption, adsorption, viscosity, surface tension, and capillarity, those properties of greatest importance in the production of oil and gas from porous and permeable formations of sedimentary origin. We shall therefore review briefly these properties.[7]

25. *Porosity and permeability.*—A geological formation must be porous in order that it may act as a container of fluid. To be a conveyor of fluid it must be permeable. The two properties are quite distinct, although permeability of a medium exists by virtue of its porous nature. Porosity is usually expressed in the percentage ratio between the space occupied by voids and that occupied by the solid matter and voids taken together, whereas if permeability were to be expressed numerically, a definite number would specify the quantity of a given fluid which can pass through a given length of the medium in a given time, and at a given pressure.[8]

There is no fixed mathematical relation between the two properties. It is not difficult to imagine two sandstones with cemented grains having the same porosity, one of which possesses restricted communication between adjoining pores on account of the manner of cementation, while the other has comparatively large openings which give freer communication. The manner of cementation is an important factor in the movement of fluid toward the well, because of its direct bearing upon the permeability of the medium.[9]

Presumably all sedimentary formations are porous when they are submitted to pressures of sufficient intensity. We shall find it convenient to classify them arbitrarily, however, as porous and non-porous, or as permeable

[7] It is of course impossible to enter here into a thorough discussion of these properties which shed so much light upon the behavior of oil and gas reservoirs. Reference to the experiments of investigators may be found in the textbooks listed in Appendix D.

[8] Here it is assumed that the "given fluid" includes the consideration of temperature.

[9] I would not infer the necessity of actually measuring the permeability of formations in our fields.

and impermeable, according to the range of pressures and the nature of the fluids usually encountered at our wells.

By the texture of a porous formation we mean not only the size of the pores, but their shape as well. The homogenous texture called for in the description of the ideal reservoir was intended to include both of these features in a state of perfect uniformity throughout the medium. In the actual reservoir, however, we are to recognize the fact that homogeneity of texture in the vertical direction may seldom exist, because of the conditions under which the deposition of the material takes place. Although the texture may thus vary in different strata of the same geological formation, there is evidence to show that homogeneity in horizontal directions is most frequently closely approached, at least for reasonable distances from the wells. This feature will become evident in connection with the *law of equal expectation.*[10]

26. *Absorption and adsorption.*—Porous solid matter, including porous sedimentary formations, possesses the ability to absorb fluids with which it is brought into contact. A sandstone will absorb oil, water, and gas. We frequently associate the idea of *absorption* of liquids with the action of a sponge. The formations, however, clearly differ from a sponge in so far as they are practically incompressible; they are, we say, competent to withstand applied external forces without appreciable deformation. With respect to the absorption of gas by porous masses, perhaps the most remarkable case is that of ammonia and charcoal.

A porous mass which has absorbed gas alone, or liquid with gas in solution by virtue of pressure, will tend to give up some of the fluid upon a reduction of the pressure. The expansion of gas in the free state causes flow to the exterior of the mass. When liquid alone has been absorbed, the reduction of the pressure will have no effect, if such reduction is uniform throughout the space entirely surrounding the mass. In any case, however, whether gas is present or not, fluid can be expelled from the porous mass by the application of unbalanced forces from at least one direction, the forces being exerted by some sort of a fluid either like or unlike that which is in the absorbed state.

We may be quite certain of the fact that the phenomena of absorption are closely associated with those of adsorption, surface tension, and capillary action.[11] In fact we can not err greatly in saying that the phenomena of absorption are the undifferentiated effects of those of the three. And as we shall find *preferential effects,* where a given solid prefers, or has a greater attraction for, one fluid than another, we may expect these preferential effects within

[10] See §§ 112 and 161.

[11] Viscosity is omitted here intentionally. We are presumably dealing with true fluids rather than with plastic substances verging between a solid and a liquid. In our case, then, viscosity only affects the rate of absorption, and not the fact of absorption itself.

absorption itself. It is a fact easily verified, for example, that a sponge will surrender gasoline or kerosene for water.

By *adsorption* we mean the phenomena which are associated with the concentration or condensation of fluids upon the surface of solid matter. It seems to be due solely to the adhesion between the molecules of the fluid and those of the solid at their contact. A film of the fluid, either liquid or gas, becomes attached to the surface of the solid, and the fluid constituting this film is known to display physical and chemical properties which are quite at variance with those of the same fluid in the free state.

Many solids adsorb certain liquids to a greater degree than they do gases. The liquid is then said to "wet" the surface of the solid. It is clear that there is no wetting where the opposite condition holds.

A given solid absorbs certain liquids to a greater degree than it does others. If given the opportunity, it will give up one liquid for another; for example, a surface which prefers water to oil will surrender the one for the other. Thus we may speak of the preferential effects in adsorption in the case of given solids. The same situation holds with respect to a given surface and various gases.

Adsorption appears to be a more fundamental property of matter than absorption. For convenience we can distinguish them by viewing one as a mass phenomenon and the other as purely a surface phenomenon. For example, with respect to the case of charcoal, a good grade of this substance *absorbs* ammonia gas to an amount equal to two or three hundred times its volume. The surface exposed in charcoal is very great, owing to its porosity, and the molecules of carbon at this surface *adsorb* the gas.[12]

Although this distinction between the two phenomena seems simple enough in theory, it is not always so simple in practical applications. In recognition of this fact some investigators have adopted the general term *sorption* to specify one or both of them in experiments, where the differentiation is not essential.

27. *Liquids, gases, and vapors.*—Fluids are classified according to their general physical properties into three divisions: namely, liquids, gases, and vapors. As distinguished from solid matter, fluids cannot retain their shape without support. They are therefore subject to flow when not prevented from doing so by the walls of their container.

Liquids are compressible to a very slight degree. Water and oil are more compressible than iron; nevertheless this amount of compression is sufficiently slight so that its neglect in engineering computations causes no appreciable error. Liquids generally increase their volume with a rise in temperature, a

[12] In this instance we can form some idea as to how dense the adsorbed gas of the film must be, and consequently how nearly its molecular state must approach that of either a liquid or a solid.

fact which may also be ignored in practice, provided its container is not one of the closed type. Furthermore, liquids possess the property of presenting a free surface to gases which are in the free state, and to other liquids with which they will not mix. In the particular case of a porous medium containing liquid and bubbles of free gas which have come into existence by virtue of a release of pressure, there are a multitude of minute free surfaces throughout its mass. With these we must contend in all natural reservoirs which contain the two classes of fluids in intimate association.

A perfect liquid may be defined as one possessing the following properties:

*a*) Absolute incompressibility;

*b*) Permanency of state throughout the range of pressure and temperature involved; that is, it does not change to a solid or gaseous form within the mentioned range;

*c*) A constant coefficient of expansion throughout any range of temperature involved;

*d*) A constant coefficient of viscosity throughout the range of pressure and temperature; and

*e*) A constant value of surface tension, when in contact with a given gas, throughout the range of pressure and temperature.

Because we shall prefer to consider the formations adjoining the reservoir medium to be perfect conductors of heat when the reservoir is ideal, we may agree that the liquid possesses a real viscosity. In this our definition of a perfect liquid differs from the one adopted in the science of hydromechanics as developed by the mathematical physicist. Whereas he is interested in the *molecular mechanics* of fluids, we are concerned with their *molar mechanics*. This fact permits us to define an ideal state which more nearly approaches the actual state.

Water and petroleum are liquids in the same physical sense. Mathematical analysis makes no distinction between them, so long as they remain in the liquid state. When both exist in a given reservoir, they are performing in a co-operative manner according to the laws of delivery.

In our problems illustrating the methods of actual calculation or the principles stated in the text, the density of water will be regarded as 62.43 pounds per cubic foot. Consequently a pressure head of one foot of water is equivalent to 0.4335 pounds per square inch, and one atmosphere of 14.70 pounds per square inch is equivalent to 33.90 feet of water, or to 29.90 inches of mercury. A crude oil of any assumed specific gravity may be readily expressed in corresponding equivalents.

Gases differ from liquids in several important respects, namely:

i. A gas confined within a reservoir fills all space not occupied by solids and liquids.

ii. Gases possess elasticity of volume to a high degree, for they may be easily compressed by the application of pressure, regaining their original volume upon a return to the original pressure.

iii. All gases appear to be miscible in any proportion.

iv. All gases appear to be soluble in all liquids, though to various degrees. Specifically, hydrocarbon gases are soluble in petroleum and in water, but to a greater degree in the former than in the latter.

Gases present a free surface to liquids, and in some instances they may be caused to present a free surface to each other. For this to happen one must have the greater density.

Work must be performed in pushing a free gas into solution, provided there is no chemical attraction between molecules of liquid and gas. And assuming the absence of chemical action between the fluids, work will be performed by the gas in emerging from solution. If further expansion is permitted after the emergence, the gas will perform work in this expansion, whether it is in one continuous mass, or finely divided in the form of bubbles throughout a mass of liquid in a porous medium.

The physicist defines a perfect gas to be one which conforms to Boyle's Law and to Joule's Law. We shall adopt his definition without qualification. In view of the fact that these laws will be discussed in a later chapter, present remarks may be confined to the following brief statement: A perfect gas possesses perfect elasticity over the entire range of conceivable pressures; consequently its molecules do not possess mass, and it does not assume a liquid form when submitted to excessive pressures. Furthermore, when no external work is performed by it in the process of expansion, it suffers no change in temperature.

While the perfect gas does not exist, some gases actually appear to behave more or less perfectly within certain ranges of pressure. It is well known that the various constituents of the gas issuing from our wells differ widely in their degree of perfection, and so we arbitrarily designate those to be permanent which do not liquefy within the range of pressure encountered at the well. Others are "wet gases," or vapors.

A vapor is distinguished from a gas by the fact that it can change from a gaseous to a liquid state at ordinary temperatures and pressures. All vapors appear to behave as gases when at sufficiently high temperatures, and probably all gases behave as vapors when at sufficiently low temperatures. There is no sharp line to be drawn between these fluids, and therefore we may in general speak of two fluids only, liquids and gases, regarding vapors as liquids when they are in liquid form, and as gases when in gaseous form.

In our problems we may either neglect the weight of fluid in the gaseous form, or treat it as having a weight equivalent to that of air, without introducing an appreciable error. In the past, when precision has so required, the specific gravity of a gas has often been expressed in terms of the density of air as unity, and the ratio of the density of water to that of air has been taken at amounts varying between 773 and 820, at normal atmospheric pressures and temperatures.

28. *Viscosity.*—The property of a fluid which pertains to its retardation in motion is called viscosity. The range of viscosity among liquids is striking; and this is true even among various crude petroleums as they issue from the well. Gases possess viscosity only to a slight degree; for example, the ratio between the viscosities of air and water is approximately one to eighty. The viscosity of liquids generally decreases with a rise in temperature, while the opposite is true with gases.

The number expressing viscosity may be obtained either by measuring the quantity of fluid passing through a given medium—for example, a capillary tube—in a given time, and under a given pressure, or by measuring the tangential force on a unit area of either of two horizontal planes of indefinite extent at a unit distance apart, one of which is fixed, while the other moves with unit velocity, the space between them being filled with the viscous fluid.[13] Measurements by the first method may be reduced to a ratio between a given fluid and one adopted as a standard for comparison. By the second method a like ratio may be determined, or a coefficient expressing a quantity in grams per centimeter per second may be obtained.

An important point to note in connection with viscosity is the fact that resistance to the motion of the fluid is implied. *When the fluid is at rest this resistance is not brought into play.*[14]

In general we may say that among fluids which wet the surface of solid matter with which they come into contact, the more viscous ones provide a greater thickness of film attached to the surface. This film appears as if it were in an adsorbed condition; its substance is not a fluid proper on account of its inability to flow. *In fact we shall not consider the substance of the film a part of the potential volume of the reservoir, inasmuch as there can be no record of it in any of the potential functions of performance measured at the orifice.* It is as though it were a part of the material composing the walls of the container, both in artificial and natural reservoirs. If we hold this view, we may say that the potential volume to be produced by the potential reservoir is independent of this film, inasmuch as any immobile fluid cannot perform the functions of a true fluid.[15]

Viscosity is due to the mutual attraction, or cohesion, between molecules throughout the mass of the fluid, and in order that this cohesion may be over-

[13] In either case the temperature at which the determination is made is to be known and stated.

[14] We are therefore to treat the effects of viscosity somewhat in the way that the physicist treats those of friction. With "static friction" there is no conversion of mechanical energy into heat. Only with "kinetic friction" is there such a conversion. By definition the two sorts of friction describe the circumstances accompanying *no motion* or *motion*, respectively, between two bodies at their contact.

[15] In the case of a porous medium we are not to confuse the content of this film with that mobile liquid held in the pores because of the lack of the necessary unbalanced pressure for its expulsion from the medium.

come, the expenditure of mechanical energy is necessary. This energy is converted into heat, and as such some of it certainly is carried out of the reservoir at once by the fluid. The remainder is held, temporarily at least, by the solid matter constituting the reservoir container. We shall find that this heat need not cause a rise in temperature, inasmuch as sensitive heat is often being converted into latent heat by the expansion of imperfect gases,[16] and this may very easily offset, or even exceed, any sensitive heat produced by the movement of the fluid. In any case the change in the amount of sensitive heat present within the reservoir is adjusted by a transference of heat in the required direction between the productive space and the space immediately adjoining, because of the absence of perfect insulation at the walls of the container. In our ideal natural reservoir the transference of heat is immediate, while in the actual one the adjustment requires time. *The potential volume of fluid within a reservoir is independent of the conversion of mechanical energy into heat.* The primary function curves for delivery from the actual reservoir are slightly disturbed by it, but eventually equilibrium is established upon a basis of a complete adjustment in temperature inside and outside the reservoir.

**29.** *Surface tension and capillarity.*—All liquids display a surface tension at their free surfaces in contact with gas or with an immiscible liquid. It may be said that the surface of a liquid behaves very much like a stretched membrane, except in the fact that an increase or decrease in the area of the liquid surface does not alter the surface tension, whereas a corresponding change in the area of a membrane does alter the tension. If we imagine a straight line drawn upon the surface of a liquid, then the portions on opposite sides of this line are pulled apart by a force which is specifically defined as the surface tension. This tension may be expressed in units of force per unit of length, and it is obvious that the direction and point of application of the force are on the surface of the liquid. The fact that the exposed layer of molecules are attracted by molecules of the same kind from one side only, whereas those within the mass of the liquid are attracted equally from all directions, leads to many interesting phenomena respecting the physical properties of free liquid surfaces.

One of the interesting phenomena due to surface tension largely, though not exclusively, is that of capillary action. When the free surface of the liquid is confined to a very small area, the surface tension, combined with the forces of attraction or repulsion between molecules of liquid and those of solid matter defining the surface, causes what is known as *capillary attraction.* The forces of attraction or repulsion between the two kinds of molecules are apparently due to preferential adsorption on the part of the solid surface. Glass, for instance, prefers water to air; on the other hand it prefers air to mercury. Indeed it may be incorrect to speak of a repulsion in any case. Might we not say rather that there always exists a preferential attraction?

[16] This conversion of heat is known as the "Joule-Thomson effect." See § 53.

As we know, if a liquid wets the surface of a glass capillary tube, this liquid will immediately run upward contrary to the action of gravity, but if it does not wet the surface, it will run downward. In either case there is, exterior to the tube, liquid whose center of mass has been displaced vertically, even though such a displacement is ordinarily very slight. Liquids, by themselves, cannot raise or lower their own centers of mass; clearly outside forces are required for this. Apparently the only outside force coming into play here is that due to the preferential attraction, for surface tension is not to be considered outside, or exterior to the mass of liquid, even though it exists on the outer side of the liquid.

While this phenomenon of capillary attraction no doubt plays an important rôle in the accumulation of oil and water in our porous formations, and possibly bears its influence upon the production of these liquids, it is one which we may largely ignore in our investigation in view of another phenomenon associated with capillarity, namely, *capillary resistance.* This phenomenon plays an astonishing rôle in the movement of fluid within, and in the production of fluid from, all porous reservoirs which contain liquid and gas in association. The resistant action is beautifully illustrated by the Jamin capillary tube, one possessing a series of alternating globules of liquid and bubbles of gas, each globule presenting a minute free surface to the bubbles adjoining it, and each such surface being capable of distortion by the application or release of pressure from one side. The globules in a distorted condition exert a small pressure in a direction opposite to that of the actual or expected movement of the fluids. A porous and permeable formation, so provided with alternating globules and bubbles, may be considered as a huge bundle of radiating capillary tubes, one which extends from the top to the bottom of the formation and spreads laterally outward in all directions from any well that might be drilled into it. The bubbles come into existence by the conversion of gas in the dissolved state into gas in the free state by the release of pressure within the reservoir.

It is the resistant action of these alternating globules and bubbles which is overcome by the hydrostatic pressure within natural reservoirs of Hydraulic and Volumetric controls, and which, on the contrary, overcomes any hydrostatic pressure within natural reservoirs of Capillary Control.

As we might infer from the statement just made, we are to find this action, which we are to call "Jamin action," in all natural reservoirs in which there is a porous medium, and in which both liquid and gas occur. In Hydraulic and Volumetric controls we shall study its minor effects, and in Capillary Control its major effects, the latter as presented in Part IV of this treatise.

CHAPTER IV

# Laws of Physics

"Experimental laws are only approximate, and if some appear to us as exact, it is because we have artificially transformed them into what I have called a principle. We have made this transformation freely, and as the caprice which has determined us to make it is something eminently contingent, we have communicated this contingence to the law itself."—HENRI POINCARÉ

30. *Introduction.*—The general physical properties of matter which are of importance in the study of reservoir performance, for instance, porosity, permeability, absorption and adsorption phenomena, viscosity, surface tension, and capillary action, are merely given a qualitative character when treated descriptively without the use of analytical equations. The equations are furnished by the laws of physics, and it is essential that these laws are thoroughly understood in their analytical aspect. They are based upon simple geometrical principles, and at any time we may wish to solve a general or specific problem concerning well production, they will return these principles to us, varied in form if we wish, to serve as a secure and reliable basis for the computations.

The laws have been definitely formulated after careful observations by our most eminent scientists of past generations, and others, equally competent, have tested them, extended the original experimentation, shown their practical application to problems of economic life, and clearly pointed out any limitations which they might have in connection with matter as we actually encounter it.

These laws are said to be true and exact only for perfect gases and liquids. There can be no denial of this fact, for the conception of the perfect fluid was made necessary because no actual fluid was found to conform to the laws with an accuracy well within the scope of our precision instruments. According to the physicist, the conception of a perfect fluid is a great aid in physics and theoretical chemistry, because many mathematical arguments can be carried out with comparative ease when applied to them, arguments which become extremely difficult when applied to actual fluids; and the most remarkable thing about such arguments is that they lead to conclusions which can be confirmed in nearly every case by experiments on actual fluids.

Let me give a list of the laws I propose to discuss briefly in connection with the production of oil, gas, and water from natural reservoirs:[1]

*a*) Avogadro's Law: At the same temperature and pressure, all gases have the same number of molecules per unit of volume.

*b*) Boyle's (or Mariotte's) Law: At a constant temperature, the product of the volume of a given mass of gas, into the absolute pressure to which it is subjected, remains constant as the volume and pressure are varied.

*c*) Dalton's Law: The total pressure exerted by a gas mixture on the walls of a vessel is equal to the sum of the pressures which each would exert if present alone.

*d*) Gay-Lussac's (or Charles') Law: If the pressure of a given mass of gas be kept constant and the temperature altered, the volume varies as the absolute temperature; or, what amounts to the same thing,[2] if the volume be kept constant, the absolute pressure varies as the absolute temperature.

*e*) Henry's Law: If the temperature is kept constant, the amount of gas dissolved in a liquid is proportional to the absolute pressure exerted by the gas upon the liquid.[3]

*f*) The following fundamental principles in the mechanics of fluids:

$f_1$ Boyle's Law in potential phase
$f_2$ Henry's Law in potential phase
$f_3$ Pressure-volume relations for liquids
$f_4$ Energy, pressure, and volume relations in general
$f_5$ Torricelli's Theorem
$f_6$ Bernoulli's Theorem
$f_7$ Time required to empty a vessel
$f_8$ Liquids compared with gases

*g*) The general principles of thermodynamics and their influence upon the production of fluids from reservoirs.

There are many other formal laws of physics which play their rôle in

[1] The following statements of the laws are modern versions formulated upon the work of the original investigators. They are taken from the various textbooks of physics listed in Appendix D.

[2] It amounts to the same thing in virtue of Boyle's Law.

[3] This law is now accepted as a special case of the more general van't Hoff Distribution Law, which may be stated as follows: If a substance possesses the same molecular weight in the gaseous and in the dissolved states, its partial pressure in the gas, which is in equilibrium with the solution, is proportional to its concentration in the latter.

There may be one pure gas or a mixture of several of such gases, as, for example, the components of a so-called "natural gas," in contact with the liquid. In the latter case there is no effect upon the ratio of solubility, or ratio of concentration, of any one of them due to the presence of the others, presupposing the absence of any chemical interaction between molecules of different kinds.

the performance of a reservoir, but we must be content with the consideration of the five major ones listed above. The others may be safely neglected in the present treatise, although it must be admitted that a complete and exhaustive investigation would necessarily include them.

The principles of fluid mechanics, which are to be discussed in chapter v, are of course based upon the principles of fluid statics and kinetics. The elements of these subjects are so thoroughly understood today that to dwell upon them here would indeed be an imposition.[4] In treating of fluid mechanics it is my intention to refer only to those topics which are of fundamental importance in the study of production, and I shall rather aim toward supplementing current textbook literature for the purpose of approaching more closely the problem at hand. In this, as in any investigation of a special nature, we are to tread the beaten paths only in the beginning, for soon thereafter we must deviate to one side, and clear a path in a new direction.

The general principles of thermodynamics and their influence upon the production of fluids are to be discussed in chapters vi and vii. Every machine, every reservoir, and every chemical reaction seems to be dependent upon these principles to a greater or less extent. While we are not to question the application of thermodynamical principles to reservoir performance in general, we are to find, I believe, that the application of mechanical principles is of far greater importance to us today.

31. *Avogadro's Law.*—This law, as stated in paragraph (*a*) above, has come to be recognized as an important foundation of molecular physics, as well as of all chemical investigations.

Figure 1 represents two perfectly rigid tanks which are assumed to be absolutely identical in all external and internal conditions, including temperature. Let *A* be filled with gas, say nitrogen, and *B* with oxygen. The law states that when both tanks are brought to the same pressure, they contain the same number of molecules. Under this condition the density of the two gases must have the same ratio as their molecular weights. Many permanent gases adhere to this law very closely throughout ordinary temperatures and pressures.

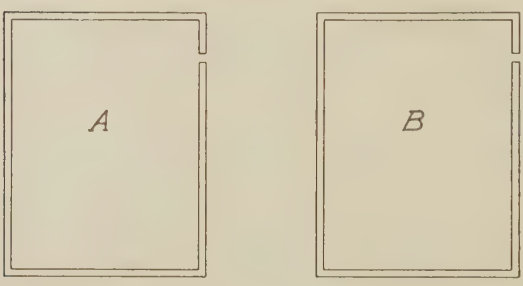


FIG. 1

[4] I refer here to fundamental propositions, such as Archimedes' Principle, regarding the buoyancy effects of bodies submerged in liquids, and as Pascal's Principle, regarding equal intensities of pressure in all directions at a point within a liquid or gas.

In place of the rigid tanks suppose we have two identical cylinders which are provided with non-leakable pistons, as shown in Figure 2. Using the same gases as before, we may arrange the cylinders with their pistons in identical positions, and provide for equal temperatures and pressures within. In accordance with the law they contain equal numbers of molecules. Now let us move the pistons so that the molecules occupy, say, one-half the space formerly occupied. If the temperatures alter in this process, we may equalize them, and again the pressures will become equal, although obviously at a higher value than before. Thus we see that all gases are equally compressible under isothermal conditions.

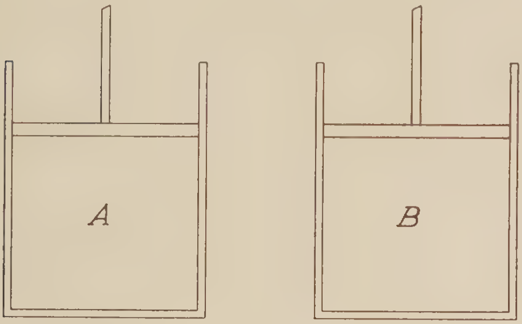


Fig. 2

These experiments are performed with two vessels in each case. Now we may consider the interior of one rigidly constructed tank, filled with a gas to a given pressure which is to be maintained at a constant value. The temperature is uniform throughout. We may imagine the contents of this tank divided into columnar sections of equal horizontal area, as shown in Figure 3. The corners *a, b, c,* etc., represent corresponding corners within the vessel, the cover having been removed, supposedly, without affecting the gas. The areas $S_1$, $S_2$, $S_3$, etc., define spaces of equal volume, and, in accord with the law, each one contains the same number of molecules. We learn from the kinetic theory of gases that the molecules are continually darting across these imaginary partitions, but we must assume that as many are entering each space as there are leaving it at every instant; otherwise unequal pressures would exist at various points within the given mass of gas—a condition which appears illogical and contrary to our experience.

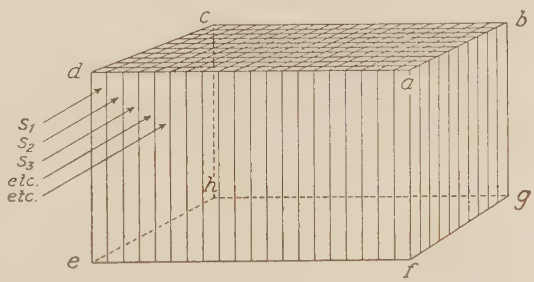


Fig. 3

The molecules of a perfect gas must not consist of incompressible matter, if the gas as a whole is to conform to the requirements of perfection. We may say that the molecules of a perfect gas have zero dimension. Now if the gas within the tank were perfect, the area of the sections in the figure might be reduced gradually in size, until they become of zero dimension and infinite in

number without conflicting with Avogadro's Law. If we may be certain that this "passing to the limit" will not conflict with the law when we deal with actual molecules, our analysis of the performance of reservoirs will be facilitated, for on a later occasion we shall desire to obtain the relations between pressure and volume during delivery, and it will be convenient to use calculus in a way which implies a summation of an infinite number of spaces having zero cross-section. Will this be permissible?

Obviously the areas of the sections in their reduction, immediately before they become zero, will be too small to define spaces which can include molecules having matter of finite dimension—they can only pierce those which happen to be in line. At some instant, then, it may so happen that the sections do not pierce equal numbers of molecules; and only by chance may two adjoining spaces include equal quanities of matter. The assertion can be made, however, that on the average the areas will define spaces which will include equal quantities of matter. We need only provide a sufficient time for this average condition to be brought about. And in expectation of its ultimate fulfillment we may perform the summation, or integration, at once. The result will be found to coincide with experience. As we shall find in the process of delivery, so do we observe now in connection with fluid in equilibrium—from the point of view of mass mechanics—that the relations between pressure and volume are analytically independent of time.

The following statement may be made in conclusion: *In virtue of Avogadro's Law the processes of integral calculus may be applied to pressure-volume conditions within a reservoir; and this is true whether we regard the gas as perfect or actual.*

32. *Boyle's Law.*—This law, stated in section 30, paragraph (*b*), is easily interpreted by means of an imaginary experiment. Figure 4 represents a cylinder equipped with a valve and pressure gauge at its head and a non-leakable piston sliding freely within it. For convenience we shall say that the position of the piston is indicated by comparison with the scale which parallels the cylinder. The divisions of this scale are equal, and it is immaterial for our purposes what length they may actually represent. With the valve open, the piston is placed at the position marked 10, say, and then the valve is closed. Let us imagine now that the apparatus is of such dimensions that it contains 10 cubic feet of air in its present condition. The pressure gauge reads zero.

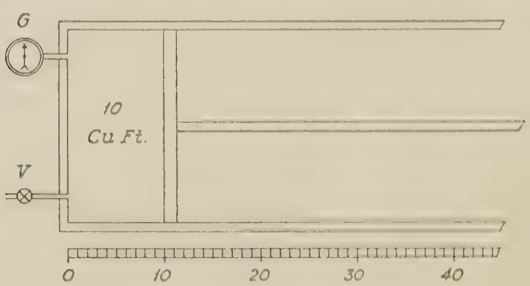


FIG. 4

On the assumption that the atmospheric pressure amounts to 14.7 pounds

per square inch, absolute, the product of this pressure and the volume is 147. The law states that whatever the position of the piston, the product of these two variable quantities remains the same, namely, 147, provided the temperature is maintained at a constant value. We shall assume the latter to be the case.

With the piston at position 5, the absolute pressure is two atmospheres, or 29.4 pounds absolute, and the gauge registers 14.7 pounds above atmospheric pressure. The product of pressure and volume is again 147, as indicated by 29.4 and 5, respectively. In the opposite direction, by placing the piston at 20 the gauge registers minus 7.35 pounds, an absolute pressure of plus 7.35 pounds. Once more the product of 7.35 and 20 is 147. In order to obtain absolute zero pressure within the cylinder, it will be necessary to move the piston in the extended cylinder an infinite distance to the right, and to obtain infinite pressure we must move the piston to zero on the scale. Both of these extreme cases are, of course, impossible of fulfillment in practice—with this or any other type of apparatus. Nevertheless, we may imagine that we have the following table of exact data:

| $p'$ Gauge Pressure | $p$ Absolute Pressure | $v$ Space-Volume |
|---|---|---|
| 0.0 | 14.7 | 10 |
| 1.6 | 16.3 | 9 |
| 3.7 | 18.4 | 8 |
| 6.3 | 21.0 | 7 |
| 9.8 | 24.5 | 6 |
| 14.7 | 29.4 | 5 |
| 22.1 | 36.8 | 4 |
| 34.3 | 49.0 | 3 |
| 58.8 | 73.5 | 2 |
| 132.3 | 147.0 | 1 |
| Infinity | Infinity | 0 |
| — 7.35 | 7.35 | 20 |
| — 9.8 | 4.9 | 30 |
| —11.0 | 3.7 | 40 |
| —11.8 | 2.9 | 50 |
| —12.3 | 2.4 | 60 |
| —12.6 | 2.1 | 70 |
| —12.9 | 1.8 | 80 |
| —13.1 | 1.6 | 90 |
| —13.2 | 1.5 | 100 |
| —14.7 | 0.0 | Infinity |

It is obvious that the gauge continually registers a pressure of one atmosphere less than whatever absolute pressure there may be within the cylinder. The experiment concerns all molecules of air within, and these are always of the same number, since it is stipulated that none can pass the piston in either direction. We see that the mass of air within the cylinder is constant, though the space it occupies is variable; in other words, mass-volume is constant, while space-volume is variable.

The curve of Figure 5 may be plotted from the points supplied in the table. Absolute pressures are taken as ordinates and space-volumes as abscissas. The equation of the curve is $pv = k$, wherein the significance of the symbols $p$ and $v$ are obvious, and $k$ is the constant of 147. The usual mathematical expression for this type of curve is $xy = k$, the equation of the rectangular hyperbola; $x$ and $y$ are merely variables, and $k$ is to be read, "some constant" or "a constant." The hyperbola expressed in this form has its asymptotes coinciding with the axes of the co-ordinates used in plotting the points of the curve, as shown.

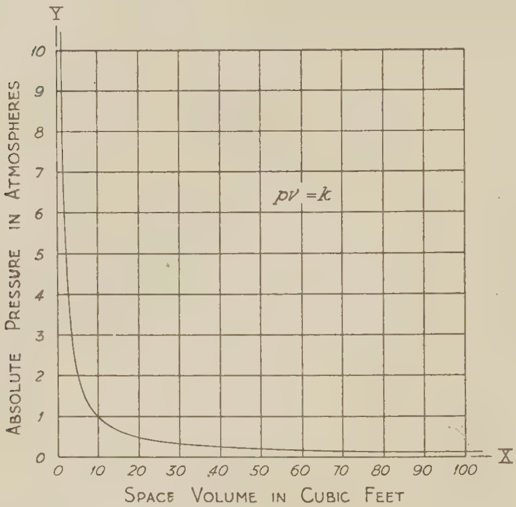


Fig. 5

There are two important points to be noted in connection with this experiment, one physical and the other geometrical:

*a*) The experiment is one which pertains properly to physics and theoretical chemistry. It is not one in the mechanics of fluid delivery, for it is quite obvious that no production is involved in any way. We shall later perform a corresponding experiment, also imaginary, on delivery, and there the space-volume will remain constant while the mass-volume varies—the present situation reversed.

*b*) By inspection of the table of values we see that one quantity increases while the other decreases, and vice versa, in order that their product may remain the same. In the later experiment both quantities increase or decrease simultaneously, and whereas we now have a rectangular hyperbola, we shall obtain instead a straight inclined line, as of Volumetric Control. (The corresponding line in Hydraulic Control will be shown to be a straight horizontal line, and that in Capillary Control a parabola.)

The data in the table indicate ideal performance. If it were possible for us to construct the perfect apparatus described, only a perfect gas would act in this exact manner. Since natural gases issuing from wells are imperfect, they do not fulfill the law with exactness. They deviate from it according to the character and amount of their imperfection. Where precision requires our doing so, we may determine the deviation of the particular natural gas in which we are interested by test in the physical laboratory. The determinations should, of course, cover the range of pressures involved in the performance of the reservoir, and it will be convenient to adopt the so-called "$pv$-$p$" curve as a basis of recording the data. Figure 6 shows such a curve. The ordinates represent the product of absolute pressures and space-volumes, while the abscis-

sas denote simply the absolute pressures. The former may be expressed either in actual amounts of foot-pounds, or in percentages based upon 100 per cent $pv$ at atmospheric pressure. Abscissas are preferably expressed in pounds. The line $A$ is that of a perfect gas, for which $pv$ is constant throughout the range of pressures. Now the actual curve obtained will deviate from this, and appear as the one at $B$, if the gas is permanent or "dry," or as the one at $C$, if vapor is present, that is, if the gas is "wet."

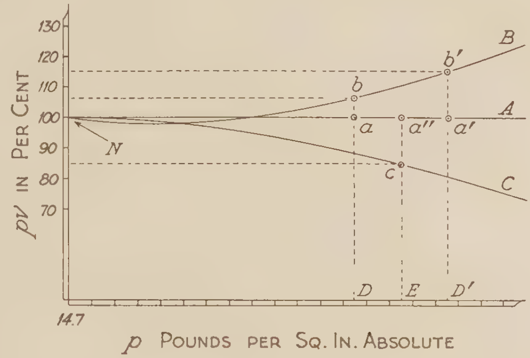


FIG. 6

If the pressure within a reservoir containing permanent gas alone has the value of $D$ at any instant, then any calculations involving the product of pressure and volume must be divided by the percentage indicated to the left of the point $b$, in order to place them upon the basis of a point $a$, the latter representing conditions "as of the same reservoir with the same pressure, but with a perfect gas." The corrected $pv$, divided by $p$, indicates the volume as of the point $N$ at atmospheric pressure. Evidently a calculation involving two pressures, $D$ and $D'$, is in reality based upon the point $N$ through the points $b$ and $b'$. In the same way, calculations for a reservoir containing permanent gas and vapor, involving a pressure $E$, require the division of the product $pv$ by the percentage to be left of the point $c$, in order that they may likewise be based properly upon $N$.

*Hereafter in the course of our investigation we are to assume that the correction for imperfection of the gas has been made, whenever we speak of the performance of the actual reservoir.* Our reservoir in the field is not ideal, but it lends itself to computations "as of the ideal" with respect to the gas in the manner outlined. Further reference to the non-fulfillment of Boyle's Law in developing our system of fluid mechanics becomes unnecessary.

33. *Dalton's Law.*—Dalton's Law, stated in section 30, paragraph ($c$), may be interpreted by referring to the diagram of Figure 7 (p. 50). Here we have a rigidly constructed tank containing two gases, $N$ and $O$, separated by a thin impervious membrane at $a$-$a$. The quantities of gas supplied by a pump beyond the orifices are such that the membrane possesses a perfectly plane surface. The pressure is uniform throughout the tank at, say, $C$ pounds per square inch. Let us imagine $N$ removed, allowing $O$ to fill the entire tank. Its pressure would then be lower, say $B$ pounds per square inch, where $B$ and $C$ are related in accord with Boyle's Law. Now we are to imagine the original conditions again established, and remove $O$, allowing $N$ to fill the tank.

The pressure is then, say, $A$ pounds per square inch, where $A$ and $C$ are related as before. The present law states that $A + B = C$, or in words, that the total pressure is equal to the sum of the partial pressures.

When both gases are in the tank, we may remove the membrane. Diffusion takes place; yet the law holds as before. The fact that the individual gases cannot be separated and made to occupy the tank alone in no way disturbs the relation between the total and partial pressures.

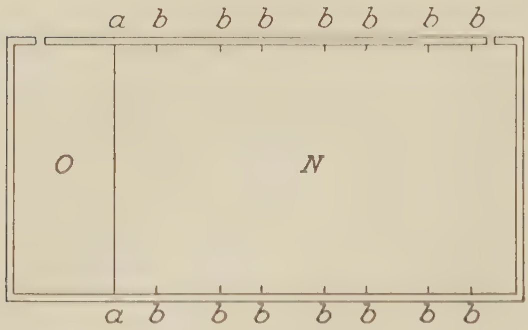


Fig. 7

A sufficient number of membranes may be imagined placed at the positions marked *b-b*, so as to accommodate the individual components of natural gas. Once more the pressure which they exert together is equal to the sum of the individual pressures, if each in turn were to occupy the entire space within the tank. The removal of the membranes while all components are in the tank permits a return to the original state of the gas as it issued from the well. Although diffused, each component continues to exert its partial pressure upon the walls of the container.

Dalton's Law holds for all mixtures of gases, provided chemical action does not take place between the components when they are brought into contact with each other. In virtue of this law it is clear that the laws of Avogadro and Boyle are extended so as to include mixtures of gases. Henry's Law is likewise extended.[5]

**34.** *Gay-Lussac's Law.*—In virtue of Avogadro's Law the present one is frequently stated as follows: *All gases expand equally for the same rise in temperature.* The statement of the law as given in section 30, paragraph (*d*), is generally preferred. If the pressure of the given mass of gas be kept constant, we may write

$$v = kT \quad \text{..........................} (1)$$

and if the volume be kept constant,

$$p = kT \quad \text{..........................} (2)$$

where $v$ is space-volume, $p$ is absolute pressure, $T$ is absolute temperature, and $k$ is a constant.

We find this law, as the others, not rigorously exact. There are in fact

[5] The extension is in accord with the van't Hoff Law.

differences in the rates of expansion for the different gases, but these differences are exceedingly small among the more approximately perfect ones. When gases are kept at a constant pressure and allowed to expand in volume, slightly different results are obtained from those found when they are kept at a constant volume and allowed to increase in pressure.[6]

Beginning with the temperature of melting ice, Gay-Lussac determined that with pressure maintained constant the increase in volume for each degree Centigrade above that temperature amounted to 1/273.1 of the original volume. On the Fahrenheit scale the change is 1/491.6 of the volume per degree. This gives the absolute zero temperature at minus 273.1 degrees Centigrade, and at minus (491.6 — 32.0), or 459.6 degrees Fahrenheit. At this temperature a given mass of gas would theoretically possess zero volume.

The law may be represented graphically by diagram, as in Figure 8, where in (*a*) is shown the increase in volume at a constant pressure, and in (*b*) the increase in pressure at a constant volume, corresponding to an increase in temperature. Where our previously mentioned assumptions with respect to heat phenomena are at variance with known local or temporary conditions within the reservoir,[7] a correction in pressure and volume can be made in accordance with the law. Ordinarily it will be sufficient to make this correction upon the basis of the straight lines shown, but if greater accuracy is required, deviation curves with $v/T$ and $p/T$ as ordinates and $T$ as abscissas can be constructed for use similar to that of Figure 6. In this way we may once again make our calculations on reservoir performance as of the ideal, now with respect to temperature changes in the reservoir. *Hereafter in the course of our investigations we are to assume*

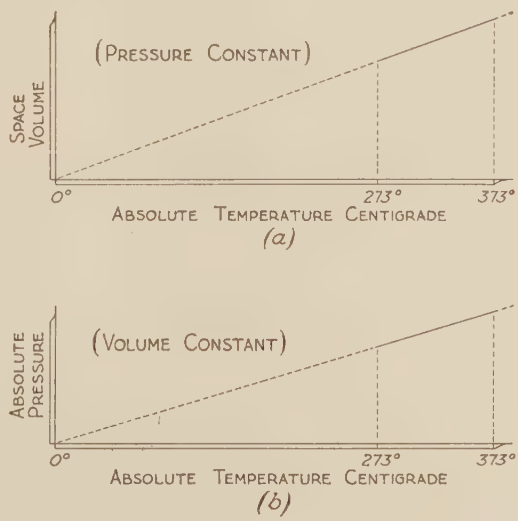


Fig. 8

[6] This circumstance is not to be confused with thermodynamical considerations that involve the performance or non-performance of external work in the two respective cases. The conditions as stated are to be expected in view of the fact that Boyle's Law is not fulfilled exactly in practice. Exact equality in the two cases infers a non-deviation in the value of Boyle's constant $k$.

[7] We are to assume the walls of the containers of fluids to be perfect conductors of heat, and the temperature within the reservoirs to be constant by virtue of a transfer of heat, either inward or outward, as the case may require, in the process of production. (§§ 22 and 28.)

*that, where corrections for alterations in temperature are necessary, such corrections have been made.* Further reference to this law in developing our system of mechanics becomes unnecessary.

35. *Characteristic equation of gases.*—The laws of Avogadro, Boyle, Dalton, and Gay-Lussac may be summed up in one formula known as *the characteristic equation of gases.* It is as follows:

$$pv = MBT$$

in which equation $p$ is the absolute pressure exerted upon the gas, $v$ the space-volume of the gas, $M$ the mass of the gas, $T$ the absolute temperature, and $B$ is a constant dependent upon the molecular weight of the gas, if pure, or upon the relative proportion and molecular weights of constituents, if the gas is a mixture. $B$ is independent of changes in the values of $p$, $v$, $M$, and $T$, although it is dependent upon the units employed in expressing them.

The characteristic equation of gases is of fundamental importance in physical chemistry. The two quantities, $M$ and $B$, are usually combined into one constant $R$, called *the gas constant,* which is the same for all gases. To unite these it is necessary to take only a definite amount of gas at standard pressure and temperature for the quantity $v$ in an experiment. For instance, if we divide the molecular weight of oxygen by its weight of one cubic centimeter at 76 centimeters of mercury-pressure and 0 degrees Centigrade temperature,

$$\frac{2 \times 16}{0.0014290} = 22{,}390 \text{ cubic centimeters}$$

is an amount which provides for a definite value of $MB$ or $R$, and, in virtue of Avogadro's Law, this amount of any gas will furnish the same $R$. Now the equation may be written in the familiar form

$$pv = RT$$

which is the basic equation in theoretical and applied thermodynamics. This amount of gas, 22,390 cubic centimeters, is called a *gram-molecule,* or briefly, a *mol* of gas. $R$ can be shown to be 1.985 calories, when $p$ is expressed in grams per square centimeter, $v$ in gram-molecules, and $T$ in degrees Centigrade, as follows:

| | | |
|---|---|---|
| 1 cubic centemeter of mercury weighs | 13.5951 | grams |
| 76 cubic centimeters of mercury weigh | 1033.23 | grams |

Now let

$p = 1033.23$ grams per square centimeter, and
$v = 22{,}390$ cubic centimeters

Then

$pv = 23{,}134{,}000$ gram-centimeters, the intrinsic energy possessed by a gram-molecule of gas on isothermal expansion.

$\frac{23{,}134{,}000}{273} = 84{,}700$ gram-centimeters necessarily added to the intrinsic energy of one gram-molecule of gas for each degree Centigrade rise in temperature. But

$$42{,}660 \text{ gram-centimeters} = 1 \text{ calorie}$$

Consequently,

$\frac{84{,}700}{42{,}660} = 1.985$ calories necessarily added per degree Centigrade rise in temperature. Then in the equation $pv = RT$, $R$ must be the quantity 1.985 calories, as determined, in order that the relation between $p$, $v$, and $T$ be satisfied.[8]

The gas equation may be expressed in purely mathematical symbols as follows: $xy = kz$, the equation of the surface shown in Figure 9. If in $pv = RT$ the three variables are successively given constant values, these values specify certain planes which are to cut the surface in the figure. If $T$ is constant, the equation may be written $pv = k$; the hyperbola of Boyle's Law, section 32. The plane for a particular value of $T$ cuts Figure 5 out of Figure 9. Again, if $p$ and $v$, successively, are constant, the equation may be written $v = kT$, and $p = kT$, respectively. The planes for particular values of these quantities cut Figure 8 out of Figure 9. These are the straight lines of Gay-Lussac's Law, section 34. Incidentally the significance of the constants $k$ in the various equations becomes clear.

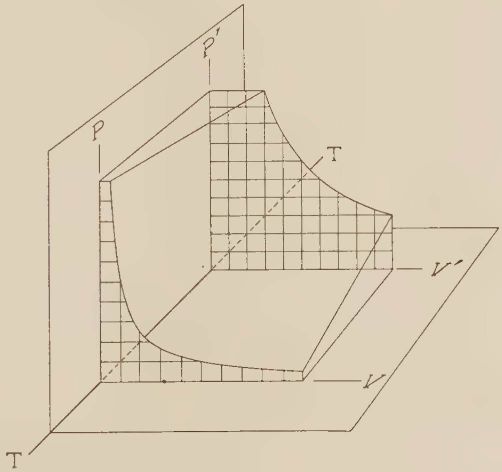


Fig. 9

The laws of Avogadro and Dalton enter into the equation indirectly, in so far as the former relates to the value of $MB$, or $R$, and the latter specifies the fact that the gas may be a pure one or a mixture of several gases.

[8] If a mass other than the gram-molecule of gas be chosen as a unit, $R$, being energy pertaining to the mass, will possess a different value. Not only will $R$ for the given gas under consideration be different because of the different unit, but also each gas will have a distinct value of $R$, because of its unique molecular weight. Thus in textbooks on technical thermodynamics we see tabulated values of $R$ for a gram-mass or a pound-mass of the various gases used in industries.

**36.** *Henry's Law.*—If the temperature is kept constant, the amount of gas dissolved in a liquid is proportional to the absolute pressure exerted by the gas upon the liquid.

Temperature is important because of the fact that the capacity of a liquid to act as a solvent of gas increases with the lowering of the temperature, and vice versa. By "amount" is meant mass-volume as defined in section 12.

We have been reminded before that all gases appear to be soluble in all liquids to a greater or less degree. The coefficient of solubility is a number which expresses the volume of a given gas dissolved in a unit volume of given liquid at a temperature and pressure adopted as a standard. The coefficients for various gases with a givên liquid, and those for a given gas with various liquids, vary considerably.

The gas in contact with liquid, under pressure, may be either a pure one or a mixture of several gases. That each individual gas in a mixture is dissolved in proportion to its own partial pressure was pointed out by Dalton; and that mixtures of dissolved gases which do not react with one another distribute themselves as though each alone were present in the system was shown by Berthelot.[9] A dissolved natural gas distributes itself between two immiscible liquids, such as oil and water, in accord with the ratio of its solubility in each liquid.

Henry's Law may be stated in the following manner: $v = kp$, or $p = kv$, wherein $v$ is the mass-volume of gas dissolved in the liquid, $p$ the absolute pressure, and $k$ is a constant whose values differ for various combinations of gases and liquids. (We here have an example of the manner in which we often employ constants. Obviously in this case the two $k$'s are mutually reciprocal, but no mention need be made concerning this fact in our general analysis. As stated previously, $k$ is to be read simply as "a constant," or as "some constant.")

If, in accordance with the second equation, we plot values of $p$ as ordinates and values of $v$ as abscissas, we obtain the straight inclined line as in Figure 10. The origin is at the left at a point indicating absolute zero pressure. This shows that, theoretically at least, the last molecule of gas will not leave the liquid upon the release of pressure until absolute zero pressure is attained. Upward to the right the line may be extended indefinitely. Figure 10 may be en-

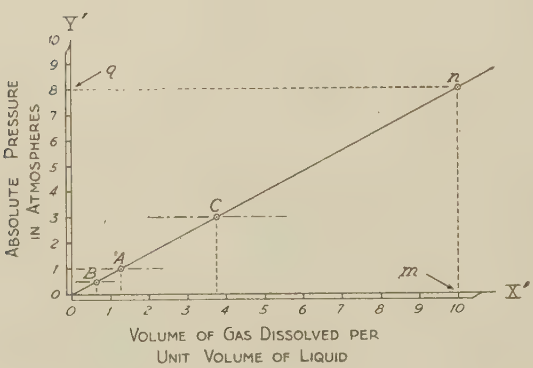


FIG. 10

[9] These two circumstances are in accord with the van't Hoff Law.

titled, "The Curve for an Unlimited Volume of Gas per Unit Volume of Liquid."

When the volume of gas per unit volume of liquid is limited in a particular experiment, say at 10 units as indicated by the point $m$ in Figure 10, we may draw a vertical line through $m$ intersecting the inclined line at the point $n$. From $n$ draw a horizontal line to the point $q$. Now the figure represents the case where a definite amount of gas is in contact with a definite amount of liquid, a case which might occur in a laboratory experiment, or in the natural course of events within a combination oil and gas reservoir in a porous formation. It is evident that $q$ units of absolute pressure are necessary and sufficient to force the entire allotted amount of gas into solution. A pressure less than $q$ allows some of the gas to remain in the free state, and one greater than $q$ tends to compress the liquid. We may therefore draw Figure 11, and entitle it "The Curve for a Limited Volume of Gas per Unit Volume of Liquid." (The vertical line at $n$ is in fact the curve $pv = k$, a rectangular hyperbola, for the compression of the liquid. The constant $k$ is such that the portion shown appears approximately straight. Naturally the curve breaks at $n$ because the liquid does not continue to expand below the pressure $q$.) Since the total amount of gas is limited, abscissas may be designated in percentages of this amount, measured from zero at absolute zero pressure to one hundred at $m$. From left to right the scale indicates the percentage of gas in solution, and from right to left, the percentage of gas in the free state.

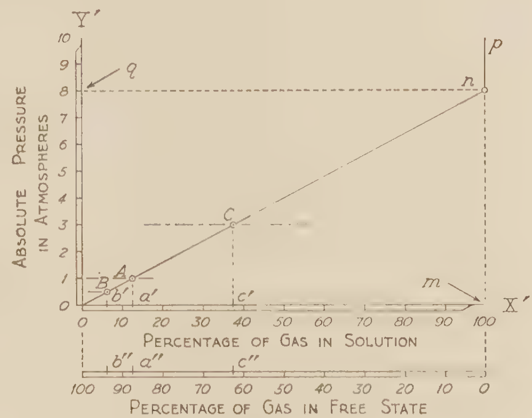


Fig. 11

The straight inclined lines, or "curves," of these two figures are ideal. By actual test with given fluids the lines may be expected to deviate somewhat, a circumstance due to the imperfection of both the gas and the liquid, particularly—so we may presume—to the imperfection of the former. For a given experiment in the laboratory or in the field we can construct a deviation curve similar to Figure 6, with $v/p$ as ordinates and $p$ as abscissas. As before, calculations involving a change in pressure may be adjusted in accordance with this curve; and thus our computations are placed upon a basis as of the ideal once again. The determination of the points for the deviation curve need take no special cognizance of Boyle's Law, for if deviations due to this law come into play their effects are directly recorded in connection with the present law.

It is interesting to note that the proposed deviation curve registers a

deviation due to the liquefaction of vapors upon the increase of pressure, and the converse vaporization of liquid upon the decrease of pressure. While it would be logical to assume that permanent gas and gaseous vapor pass into and out of solution with respect to the liquefied vapor, we need in fact take no particular account of the circumstances, inasmuch as *the curve will care for all deviations, irrespective of their sources.*

The variation in the dissolving power of a liquid with changes in temperature does not appear to be strictly mathematical. This is to say, we cannot devise a formal law of physics relating to the solubility of a gas in a liquid at constant pressure and variable temperature in a simple verbal or mathematical expression. For any given gas-liquid system we may, however, determine such a variation by actual test in the laboratory, and give it a graphic expression in the form of a curve. Furthermore, we can construct a deviation curve wherein ordinates represent the coefficients of solubility at a standard pressure, and abscissas represent temperature. Such a curve may be used in the manner of Figure 6.

In virtue of the two deviation curves here described *it shall be understood hereafter that the correction for any changes in the solubility of the gas in the liquid at various pressures and temperatures has been made.* Further reference to deviations from Henry's Law becomes unnecessary in developing our system of fluid mechanics.

The curves in Figures 10 and 11 pertain to a reservoir with a gas, either pure or mixed, and one liquid. Now let us assume that we are concerned with a reservoir which contains a gas and two immiscible liquids, such as oil and water. In Figure 12 the lines $A$ and $B$ pertain to the solution of the gas in the separate liquids. These lines indicate the fact that the gas is more soluble in the case of $A$ than in that of $B$. If we adopt atmospheric pressure and a definite normal temperature as standard conditions for the purposes of a comparison, we may say that the ratio of solubility of the gas in the two liquids is $S_1/S_2$, or, if we prefer to adopt a higher pressure as the standard, the ratio is $S_3/S_4$. These ratios are equal when the fluids are perfect, but not otherwise. Where there is a deviation in the ratio within a given range of pressures, we can adopt any determined ratio as a standard and construct the deviation curve in the usual manner.

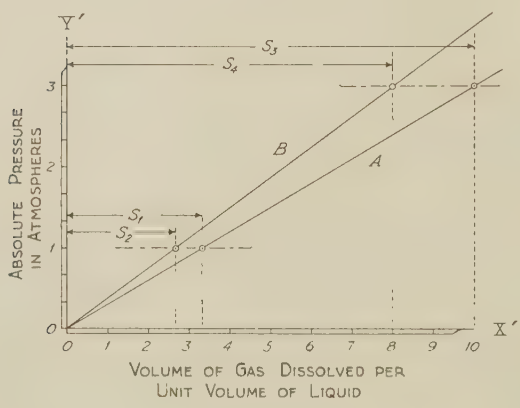


Fig. 12

Figure 12 may also be used in a situation which involves two reservoirs. Let the line $A$ represent the solubility of a gas in a liquid in one reservoir, and

the line $B$ the solubility in another. Either the gas, the liquid, or both, are presumably different in reservoirs which we may readily imagine to be in different parts of the same stratum, in different strata in the same field, or in different fields. With the ratio of solubility determined as before, we may easily carry our computations from one locality to another. These principles may be extended to cover two or more reservoirs, each containing two immiscible liquids.

CHAPTER V

# Mechanics of Fluids

"In relation to the sensible world (according to Kant) we have logical certainty only of the propositions of geometry and mechanics, the last distinguished from the first on account of their requiring, besides the three dimensions of space, the fourth one, time, and the notion of mobile matter."—ROBERT VON HELMHOLTZ

37. *Introduction.*—The laws of Avogadro, Boyle, Dalton, Gay-Lussac, and Henry are formal expressions which concern the functions of fluids in *absolute phase.* Experiments upon which they are based necessarily refer to pressure measured from absolute zero, space-volume as made up of a definite number of molecules that are confined within an apparatus in the form of a closed container, and temperature, also measured from absolute zero, as a property of fundamental character. On the other hand, the delivery of fluid from a reservoir has been said to concern the functions of fluids in *potential phase.* Pressure is measured from a potential zero which might be located at any position on the absolute scale, and mass-volume involves molecules which are being transferred from the interior to the exterior of an apparatus in the form of a reservoir with an open orifice. Temperature is merely an incidental feature, one ordinarily assumed constant, but should it be otherwise, only a slight correction aside from the main problem need be made. The problems confronting the physicist in his determination of the physical properties of fluids are by no means identical with those confronting the production engineer. They are related, to be sure, inasmuch as the latter problems are dependent upon the former ones for their resolution.

Of the five physical laws discussed in the preceding chapter those of Boyle and Henry are of the greatest importance to us. Our first task is to transform them into the potential phase, for otherwise they cannot be correctly applied to the performance of reservoirs. Allowances for the two distinct situations in the nature and behavior of volume are easily made. I cannot claim to reveal anything new and unfamiliar in the transformations. The mathematical forms which the laws assume are simple; yet I do not believe that there are any propositions in connection with the delivery of fluid which suffer more careless treatment and more frequent misapplication or misinterpretation than these. But perhaps this is not surprising when one considers some of the possibilities of confusion in the mechanics of gases.

Eight propositions, listed in section 30, are to be investigated in the present chapter. For the present let us believe that all of them relate specifically to reservoirs in Volumetric Control. The discussion will at least be confined to this class of reservoirs because of our familiarity with them. A few remarks concerning Capillary Control, however, will not be out of place.

That the relations between pressure, volume, and energy are independent of time in every way, is a fact which we cannot afford to ignore in the collection of data for problems involving the forecast of production from wells. Some engineers have already recognized this independence, in so far as they have adopted a so-called "graphic solution by Boyle's Law" based upon pressure-volume data of production.

A most important topic to be discussed in the present chapter is the relation between the mechanics of liquid and the mechanics of gases. In our conception of a potential reservoir they are identical. This means that a reservoir producing liquid alone behaves in exactly the same manner as one producing gas alone. Combination reservoirs, which produce both liquid and gas, require special attention in virtue of the fact that the gas dissolves in the liquid.

38. *Boyle's Law in potential phase.*—In section 32 we performed an imaginary experiment for Boyle's Law in absolute phase. Let us now perform one in potential phase.

Figure 13 ($a$) represents the interior space of a rigidly constructed gas tank which is properly equipped with a valve at the orifice and a pressure gauge. The cover and equipment are not shown. Atmospheric pressure is assumed to be constant, while the tank is repeatedly filled with air and allowed to produce into the atmosphere. Thus the potential phase is identical with the atmospheric phase, as mentioned in section 11.

We are to begin with the orifice open, and with the pressure inside equal to that outside of the tank. For convenience we will assume that there are now 2 cubic feet of air at atmospheric pressure within the vessel, a volume represented by $A$. It is evident that under these conditions there are really 2 cubic feet within, but none is subject to production; in other words, the absolute volume is 2 cubic feet, while the potential volume is zero. The potential pressure here indicated directly by the gauge is zero.

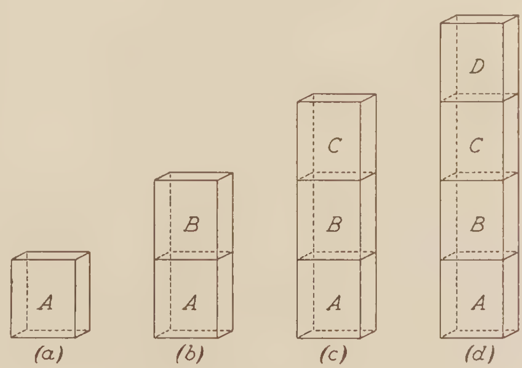


Fig. 13

First we force a volume $B$, equal to $A$, as shown in Figure 13 ($b$), into

the tank. The absolute volume contained is 4 cubic feet of free air, and the potential volume is 2 cubic feet. The pressure gauge now registers 1 atmosphere, or 14.7 pounds per square inch. Secondly we force volumes $B$ and $C$, both equal to $A$, as shown in Figure 13 ($c$), into the tank. The absolute volume is now 6 cubic feet of free air, while the potential volume is 4 cubic feet. The gauge registers 2 atmospheres, or 29.4 pounds per square inch. Next we force volumes $B$, $C$, and $D$, all equal to $A$, as shown in Figure 13 ($d$), into the tank. The absolute volume is 8 cubic feet of free air, while the potential volume is 6 cubic feet. The gauge registers 3 atmospheres, or 44.1 pounds per square inch. In this manner we may continue indefinitely, except as we may be restricted by the capacity of the air pump, and the capacity of the tank to withstand high pressures. The following table may be set up:

| Potential Pressure | Potential Volume |
|---|---|
| 0.0 | 0 |
| 14.7 | 2 |
| 29.4 | 4 |
| 44.1 | 6 |
| 58.8 | 8 |
| 73.5 | 10 |
| 88.2 | 12 |
| 102.9 | 14 |
| 117.6 | 16 |
| 132.3 | 18 |
| 147.0 | 20 |

Throughout the experiment the space-volume within the tank has remained constant, and the mass-volume has been varied. It is to be noted that the quantities in the two columns increase simultaneously, herein differing from the quantities in the table of section 32. The curve, with potential pressures as ordinates and potential volumes as abscissas, is shown in Figure 14. It is a straight inclined line with the equation

$$y = kx$$

or as we shall prefer to write it in the symbols for the potential functions of performance:

$$P = KVo$$

It is not material whether the origin is placed on the right or on the left margin of the plat.

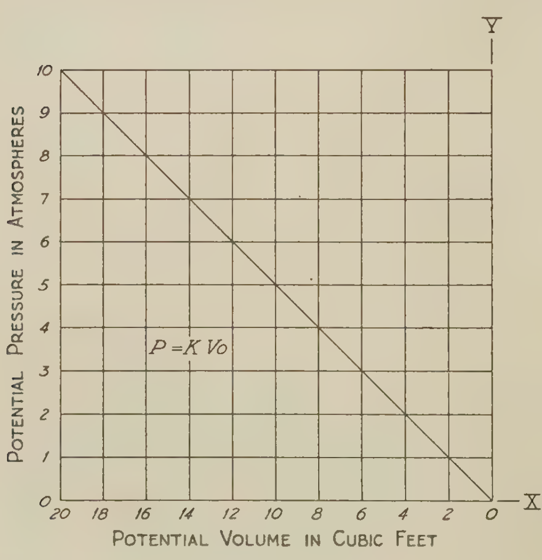


Fig. 14

The present plan is to place it at the right as shown, in order that the curve may have the appearance of decline from the upper left to the lower right corners upon production from the reservoir, in accordance with the usual custom prescribed by field practice.

The straight-line relation between pressure and volume is easily seen by inspecting the quantities in the table. If we do not know the value of the constant $K$ which must appear in the equation, or if we do not care to concern ourselves with it, we can either write simply that

$$P \text{ varies as } Vo$$

or, since the form of an equation is so convenient in algebraic operations, *we may adhere to the constant and ignore the value which it possesses.* Let us choose the latter method, thus justifying ourselves in reading $K$ as a constant, or as some constant. (See sections 32 and 36.)

If the tank were compelled to produce against a line pressure of, say, 5 pounds above atmospheric pressure, or even 5 pounds below atmospheric pressure, the potential pressures in the table would each be 5 pounds less, or more, respectively, than those shown. Clearly the straight-line relation between the functions is not altered by so doing. The fact is that the various values of the constant back pressure merely provide for distinct potential reservoirs.[1]

We shall find this experiment to be one in Volumetric Control, where, as we here see, the hyperbola of Boyle's Law in absolute phase becomes a straight line in potential phase.

**39.** *Henry's Law in potential phase.*—Any problem concerning the gas issuing with liquid from a reservoir, whether the reservoir is of the open or closed type, depends upon Henry's Law for its resolution.

[1] To take another example let us consider one that perhaps more closely approaches field problems: If a gas tank undergoes a decline in pressure from 1,000 pounds to 500 pounds per square inch, as shown by an attached gauge, exactly 50 per cent of the gas *to be produced into the atmosphere* has been produced in this decline. If production has taken place against a line pressure, say of 50 pounds per square inch above atmospheric pressure, the volume of gas produced is not $\frac{500}{1,000}$, but $\frac{450}{950}$, or approximately 47.4 per cent. In either event of production it would be improper to say that $\frac{514.7}{1,014.7}$, or approximately 50.7 per cent of the gas, remains to be produced after the specified decline that is recorded by the gauge. *There is no reason for including the pressure of the atmosphere in stating the fraction, unless production takes place into a perfect vacuum.*

It is not the gauge pressure, nor yet the absolute pressure, that should be used in computations, but only the potential pressure, granting that under proper conditions of production this pressure may be equal to either one of the other two. (It is essential to remember that in selecting the gas tank as the container in the present problems we stipulate Volumetric Control.)

Let Figure 15 represent a closed tank containing liquid and gas under compression, with an orifice at $O$. When the surface of the liquid is at $a$, the absolute closed-in pressure at $O$ is, say, $p_1$ pounds per square inch. During production the surface lowers to a position at $b$, and now the absolute closed-in pressure at $O$ is, say, $p_2$ pounds per square inch. What change has taken place regarding the amount of gas dissolved in, and the amount of gas produced with, a unit volume of liquid, supposedly such a volume as $U$ immediately at the entrance to the orifice?[2]

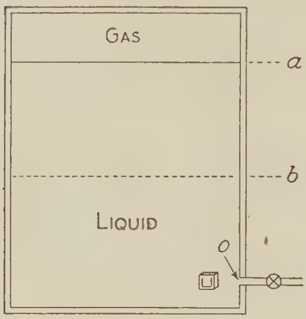


FIG. 15

The equation expressing Henry's Law is given in section 36 in the form

$$v = kp \quad \text{.............. (3)}$$

where we may now say that $v$ is the mass-volume of gas dissolved in the unit volume of liquid $U$, $p$ is the absolute closed-in pressure exerted upon $U$, and $k$ is some constant.

The following equations may be written:

$$v_1 = kp_1 \quad \text{........................ (4)}$$

and

$$v_2 = kp_2 \quad \text{........................ (5)}$$

two equations which may be united by dividing the former by the latter thus:

$$\frac{v_1}{v_2} = \frac{p_1}{p_2} \quad \text{........................ (6)}$$

This equation between ratios may be interpreted as follows: *The amount of gas dissolved in a unit volume of liquid at the interior of the orifice, therefore the amount of gas produced per unit volume of liquid, at two instants during the life of the reservoir bears the same proportion as the absolute closed-in pressure at the instants.* In the course of production the lowering of the pressure permits dissolved gas to pass into the free state, and that portion of the latter which is not swept through the orifice ascends to the space above the liquid, where it continues to expand. That gas which is swept out with the liquid is not an erratic amount regulated only by chance from one instant to another, but it is a definite, mathematically declining amount in accordance with the foregoing equations.

Figure 16 illustrates a pressure-decline curve which includes the points $a$ and $b$ of $p_1$ and $p_2$ pounds per square inch absolute closed-in pressure in ac-

[2] The amount of gas dissolved in $U$ is less than the amount dissolved in any unit volume at a greater distance from the orifice. This feature will be considered in connection with the pressure gradients for the various reservoirs of the three controls.

cordance with the preceding example. In virtue of Equation 3 we may select vertical scales so that the same length ordinates represents both $p$ and $v$; therefore $v_1$ and $v_2$ are as shown. The axis $X$ marks a potential zero $O$ at the left which is determined in vertical position by the value of the constant back pressure $C$. The relation between $v$ and $p$ is not disturbed by the position of the axis $X$, whether it is maintained at one place or altered from time to time, since both of these quantities refer to their own absolute axes, immovable, and here coincident, at $X'$.

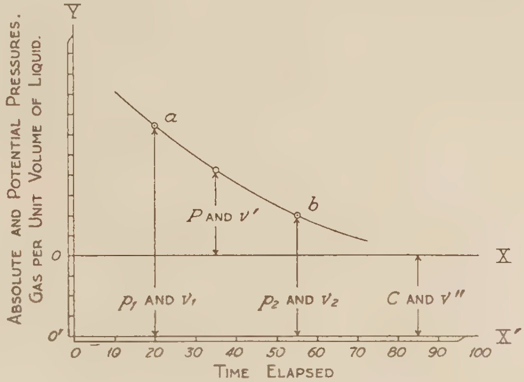


Fig. 16

At any time during production we have

$$p = P + C \dots (7)$$

in which expression $p$ is the absolute closed-in pressure, as before, $P$ the potential pressure, and $C$ the constant back pressure.

By substituting this value of $p$ into Equation 3 we obtain

$$v = k(P + C) \dots\dots\dots\dots (8)$$

The mass-volume $v$ itself may be arbitrarily divided into two parts, say, $v'$ the mass-volume liberated from solution immediately at the orifice, and therefore that amount of gas which is produced in the free state at the orifice, and, say, $v''$ the mass-volume held in solution at the orifice, and therefore still in solution at the orifice during production. Now we may write

$$v' + v'' = k(P + C) \dots\dots\dots\dots (9)$$

In virtue of Equation 3 this one may be split up into

$$v' = kP \dots\dots\dots\dots (10)$$

and

$$v'' = kC \dots\dots\dots\dots (11)$$

If we are to measure only the gas in the free state during the course of production, we shall find that its amount varies, per unit volume of liquid, in direct proportion to the variation in the potential pressure, as we see in Equation 10. That which remains in solution, per unit volume of liquid, is constant in amount,[3] as indicated in Equation 11.

The discussion pertaining to the equations is conveniently based upon the assumptions that the constant back pressure always retains the same value,

[3] The product of the two constant quantities $k$ and $C$ is itself a constant.

and that the measurement of the gas takes place while the fluids are yet subject to the constant back pressure after production. Let us suppose that Figures 10 and 11 pertain to the fluids in the reservoir of Figure 15. The amount of gas in solution, and the amount of gas in the free state, varies with the lowering of the pressure in accordance with the inclined line. The point $C$ denotes the position of the potential zero on the scale of absolute pressures. The equations above refer to a gradual descent from some high point on the curve to $C$, and as just stated the measurement of the gas takes place at $C$. If, subsequently to production at the orifice, the fluids are subjected to the pressure of the atmosphere, a further descent to the point $A$ is noted. Measurement of the free gas per unit volume of liquid at $A$ cannot refer to the reservoir as of production at $C$, for the quantities $v'$ and $v''$ are altered in value, maintaining a sum, however, equal to $v$. Again, if the fluids are subjected to a pressure below that of the atmosphere by being placed in a vaccum chamber, then the descent reaches some point $B$, corresponding to the amount of vacuum. Quantities $v'$ and $v''$ now possess still different values.

Equation 3, we will say, expresses Henry's Law in absolute phase, whereas Equation 8 expresses the law in potential phase. Equation 9 calls our attention to the fact that, if the measurement of the ratio between free gas and liquid is taken at some pressure other than the constant back pressure for production, such a measurement is not correct for the reservoir, unless adjusted to read "as of the constant back pressure" by means of the law in absolute phase. The quantity $v$, as stated previously, is not affected by subjecting the fluids to a pressure lower than the constant back pressure, but inasmuch as $v$ includes the gas which remains in solution, it is not the quantity usually referred to in our literature respecting the proportional production of gas.

I firmly believe that we may profit by properly interpreting and applying this law in our investigations upon combination wells in the field. No doubt the confusion which has existed heretofore is due to the fact that *both the absolute and potential axes of pressure are involved in problems that concern gas-oil ratios.*

**40.** *Reservoirs of liquid alone.*—An important mathematical feature of any solution tank is the area of the free surface at the top of the liquid. If the tank is kept full, or if the liquid is maintained at any constant level during production, it is obvious that the area of the free surface remains constant, irrespective of the form of the tank. But where the surface alters in position, either in filling or in producing, only the tank with the proper geometrical form will provide a constant area of free surface. Many of such forms may be readily imagined, although the most familiar ones are of a rectangular or circular base and vertical sides. *To insure a constant area the base may in fact be of any shape, geometrically regular or irregular, and the sides need only be generated by a straight line held in one direction, while being passed around the edge of the base.* Thus the sides are not necessarily vertical.

The type solution tank selected to represent a reservoir in Volumetric Control (section 5) is conveniently of the simplest design. Its sides are vertical, and its orifice is at the level of the bottom of the tank. Figure 17 shows a vertical section of such a tank, which, if desired, may be imagined to have a circular base. $A$ is the area of the surface, $y$ is the position of $A$ at any instant, furnishing a head $h$ at the orifice $O$. The levels $a$ and $b$ denote two definite values of $y$ which furnish two heads, $h_1$ and $h_2$, respectively. If we say that $P$ is the pressure at $O$ in pounds per square inch, due to $h$, then

$$P_1 = kh_1 \quad \text{(12)}$$

and

$$P_2 = kh_2 \quad \text{(13)}$$

or in general,

$$P = kh \quad \text{(14)}$$

Since $A$ is constant,

$$A \times h_1 = Vo_1 \quad \text{(15)}$$

and

$$A \times h_2 = Vo_2 \quad \text{(16)}$$

or in general,

$$A \times h = Vo \quad \text{(17)}$$

Solve for $h$ in this equation and substitute the value into Equation 14:

$$P = \frac{k}{A} Vo \quad \text{(18)}$$

and replace the constant $k/A$ by $K$, thereby obtaining

$$P = KVo \quad \text{(19)}$$

This equation shows the relation between potential pressure and potential volume in the solution tank of Volumetric Control. It is identical with the equation between the same functions in the rigidly constructed gas tank, as shown by Boyle's Law in potential phase (section 38), and its curve is the inclined straight line as shown in Figure 14.

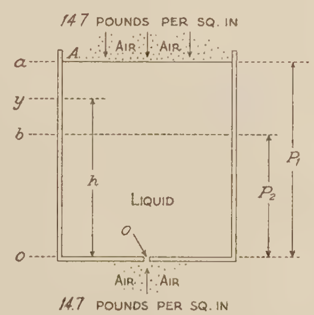


FIG. 17

There are solution tanks of other forms which will require attention. Four of special design are familiar sights about industrial plants that treat with liquids, namely, the **V**-shaped tank, the pyramidal or conical tank, the cylindrical tank with its axis horizontal, and the spherical tank. It is obvious that the area of the free surface in these tanks is not constant as its level is caused to raise or lower in filling or producing. These reservoirs, together with all other forms

of artificial and natural ones which present a variable area of free surface, are to be grouped as a class, and designated as reservoirs in Sub-Volumetric Control. Equation 19 does not hold for the relation between the potential functions of pressure and volume when these reservoirs contain liquid.

41. *Pressure, volume, and energy.*—We may say that pressure and volume, as functions of a reservoir, are independent of time. This merely means that in the equation, say for Volumetric Control,

$$P = KVo \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

not one of the three quantities—two variables and one constant—require a consideration of time in their measurement, or in their definition. A reservoir may be permitted to produce with an orifice of any size, and in any physical condition, either continually the same or altered ever so erratically in the life of the reservoir, and the relations between these functions are always normal; that is, they show no erratic features due to the state, or a change in the state, of the orifice. We can approach any artificial, and entirely accessible, reservoir at any time we choose, measure the potential pressure and the potential volume, and ignore all facts concerning the regularity or irregularity in the rate of production in connection with these measurements. And if the reservoir is not entirely accessible, as are some artificial and all natural ones, we need only measure for the following equations to suit Equation 20:

$$\frac{P_1}{P_2} = \frac{Vo_1}{Vo_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

and

$$Vo' = Vo_1 - Vo_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

wherein $P_1$ and $P_2$ are two measurements for potential pressure at two instants, $T_1$ and $T_2$, during the course of production.

$Vo_1$ and $Vo_2$ are the potential volumes at the same two instants, respectively. These are not directly measurable, since the reservoir is not entirely accessible. They are as yet unknown quantities.

$Vo'$ is the volume of fluid which has been withdrawn from the reservoir in the interval between $T_1$ and $T_2$.

Now the two equations preceding contain two unknown quantities, as noted; therefore they are necessary and sufficient for the determination of $Vo_1$ at $T_1$, and $Vo_2$ at $T_2$. In this operation $T_1$ and $T_2$ serve only incidentally in the way of calendar dates, and it is immaterial for a given reservoir whether, in the present point of view, the determined values of $Vo_1$ and $Vo_2$ pertain to an interval of a day, a month, a year, or of any other length of time.

While this discussion relates specifically to Volumetric Control, the same principle is indeed true for reservoirs of all controls.

Since $P$ and $Vo$ are independent of time in the manner described, then their product $E$ must be likewise independent of time, for no contingency

resting upon time is introduced by multiplying them. Now it is evident that the three following relations,

$$P = KVo \quad [20]$$

$$P = KE \quad (23)$$

and

$$Vo = KE \quad (24)$$

alone of all the derived primary function relations listed in section 9, are independent of time. Others in the list are involved with time in the fact that one or both of the functions require a mention of time as a matter of definition. Each control has its corresponding equations in these functions, and upon these we may most advantageously base our computations concerning the future performance of a reservoir, in so far as the recoverable amounts of oil and gas, singly or in combination, are to be learned in advance.

42. *Torricelli's Theorem.*—All solution tanks show a definite relation between pressure and velocity of flow from the orifice at all instants during the life of the reservoir. This relation is known as Torricelli's Theorem, and it may be expressed as follows:

*From a small orifice in the side of a vessel containing liquid, the theoretic velocity of flow is the same as that acquired by a body falling freely from rest, through a vertical distance equal to the head of the liquid on the orifice.*

This proposition states[4] that

$$v = \sqrt{2gh} \quad (25)$$

where $v$ is the lineal velocity of the liquid at the orifice, as at $O$ in Figure 18, say in feet per second; $g$ is the value of acceleration due to gravity, 32.17 feet per second per second; and $h$ is the head of the liquid upon $O$, expressed in feet.

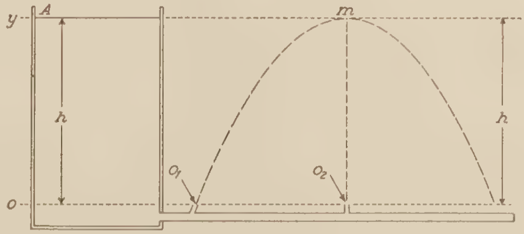


Fig. 18

The exact fulfillment of this equation in practice would require the following conditions:

*a*) No resistance offered by the air against flow.

[4] For the falling body we have

$$h = \tfrac{1}{2}gt^2 \quad (i)$$

and

$$v = gt \quad (ii)$$

where $h$ takes the place of the usual symbol $s$, the distance through which the body falls, and $t$ is time in seconds. $v$ and $g$ are as described above.

From (i) we see that

$$t = \sqrt{\frac{2h}{g}} \quad (iii)$$

Now this value of $t$ when substituted into (ii) gives

$$v = \sqrt{2gh} \quad [25]$$

*b*) No friction at the orifice, either between the molecules of liquid and those of the solid matter, or between the molecules of liquid themselves.

These conditions are such that no mechanical energy can be converted into heat, for any amount of heat produced would mean a reduction in the amount of energy possessed by the liquid by virtue of its position or its velocity.

Inasmuch as the production of heat cannot be entirely avoided in an actual experiment, although it can be reduced to a very small amount, computations requiring precision may be based upon a modified equation which contains a constant $c$ determined by experiment. Thus

$$v = c\sqrt{2gh} \qquad (26)$$

is the form usually employed in engineering practice.

If the vessel in Figure 18 were broad and shallow, or if it contain a porous medium, such as sand, the head $h$ will most likely not be the same during flow as it is when the orifice is closed in. Under these circumstances either we may use a different value for $c$, or we may simply introduce another constant $c'$ into the equation, one also to be determined by experiment with the given vessel. In the latter we would have the following equation:

$$v = cc'\sqrt{2gh} \qquad (27)$$

A hollow vessel containing a viscous liquid may be treated likewise.

Aside from experience there is no fundamentally logical reason for this relation between pressure and velocity—a fact which has long been recognized by scientists. We say that the equation is true, simply because experiment shows it to be so. Nor is the relation confined to liquids, for we find that gases obey it as well. Do we not employ this relation in the measurement of gas by pitot tubes and flow-meters? If we are told that the relation as stated by the equation may be proved by Bernoulli's Theorem, or general equation of energy for steady flow, we might ask if there is a fundamentally logical reason for this very equation, and we would be forced to admit that there is none. Like Torricelli's Theorem it is based upon experience. Consequently we can only cite the derivation of Torricelli's Theorem from Bernoulli's Theorem as evidence of consistency in our system of fluid mechanics.

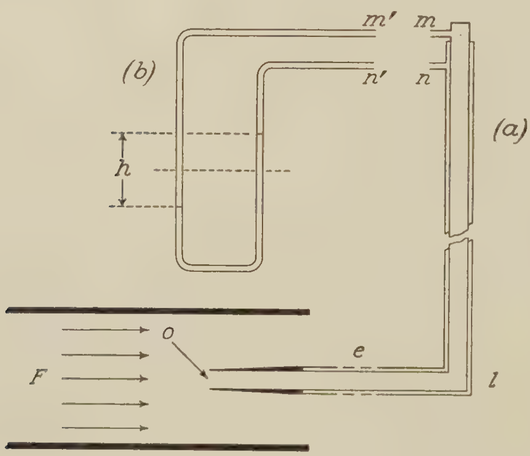


Fig. 19

The pitot tube is a simple apparatus whose operation is based upon Torricelli's Theorem. Figure 19 (*a*) shows such a tube consisting of two concentric

tubes properly formed into an orifice piece at *O*, and turned through a right angle at *l*. The apparatus is pointed toward the moving fluid which comes to a complete stop immediately in front of the orifice. The velocity head is thus converted into an equivalent pressure head that is transmitted to *m*. Perforations at *e* surrounding the tube transmit any static pressure head which the fluid may possess in addition to the velocity head, and this pressure is transmitted to *n* along the chamber between the two tubes. Figure 19 (*b*) represents a manometer tube which may be attached to the pitot tube to register the differential between the two heads. Now the velocity of motion may be computed by Equations 25, 26, or 27, as necessary.

43. *Bernoulli's Theorem.*—An important equation, often called Bernoulli's Theorem, is obtained by applying the theory of energy to a portion of a steadily flowing stream. This portion is imagined to have boundaries at plane sections across the line of flow, one an upstream section as at *A*, in Figure 20, and the other a downstream section as at *B*. If at either of the sections,

$W =$ the weight of the fluid passing per unit of time,
$z =$ the height of the center of the section above a datum plane $X$,
$v =$ the lineal velocity of flow,
$g =$ the acceleration due to gravity,
$p =$ the intensity of pressure at the center of the section, and
$w =$ the density, or the weight of a unit volume, of the fluid,

then the energy[5] passing the section per unit of time is

$$E = W\left(z + \frac{v^2}{2g} + \frac{p}{w}\right) \qquad (28)$$

If we ignore the possibility that some of the mechanical energy within the portion of the stream bounded by the sections *A* and *B* may be converted into heat, we may say that *E* has the same value at *A* as at *B*.[6] If any turbulent flow between the sections is sufficiently minimized, and if the viscosity of the fluid is low, as that of air, or even water, no appreciable

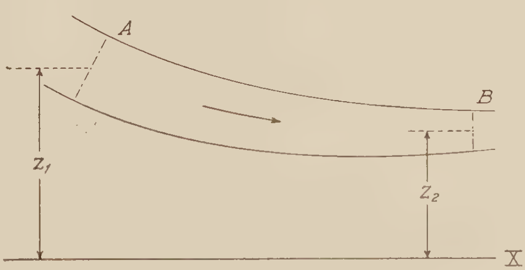

FIG. 20

[5] The added quantity *H* or *Q* for heat, usually appearing on the right-hand side of the equation, is omitted, inasmuch as we here consider only mechanical energy.

[6] It is immaterial whether the fluid is a liquid or a gas. In the latter case the specified "steadily flowing stream" between the sections obviates changes in density due to pulsating flow. It is clear that with pulsating flow there would be alternating accumulations and depletions of energy between the sections, thus unequalizing the amounts of energy passing the sections at given instants.

error is introduced into our computations by ignoring the heat produced within the portion of the fluid.

The three terms of the equation may be described as follows:

$z$ is the elevation head, and $Wz$ is the energy of position, possessed by the fluid in virtue of the fact that its mass is elevated above the datum plane. This is the physicist's *potential energy.*

$v^2/2g$ is the velocity head, and $Wv^2/2g$ is the energy possessed by the fluid in virtue of its velocity. This is the physicist's *kinetic energy.*

$p/w$ is the pressure head, and $Wp/w$ is the energy possessed by the fluid in virtue of its being compressed. This head is generally equivalent to the weight of a definite mass of elevated fluid; therefore the energy due to it is included in the physicist's potential energy.

*In our analysis it seems preferable to retain the three forms of mechanical energy in their individual identity as energy due to elevation, energy due to velocity, and energy due to pressure, respectively.*

If we apply a system of units to the terms above, for example, the foot-pound-second system, it is clear that $z$ is expressed in feet. The second term reduces to

$$\frac{v^2}{2g} = \frac{\dfrac{(\text{feet})^2}{(\text{seconds})^2}}{\dfrac{(\text{feet})}{(\text{seconds})^2}} = \text{feet} \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

and the third term reduces to

$$\frac{p}{w} = \frac{\dfrac{(\text{pounds})}{(\text{feet})^2}}{\dfrac{(\text{pounds})}{(\text{feet})^3}} = \text{feet} \ldots\ldots\ldots\ldots\ldots\ldots (30)$$

herein displaying the fact that *all three terms are co-dimensional in feet.*

The principle of Bernoulli's Theorem is important in connection with flow-meters. Diagrammatic sketches of the venturi and orifice types of these meters are shown in Figure 21 ($a$) and ($b$), respectively. The datum plane is conveniently passed through their centers, thus reducing the term $z$ to zero. The general equation is now

$$E = W\left(\frac{v^2}{2g} + \frac{p}{w}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

$A$ and $B$ represent the sections of the stream, as before. Since both types operate upon the same analytical principle, they must be similarly equipped with a differential gauge or manometer to register the difference in velocity head at the sections, and a gauge or manometer to indicate the static pressure head.

In consideration of the fact that the two heads in this equation are co-dimensional in feet as shown above, and that therefore either one of them

may be converted into the other—in so far as the operation of a flow-meter and our computations upon it are concerned—we may replace the term $p/w$ by its equivalent $h$ in feet, and write

$$\frac{v^2}{2g} = h \quad \ldots\ldots (32)$$

or

$$v = \sqrt{2gh} \quad \ldots\ldots (33)$$

an equation which will be recognized as Torricelli's Theorem. This is the

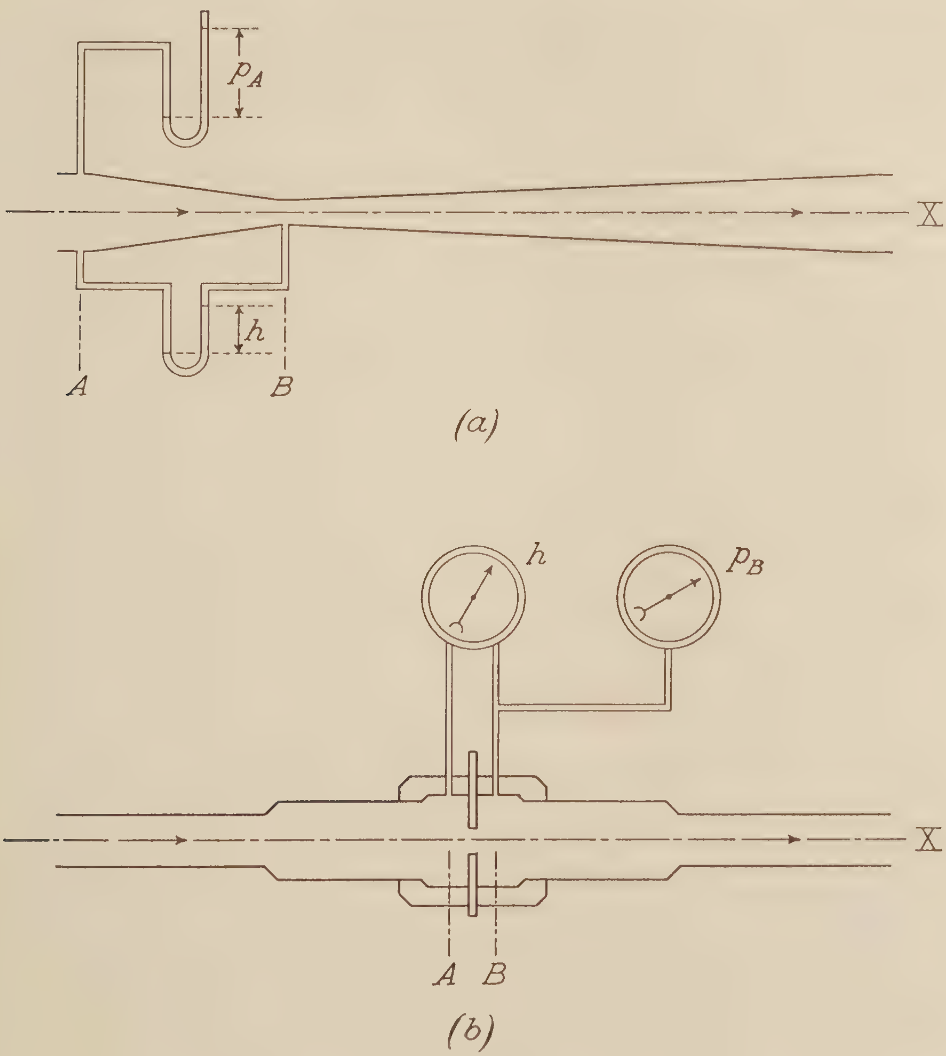


Fig. 21

derivation of the one theorem from the other, as mentioned in the preceding section.

[ § 43

**44.** *Time required to empty a vessel.*—There is a definite relation between the pressure which the fluid bears upon the orifice at any instant[7] and the time required to empty the vessel which contains it. Let us consider the vessel shown in Figure 22. It contains a liquid of low viscosity, and its form will be regarded as unspecified at first. At some instant the height of the liquid stands at $H_1$ above the level $X$ of the orifice. While production take place, the level of the free surface lowers, and its area $A$ varies, but the area of the orifice retains a constant value $a$. For any value of $A$ the height of the liquid is, we will say, $y$, and the time required for the level to lower by an amount $dy$ we will specify as $dt$. During this time $dt$ the quantity of liquid passing through the orifice is

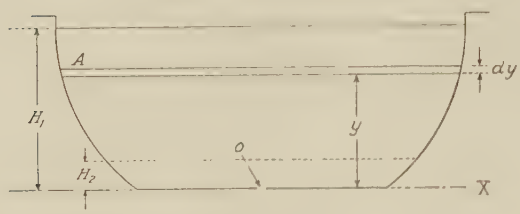

FIG. 22

$$Ady \qquad (34)$$

but the rate of flow at the instant $t$ corresponding to the pressure $y$ is, from Equation 25 (p. 67),

$$a\sqrt{2gy} \qquad (35)$$

Consequently the volume displaced in the time $dt$ is

$$a\sqrt{2gy}\,dt \qquad (36)$$

Now we may equate the expressions 34 and 36 thus:

$$Ady = a\sqrt{2gy}\,dt \qquad (37)$$

and solve for $dt$ to obtain

$$dt = \frac{Ady}{a\sqrt{2gy}} \qquad (38)$$

a differential equation between $dt$ and $dy$ in the proper form for integration in the general case. In order to integrate this expression for any particular case it is essential that the value of $A$, if variable, be expressed in terms of $y$. In any vessel of geometric form this can be done very easily.

[7] Specifically, we mean the "closed-in" pressure upon the orifice at the instant. Authors of textbooks on hydraulics seem to consider it unnecessary to mention this fact in particular, since the visible "head" of liquid at the orifice can obviously mean no other pressure than that measured by closing the orifice.

Let us consider the simplest case where $A$ is constant, as, for example, in the solution tank of Volumetric Control.[8] Now

$$dt = \frac{A}{a\sqrt{2g}} y^{-\frac{1}{2}}\, dy \quad\ldots\ldots\ldots\ldots (39)$$

and by integration,

$$t = \frac{2A}{a\sqrt{2g}} y^{\frac{1}{2}} + C \quad\ldots\ldots\ldots\ldots (40)$$

in which $C$ is the constant of integration. If we say that $t = 0$, when $y = 0$, then the value of C is zero. In other words, it will evidently be convenient to measure $t$ as time remaining, particularly in the case where production is allowed to take place from a head $H_1$ to a head $H_2$ equal to zero. Thus $t$ will express the time required to empty a vessel, and since we may wish to know the length of this time at any or all instants during production, we shall prefer to speak in terms of time remaining, rather than in terms of time elapsed.

Equation 40 gives a theoretic value which may not be sufficiently precise in engineering practice, on account of the loss in mechanical energy due to friction or resistance to flow. We may therefore introduce the constant $c$ or $cc'$ of Equations 26 and 27 (p. 68), and thereby obtain either

$$t = \frac{2cA}{a\sqrt{2g}} y^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots (41)$$

or

$$t = \frac{2cc'A}{a\sqrt{2g}} y^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots (42)$$

in accordance with our desire.

Since abscissas are so frequently expressed by the variable $x$, let us replace $t$ by $x$ in these equations. Furthermore, the fractional coefficient may be replaced by, say $k'$; and thus we obtain

$$x = k'y^{\frac{1}{2}} \quad\ldots\ldots\ldots\ldots (43)$$

which may be squared and reversed in order as follows:

$$y = kx^2 \quad\ldots\ldots\ldots\ldots (44)$$

This is the equation of the parabola, and its curve is shown in Figure 23 (p. 74). *The path from* $H_1$ *to zero is that traveled by the pressure on the orifice, with respect to time, during the course of production, uninterrupted and uninterfered with,*[9] *from the solution tank in Volumetric Control.* The full curve for the equation extends into the right-hand portion of the plat, where

[8] The sides of this tank are vertical, and the base is either circular or rectangular. In chapter xxii we shall deal with solution tanks in Sub-Volumetric Control, wherein the free surface of the liquid is not constant, but variable. See § 40.

[9] That is, where the performance of the reservoir is ideal.

we shall say that it is imaginary, in so far as any significance it can have with respect to production. It is to be noted that the axis of the parabola is vertical, and that its vertex is at zero. Whenever $y$ is expressed in terms of any power of $x$ greater than unity, the attitude of the curve is as here shown.

There is nothing to prevent our neglecting to place the axis of the curve upon the plat and using a scaled vertical line through the point $H_1$. We might also scale our horizontal axis from left to right in time elapsed, *but these acts on our part do not liberate us from the fact that the geometric axis of the curve is at the right, and that time remaining is the true function of delivery.*[10]

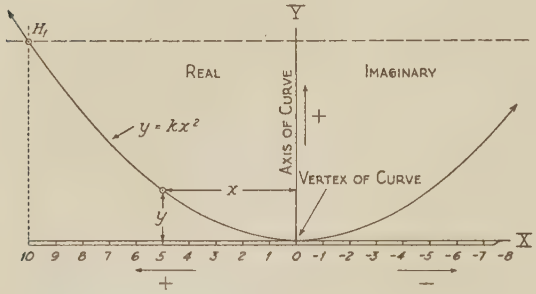


FIG. 23

Having satisfied ourselves that the pressure-time curve for delivery from the type solution tank is parabolic, let us now consider the type gas tank. We shall say that $A$ represents the area of the entire metal surface within the tank, an area which is a free surface between the fluid and the tank during the process of production. We are to imagine a shell of the fluid, $A$ in area, and of some small unit thickness, such a shell obviously being located in direct contact with the metal surface. For a decline of $dy$ in the pressure $y$ an amount of gas equal to

$$Ady \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [34]$$

escapes from the shell. Furthermore, the volume which thus escapes is also

$$a\sqrt{2gy}\,dt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [36]$$

and it thus becomes clear that we may continue with Equations 37, 38, 39, and so on, as above, and come to the conclusion that Figure 23 also represents the path of decline in pressure upon the delivery of gas from a tank of any form whatever.[11]

*The curve of Figure 23 may be easily verified by test with the two tanks of Volumetric Control in the laboratory.*

45. *Liquids compared with gases.*—The above-mentioned functions of pressure, velocity, and time are mathematically analogous to our potential functions. There is merely a difference with respect to velocity, which is

[10] See § 17.

[11] In a solution tank the "head" of liquid at the orifice is the potential pressure of the reservoir. In a gas tank it is likewise essential that we deal with the potential pressure. No pressure other than this one, in either tank, will satisfy the given equations.

lineal. In our system of mechanics Equations 26 and 27 (p. 68) may be written with the symbols mentioned in section 18, thus:

$$Vc = KP^{1/2} \quad \text{(45)}$$

where the value of $K$ is obvious, and Equations 41 and 42 (p. 73) may be written thus:

$$T = KP^{1/2} \quad \text{(46)}$$

or preferably,

$$P = KT^2 \quad \text{(47)}$$

The $K$'s are again obvious. Equation 45 is, of course, merely our expression of Torricelli's Theorem in its application to the reservoirs which we have considered.

The fact that the mechanics of liquids and the mechanics of gases are identical, with reservations, has long been recognized by the physicist. What are these reservations? I believe the situation has been admirably stated by W. N. Bond[12] as follows: *"Whenever surface tension, compressibility, and the mean free path of the molecules need not be considered, the flow of a gas corresponds exactly to the flow of liquid through the same system."* It must be admitted that the three qualifying conditions quoted must be satisfied in our present system of mechanics, if we are to say that in the production of oil and gas from natural reservoirs the mechanics of the two fluids are identical. To satisfy this statement of the situation we shall dispose of the conditions in the following manner:

*a*) The effects of *surface tension,* or *capillary resistance,* when subordinate, have no influence upon the fundamental and derived primary function curves of delivery from the reservoir.[13] When they are predominant, we will build up a system of mechanics which will care for them. In other words, we will divide and classify all reservoirs into Hydraulic and Volumetric controls on the one hand, wherein the effects of surface tension are subordinate, and into Capillary Control on the other, wherein these effects are predominant.

*b*) *Compressibility* has already been disposed of in virtue of the fact that we have defined our volume, velocity, and acceleration, not in linear units, but in mass units.[14]

*c*) *Molecular mechanics* are aside from our particular problems, for we deal with masses of fluids regardless of molecular activity. It is true that the study of the formal laws of physics require their consideration, but at each time we touch upon these laws we immediately advance into the domain of systematized molar mechanics.

[12] W. N. Bond, *An Introduction to Fluid Motion,* p. 10.

[13] See § 24.

[14] So long as we express quantities of gas in cubic feet at atmospheric pressure (and presumably at a standard temperature) we can ignore the actual space occupied by the gas under compression.

Since the classification and analyses of reservoirs is to be based upon the primary functions of performance, two further qualifying conditions must be considered, namely:

*d*) The *form and position of the reservoir* are ideal in a manner which is precisely defined. These matters need be considered only in the case of liquid in reservoirs of Volumetric Control. They are of particular importance in the study of artificial reservoirs, and they cannot be completely ignored in natural reservoirs. We are not concerned with them in the case of gas in all three controls, nor in the case of liquid in Hydraulic and Capillary controls.

*e*) The *system of mechanics* pertains alike to liquids and gases in reservoirs of the same control, when these reservoirs produce either fluid exclusively, and it pertains to the liquid alone, when both fluids are produced. In combination reservoirs—those which produce both liquid and gas—the mechanics of the liquid prevail, and the mechanics of the gas are modified in accordance with Henry's Law in potential phase. While the modification itself is independent of the control, the results differ in the three controls because of the fact that the basic principles of the mechanics of the liquid differ in them.

When we make the assertion that the mechanics of the two fluids are identical, we are to assume the foregoing qualifications, and we are to intend to refer specifically to the following important propositions:

i. The general principles of fluid statics and fluid kinetics

ii. The relations between pressure and volume in potential phase

iii. The relations between pressure and energy, and between volume and energy, in potential phase

iv. The relation between velocity and pressure in potential phase

v. The total mechanical energy as the sum of the energies due separately to the elevation, the velocity, and the pressure heads

vi. The relations between pressure and time in the process of production

vii. All other relations which follow directly from the preceding ones

CHAPTER VI

# Thermodynamics

"If we were acquainted with all the forces of Nature and knew what is the state of matter at a certain moment of time, we should be able to deduce by means of mechanics its state at every subsequent moment, and to deduce how the various natural phenomena follow and accompany each other. The highest goal the natural sciences must strive to attain is the realization of . . . . the reduction of all natural phenomena to mechanics. We shall never attain the goal of the natural sciences, but even the fact that it is recognized as such offers a certain satisfaction, and in approximating to it lies the highest pleasure to be derived from the study of natural phenomena."—GUSTAV KIRCHOFF

46. *Introduction.*—We may say in a word that a reservoir, whether artificial or natural, is a machine. It possesses accumulated energy which is capable of being converted into effective work, namely, that work which results in the delivery of the fluid from an orifice with which it is provided. This notion, it seems to me, is the broadest one we may conceive in regard to an oil or gas reservoir in particular; it therefore promises to furnish a most satisfactory basis for the ultimate analysis of well performance. I use the word "ultimate" advisedly, for I am thinking of the empirical data we are obtaining by experimentation upon reservoir performance today in our fields and laboratories—data which too frequently depend upon factors that are now little understood—and of our future investigations of the same sort. At some time I dare hope we can say that the subject of fluid mechanics as treated in three controls is well founded in theory, abundantly and accurately illustrated in practice, and thoroughly understood by those whose affair it is to know about wells. Then I believe we can say that we are actually prepared to delve more deeply into Nature's secrets, since we are truly required to do so in view of scientific and economic advantages to be gained thereby.

To say that a reservoir is a machine immediately suggests the science of thermodynamics as an aid to the understanding of the behavior of oil and gas wells. It is my intention to consider briefly the nature and scope of this science in the present chapters, to show its relation to fluid mechanics in general, and its relation to our system in particular.

First we may ask, what is thermodynamics? We are told by the physicist that it is a branch of physical science which treats of energy in its conversion from one form to another. Whereas the science of mechanics deals with

energy only in the mechanical forms which are appropriately represented by the three terms in Bernoulli's equation, thermodynamics has a broader scope, in that it includes energy in the form of heat, and therefore it involves the interconversion of mechanical and heat energy. The chemist, we find, goes one step farther. He tells us that we may include another form of energy, namely, chemical energy, and that therefore the science involves the interconversion of this energy and heat. As a step still farther, some may wish to include electrical energy, and say that the science involves the interconversion between this form and either mechanical, heat, or chemical energy, depending upon the nature of their problem. It is clear that the physicist's conception of thermodynamics is the one which is appropriate in our present analysis.

We cannot deny the existence of heat phenomena in the production of oil and gas. There is internal and external friction to be reckoned with; natural gases lower their temperature when they are permitted to expand, and consequently the potential volume of production from their reservoirs may be diminished in amount;[1] moisture at the bottom of a well freezes, paraffine precipitates, the viscosity of oil becomes altered, and so on. But these events are specific; let us consider generalities of analytical importance.

47. *Thermodynamics versus mechanics.—The science of thermodynamics permits the physicist to study energy changes within a system without requiring, in any way, a knowledge of the molecular mechanism of the process under investigation.* This is a very great advantage. It means that the conclusions of thermodynamics are quite general, and that they will remain equally correct regardless of any molecular hypothesis we may have formed in respect to the process. While our system of mechanics is equally independent of molecular mechanism, we must remember that its scope is limited to those energy changes which are not intrinsically of a molecular nature.

In thermodynamics a process may or may not involve an interconversion between mechanical and heat energy, that is, it may involve energy in one form only; it may or may not involve a change in the physical state of the fluid; and it may involve fluids of any degree of imperfection. All the circumstances which we might meet in actual fluid systems appear to be treated with equal facility, for thermodynamics offers a general method of resolving the various problems.

We should expect of course that the basic nature of problems in thermodynamics and mechanics requires a proper distinction between the circumstances which call for the methods of the one or of the other science. To illustrate the distinction we may refer once more to the problems of different

[1] The production certainly will be diminished in any reservoir of the closed type—where the source of energy lies in the gas pressure—provided there is insufficient time, or an insufficient source of heat, for the necessary transfer of heat to the interior of the reservoir through the walls of the container.

phase in connection with Boyle's Law. Inasmuch as the ordinates and abscissas in Figures 5 and 14 represent pressure and volume, respectively, it follows that the areas subtended by the curves, either in their entirety or in any part thereof, represent the product of these two quantities—that is, energy. Assuming the privilege of indulging in advantageous repetition at this point, I may arrange the features which pertain to the two figures in the following manner:

| FIGURE 5 | FIGURE 14 |
|---|---|
| *a*) Mass-volume is constant; space-volume is variable. | *a*) Space-volume is constant; mass-volume is variable. |
| *b*) Pressure is measured from absolute zero. | *b*) Pressure is measured from a potential zero whose position on the absolute scale is determined by the constant back pressure. |
| *c*) The curve is hyperbolic, with the origin at the left. (A rectangular hyperbola in this instance of isothermal expansion.) | *c*) The curve is parabolic, with the origin at the right. (An inclined straight line in this instance of Volumetric Control.) |

Now we may say that any specified area subtended by the curve in Figure 5 represents absolute energy, and any subtended in Figure 14 represents potential energy, both in accord with their previous definitions. While the first area by integration is a *logarithmic function* of absolute pressure and volume, the second area by the same process is a *parabolic function* of potential pressure and volume. The fact that problems in thermodynamics pertain to the situation in Figure 5, whereas problems in our system of mechanics pertain to that in Figure 14, cannot be too strongly emphasized. If we reckon with a change from left to right in the two figures, the subtended (entire or partial) areas represent energy withdrawn from the respective fluid systems. There is this fundamental difference to note: The thermodynamists are less interested in the value of the total absolute energy in the system, under specified conditions of pressure and volume, than they are in the difference between two values under different specified conditions of the same. The mechanists—we, in particular—are equally interested in the total potential energy in the system under any and all specified conditions of pressure and volume, and in the difference between two values under different specified conditions of the same. Our knowledge of past, present, and future performance of a reservoir may with advantage be made dependent upon both these situations with respect to the potential energy, whether we deal in ideal, theoretic, or actual performance.

**48.** *Thermodynamics versus mechanics* (*continued*).—The foregoing fundamental differences in the natures of the two sciences seem to call for distinct methods for treating their mutual sub-topics in computations. It at least

can be claimed that distinct methods have grown up with us, and no doubt they are to remain with us. Let us consider three of these sub-topics in particular:

*a*) Heat and temperature

*b*) Liquefaction, vaporization, and other molecular changes due to imperfections of fluids

*c*) Time

These we shall take up in the order given.

The four quantities in the characteristic equation of gases,

$$pv = RT$$

and a quantity $Q$, representing heat energy, form the basis of the derivation of equations and their practical application in thermodynamics. It has been said that $pv$, the mechanical energy of the system, may be converted into $Q$, and that $Q$, conversely, may be converted into $pv$. We may without hesitation attribute our progress in the generation of power by steam engines of the reciprocating and turbine types to the efficient methods made possible by an understanding of the relations between the five quantities. In the science of mechanics we may either assume the absence of the conversion of $pv$ into $Q$, or we may admit its presence and account for its effect in a manner dependent upon the nature of the problem. When we are concerned with the mechanics of a fluid mass, such as that undergoing a process of production from a reservoir, we should differentiate between reservoirs of the open and closed types, and recognize the fact that in the former the conversion has no effect of an analytical nature upon production, while in the latter it has an effect only of temporary duration. Rapid production here may alter the paths of the primary functions of performance toward the state of equilibrium, but given sufficient time, the same state of equilibrium will be reached regardless of the absence or presence of a conversion, inasmuch as there will be a compensating inflow or outflow of heat through the walls of the container. The length of time required will depend upon the conductivity of the material composing the walls. With this in mind preference was given to the assumption that the formations adjoining the reservoir medium are perfect conductors of heat in the ideal natural reservoir defined in section 22. *It is clear that this assumption more nearly approaches actual conditions, and it has the advantage of placing our computations for delivery from reservoirs of both the open and closed types upon the same basis.* Both types are to be found even among natural reservoirs.[2]

In both types of reservoirs our attention to internal and external friction

[2] I am not including here the thermodynamical problem of raising oil from the bottom of the well by means of a gas-lift or an air-lift process. This is a problem apart from the present one concerning the performance of the natural reservoir.

may be confined to these as forces,[3] and we may safely ignore the heat which they produce.

When we are concerned with the mechanics of a fluid in regard to lineal movement, generally the conversion of mechanical energy into heat is sufficiently slight to be satisfactorily treated by the introduction of empirically determined constants into the equations. We observed a provision for such a conversion in the equations of Torricelli's Theorem, discussed in section 42.

Temperature corrections which properly account for changes in the density of the fluid are, in mechanics, subject to the treatment outlined in section 34, Gay-Lussac's Law.

Thermodynamics treats with liquefaction and vaporization phenomena in a manner which may be adopted in mechanics, if desired. It will once more be necessary to transpose quantities in absolute phase to corresponding ones in potential phase. Frequently these phenomena may be ignored without introducing appreciable errors into the computations, but where they must receive consideration an allowance for their analytical effects can be made without difficulty. The effects of vapors will be discussed in sections to follow immediately.

A fertile ground for discoveries regarding the physical properties of specific fluids is offered by the science of thermodynamics. These concern molecular changes which are due to the imperfections of the fluid—changes that are usually not so evident as liquefaction and vaporization. In mechanics these imperfections are merely phenomena which are to be ignored, or to be properly taken into account through actual experiments in accordance with the methods outlined in sections 32 and 36, on Boyle's and Henry's Laws.

We have already noted that the mutual relations between pressure, volume, and energy are independent of time in potential phase, and it is clear that the same situation exists with respect to these relations in absolute phase. The physical thermodynamist is not often interested in time as a function of his process, but in case he is, we find him borrowing time-laws from the chemical thermodynamist; for example, Newton's law for cooling, or the mass-action law for the rate of progress in chemical reactions. As for the mechanist we have seen that the independence of the mutual relations between the three functions on the one hand, and time on the other, does not prohibit expressing the relation between pressure and time in ideal performance, as shown in section 44. In virtue of pressure-time relations and pressure-volume relations, volume-time and energy-time relations may be easily determined. This will be our method of treating time in the process of production.

[3] As forces the friction heads are of an intensive nature, and are therefore to be expressed in pounds per square inch, grams per square centimeter, and so on. Thus we shall speak of the internal friction head and the external friction head, quantities that are co-dimensional with the pressure of the fluid at the orifice. (Being co-dimensional with this pressure they may be mathematically subtracted from this pressure, wherever our computations necessitate our doing so.)

It is a historical fact that the science of mechanics of fluids has been built mainly upon the original investigations of Galileo, Guericke, Torricelli, Pascal, Boyle, and the Bernoullis—James, John, and Daniel, the latter being the author of the aforementioned theorem—and others which have been named in section 3. Their work soon brought to light the imperfection of actual gases and the general behavior of vapors. A struggle to evolve methods of satisfactorily treating with these fluids resulted in the establishment of the science of thermodynamics by such investigators as Carnot, Hermann von Helmholtz, and Clausius, who erected their structure largely upon the previous work of Dalton, Joule, and Sir William Thomson. The problems which these scientists encountered primarily concerned definite masses of gas and vapor as these were allowed to expand in one way or another. In other words, they dealt in absolute phase, and not in potential phase.

It may be said that nearly all our recent knowledge of gas behavior has come to us through the thermodynamist, rather than through the mechanist, and as a consequence we have a tendency to lean toward the methods of thermodynamics in any problem concerning the mechanics of gaseous fluids, including their production from artificial and natural reservoirs, thus leading ourselves astray through a lack of the proper conception of the potential phase wherein the mechanics of liquids and gases are identical.

With this account I believe I explain the fundamental causes for misconceived notions regarding the behavior of gas wells, and misconceived methods of computing their performance.[4]

49. *Laws of vaporization.*—I would not infer that thermodynamics offers to reveal little about the performance of reservoirs. Quite the opposite is true, but we must learn to distinguish between the problems of this science and those of mechanics in order that we may apply the proper principles at the proper time. And if we must study thermodynamics to some extent, so as to better understand the behavior of oil and gas wells, let us do so, but at the same time let us be on guard against possible confusion; where a transposition from absolute to potential phase is necessary, let us not fail.

By virtue of thermodynamics we know something of the behavior of vapors in closed containers which are subject to variation in pressure intensity. We are to consider briefly the following three cases: (*a*) a single vapor with its own liquid; (*b*) two or more vapors with their own liquids; and (*c*) a vapor in the presence of a permanent gas.

*a*) Suppose, as represented in Figure 24, that a few drops of water are placed in a cylindrical vessel, the space-volume of which may be varied at will by raising or lowering the piston. Manipulate this apparatus to the ex-

[4] I refer here to those notions and methods which lead to computations concerning the total amount of gas to be produced by the well, and the percentage recovery of gas from the productive formation, by means of pressures in the absolute phase.

clusion of air, and place the piston as at $A$. In a few moments the water will evaporate and entirely fill the space with aqueous vapor. Now this vapor will exert a certain pressure upon the walls of the vessel, and when the volume is altered by moving the piston to any position $B$ the product of the pressure and the volume will remain constant in accordance with Boyle's Law, provided the temperature is maintained at a constant value. This vapor is said to be in an unsaturated condition, and in such a state it behaves as a gas.

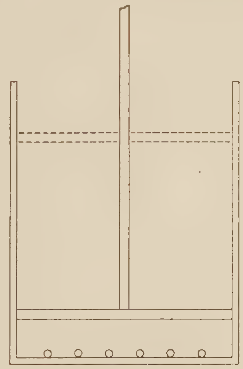

FIG. 24

If a comparatively large quantity of water is now placed in the vessel, as shown in Figure 25, only a part of it will evaporate. Alteration in the volume occupied by the vapor, say as the piston is moved from $A$ to $B$, will produce no change of pressure; the volume will determine only the comparative number of molecules in the one or the other state. An excess of molecules will leave or re-enter the liquid as the volume is increased or decreased, respectively. Under these conditions the vapor is said to be saturated, and in such a state it does not behave as a gas, for the pressure of a saturated vapor in the presence of its own liquid is independent of the volume occupied by the vapor. Pressure becomes dependent upon temperature alone. Whereas with gases and unsaturated vapors we are free to alter the pressure $p$ in the equation

$$pv = RT$$

by varying either $v$ or $T$, with saturated vapors $p$ may be altered only by varying $T$. For a given temperature, then, a saturated vapor exerts a definite pressure upon the walls of the vessel which confines it. This pressure is called the *vapor pressure*, or sometimes the *vapor tension*, of the fluid at the temperature. This pressure is the maximum pressure attainable at the temperature, while the vapor is in contact with its own liquid. To attain a greater pressure within the vessel all the vapor must be liquefied; that is, the space occupied by the vapor must be reduced to zero. Thereafter an increase in the pressure tends to compress the liquid which is alone present.

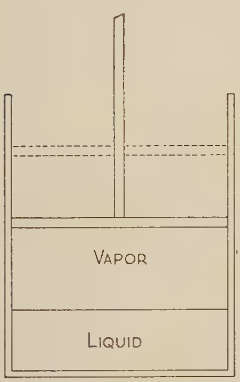

FIG. 25

The various vapors with which we are familiar possess different vapor pressures at the same temperature, and for every vapor there is a certain critical temperature above which no amount of pressure will force it to become liquefied. This is merely another way of saying that, at sufficiently high temperatures, all vapors behave as permanent gases (section 27).

Figure 26 represents a pressure-volume diagram for a vapor. The broken curve *DCBA* shows the isothermal expansion when the pressure and volume are altered. Taken in the reverse order, *ABCD,* the curve shows the isothermal compression of the same vapor at the same temperature. *AB* is a rectangular hyperbola for the unsaturated vapor which behaves as a permanent gas, from absolute zero pressure up to a definite value $n$ which represents the vapor pressure at the assumed temperature.[5] At *B* the curve breaks on the attempt to further increase the pressure by diminishing the space occupied by the vapor. Instead of continuing on the path of a permanent gas toward *B′*, the curve proceeds along the straight horizontal line to *C,* where all the vapor becomes liquefied. Here the curve breaks again, and the path extends toward *D*. *CD* is the rectangular hyperbola for the compression of the liquid which alone is present. On expansion a permanent liquid would possess, presumably, the curve *DCC′*.

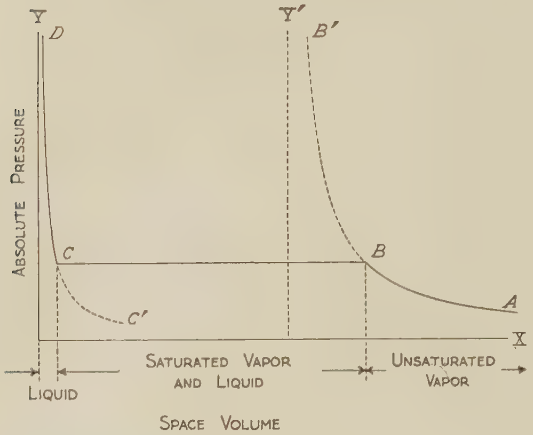


FIG. 26

*b*) In regard to the second case, where the vessel holds two or more vapors with their own liquids, Regnault found that the vapor pressure of the mixture is equal to the sum of the vapor pressures of the constituents, when these do not dissolve each other. On the other hand, when the liquids dissolve each other, the vapor pressure of the mixture is less than the sum of the individual vapor pressures of the constituents, and in some cases the vapor pressure may even be less than that of any of the constituents.

*No matter how complicated a chemical compound a given petroleum may be, that petroleum certainly possesses a definite vapor pressure,* and its value for the temperature known to exist within the natural reservoir can be determined by experiment in the laboratory. We may therefore treat petroleum as a homogeneous fluid in processes of production from natural reservoirs.

Where we deal with a natural reservoir which contains water and petroleum of various constituents, then the vapor pressure of the two is equal to the sum of their individual vapor pressures at the same temperature.[6]

[5] If $n$ is measured and expressed in reference to absolute zero pressure, its numerical value is the same as that of the point *B*.

[6] The liquefied vapors of petroleum dissolve one another, and these in turn are dissolved in the permanent liquid of petroleum. Water and these constituents of petroleum are, as we know, immiscible.

*c*) In the discussion concerning Dalton's Law, section 33, our attention was confined to the behavior of permanent gases only. We must now observe that this law applies to mixtures of permanent gas and vapor as well. A restatement of the law may be made as follows: *The total pressure exerted by a mixture of gas and vapor on the walls of a vessel is equal to the sum of the pressures which each would exert if present alone.*

Regnault verified Dalton's Law, and found it to apply to both saturated and unsaturated vapors. The pressure exerted by the vapor is the same whether the space through which it is distributed is otherwise empty or is occupied by one or more permanent gases. It is therefore clear that Regnault's experiments included the third case.

The following difference between the behavior of permanent gas and vapor is observed: Permanent gas is forced into solution by pressure in accord with Henry's Law, and the pressure required to force all of a given quantity of the gas into a liquid depends upon the solubility of the gas in the liquid at the existing temperature. Vapor is forced into solution, that is, it is liquefied, by pressure in accord with the laws of vaporization, and the pressure necessary to force all of a given quantity of vapor into the liquid form is the vapor pressure at the existing temperature, accompanied by a sufficient diminution of volume to accomplish complete liquefaction.

*If petroleum and its associated permanent gas and vapor are together in a reservoir, the critical pressure required to force all the permanent gas into solution is the same pressure required to accomplish the liquefaction of all the vapor, because there must be a diminution of volume sufficient for the latter.* So long as there is any permanent gas undissolved, some of the vapor molecules are disseminated throughout the space which this gas occupies, exerting its own vapor pressure in addition to the pressure of the gas. Not until all permanent gas is in solution is there sufficient diminution of volume to force all the vapor into the liquid state.

**50.** *Mechanics of vapors.*—From experience we can say that most of our natural combination reservoirs contain vapors and permanent gases in the presence of petroleum. What effect these vapors may have upon the primary functions of performance is a matter of concern to us. What is the nature of their effects, and how intense are they? Let us return once more to Boyle's Law.

The imaginary experiment described in section 32 was performed with a permanent gas, one assumed perfect for the purpose of demonstration. If into our container of Figure 4, already holding the permanent gas (air), there is introduced a vapor, the pressure corresponding to any volume of the gas will be increased throughout the experiment by the constant vapor pressure at the existing temperature. The isothermal line for the mixture bears a simple relation to that of the permanent gas alone.

In Figure 27 the curve $CD$, referred to the axes $X$ and $Y$, is the same curve as the one in Figure 5, the horizontal and vertical scales having now been altered for convenience. After adding the vapor to the gas within the apparatus,[7] a repetition of the previous experiment gives data which, when plotted, appears as the curve $AB$. This curve is the same as the curve $CD$, shifted vertically upward through a distance $a$ corresponding to the vapor pressure, in conformity with the straight horizontal line $BC$ of Figure 26.

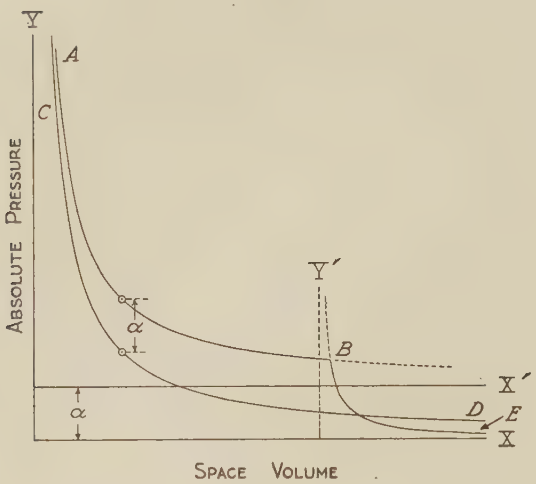


FIG. 27

The equation of the curve $CD$ with respect to $X$ and $Y$ is

$$pv = k$$

and this is likewise the equation of the curve $AB$ with respect to an axis $X'$ at a distance $a$ above $X$ with the same $Y$. *The expansion or compression of a mixture of permanent gas and vapor at constant temperature takes place according to the same mathematical equation with respect to a new definitely located axis for pressure as the equation of the permanent gas alone with respect to the original axis.* If we will take into account this shifting of the horizontal axis, we may say that Boyle's Law is unaffected by the presence of a vapor with the permanent gas.

The equation of the curve $AB$ with respect to the axes $X$ and $Y$ is

$$(p - a)v = k$$

and thus should it happen that we are unaware of the value of the vapor pressure in such an experiment with a mixture, we simply take two observations with the apparatus for the determination of the specific equation, instead of one observation as in the previous case.[8]

At some point $B$ on the curve the volume may have become sufficiently

[7] In such an experiment upon a mixture the volume of the condensed vapor is sufficiently small in comparison with the space within the apparatus so that its effect in diminishing the space occupied by the remaining gaseous fluid may be ignored without introducing an appreciable error.

[8] For the equation

$$pv = k$$

a single observation on $p$ and $v$ is sufficient to determine $k$; therefore it is sufficient to determine the specific equation for the apparatus. Now another quantity, $a$, is introduced into the equation; consequently two observations on $p$ and $v$ are required.

great so that the condensed vapor has entirely evaporated. The vapor upon further expansion is unsaturated, the point $B$ of Figure 26 is surpassed, and the curve breaks toward $E$. $BE$ is a rectangular hyperbola for the mixture which then behaves as a permanent gas.

The foregoing experiment is, of course, one in absolute phase. Let us now consider a corresponding experiment in potential phase. In Figure 28 the line $CD$ is identical with that of Figure 14 for permanent gas alone. By the introduction of the vapor this line is shifted vertically upward through a distance $\beta$ corresponding to the vapor pressure. The same situation holds with regard to the equations of these lines. $CD$ with respect to the axis $X$, and $AB$ with respect to an axis $X'$ at a distance $\beta$ above $X$, have the equation $P = KVo$, and $AB$ with respect to the axis $X$ has the equation $(P - \beta) = KVo$. Ordinarily we will not know the value of $\beta$ except by experiment under the conditions imposed upon production, since the position of the axis $X$ is only determined by the value of the constant back pressure. The introduction of the quantity $\beta$ requires one additional observation for the determination of the specific equation pertaining to the reservoir. As explained in section 41, the inaccessible reservoir requires two observations when vapor is not present; it is therefore clear that three are required when vapor is present.

If the value of the constant back pressure is greater than that of the vapor pressure, both measured on the absolute scale, then $\beta$ does not appear in our curve. But if this back pressure is less than the vapor pressure, the curve breaks at the point $B$, and proceeds toward some point $E$. $BE$ is a straight inclined line standing at a new angle.

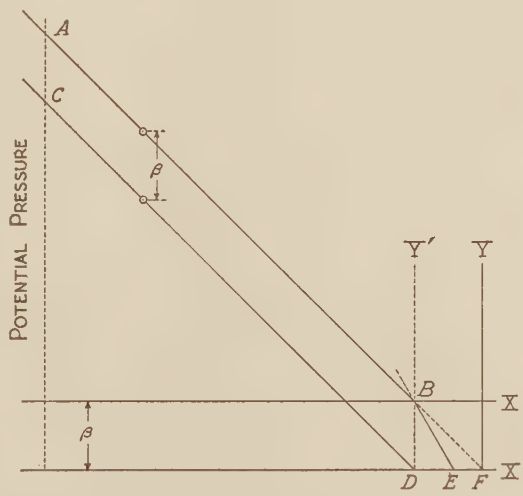


Fig. 28

The effects produced by vapor are not negligible, but their simplicity in view of our system of mechanics permits us to proceed onward without specifically mentioning them at every turn. *It is obvious that we cannot cite the presence of vapors as an explanation of any peculiar features in the performance of a combination oil and gas well, for the laws of delivery are not subject to any analytical disturbance by them.* All curves are the same with or without them; one additional observation

completely cares for any effects which they may have, and if the constant back pressure is sufficiently great, this one additional observation will show the effects nullified.[9]

[9] If we were further along in our analysis of production at this point, we might explain this fact by showing the axis $X$ to be necessarily above the axis $X'$ in Fig. 28, as determined by the value of the constant back pressure. Thus $B$ becomes "submerged," meaning simply that the constant back pressure against production, being in excess of the vapor pressure of the fluids, does not permit the existence of *unsaturated vapor* within the reservoir. The entire process of production takes place under the conditions of a *saturated vapor.*

## CHAPTER VII

# Thermodynamics *(Continued)*

"In the most general sense, thermodynamics is the science that deals with energy. Since all natural phenomena, all physical processes, involve manifestations of energy, it follows that thermodynamics is one of the most fundamental and far-reaching of the physical sciences."—G. A. Goodenough

**51.** *Effective and non-effective work.*—Admitting that a reservoir capable of producing fluid possesses energy or capacity to perform work, as stated in section 15, and that the reservoir in the nature of a machine does perform effective work which results in the delivery of the fluid, as stated in section 46, we are at once curious to know more than merely the source and nature of the energy accumulated within the reservoir. It would no doubt be advantageous to know something concerning the application and disposition of this energy in the process of production. The science of thermodynamics can enlighten us upon this subject, inasmuch as it is concerned with the conservation of energy, the reduction of energy to forms and conditions where it becomes unavailable for the performance of work, gas expansion when no external work is performed, and gas expansion when external work is performed.

There are, as we know, two fundamentally important formal laws of thermodynamics. The first one is a statement of the conservation of energy, as follows:

*If or whenever heat is converted into work of any kind, or work into heat, there is always a definite quantitative relationship between the heat which has disappeared as such and the work which has been done, or vice versa.*

It was upon the basis of this law that Joule performed his classical experiment which determined the mechanical equivalent of heat, a value subsequently corrected to

$$42{,}660 \text{ gram-centimeters} = 1 \text{ calorie}$$

This is expressed by saying that the work expended in raising a weight of one gram to a height of 42,660 centimeters will, if spent in friction, produce as much heat as will raise the temperature of one gram of water one degree Centigrade. This equivalent was used in computing the value of $R$ in the characteristic equation of gases, section 35.

Let us rewrite this law in terms which make it more directly applicable to the performance of reservoirs:

*If or whenever fluid is produced from one reservoir to another, there is always an identical quantitative relationship between the energy which has left one reservoir and that which has entered the other.*[1]

We may conceive the first reservoir to be the natural one holding oil and gas below the surface of the ground, and the second one to be all space exterior to the orifice of the first reservoir, or, specifically if desired, to be a properly constructed artificial container which is located at some definite place near the well. In view of this restatement we can appreciate the dissipated forms of energy listed in section 16. There is an exact equality between the amounts of energy before and after dissipation. Before dissipation this energy exists as "effective energy," or as that which has been given the name of "potential energy" in the present system of fluid mechanics. Its amount is equal to the product of potential pressure and potential volume, multiplied by an appropriate constant for each control.[2] After dissipation the same effective energy continues to exist as mechanical energy, diminished slightly in quantity by virtue of a conversion into heat. For reasons stated in sections 28 and 34 we are to consider the eventual performance of the natural reservoir as unaffected by this conversion.

Let us now consider the second law of thermodynamics. It is as follows:

*It is impossible for a self-acting machine working in a cycle, unaided by any external agency, to convey heat from a body at a low temperature to one at a higher temperature; or heat cannot of itself pass from a cold to a less cold body.*

By similarly rewriting this law we have:

*It is impossible for a self-acting reservoir working in a cycle, unaided by any external agency, to convey fluid from itself at a low pressure to another reservoir at a higher pressure; or fluid cannot of itself pass from a low-pressure to a high-pressure reservoir.*[3]

Now the first reservoir, if it has pressure and volume at all, certainly has mechanical energy; yet this energy is not available as effective energy in a process of production.

Of course we well know that fluid can of itself pass only from a high-pressure to a low-pressure reservoir. But this does not mean that all the mechanical energy which it possesses is necessarily effective in production. In fact we find that the pressure of the low-pressure reservoir determines a definite amount of non-effective energy in the high-pressure reservoir, accord-

[1] In thus rewriting the law it clearly becomes one of restricted application. It now applies only to reservoirs.

[2] See footnote 10, § 15, page 24.

[3] Again the rewritten law is clearly one of restricted application.

ing to the volume of fluid which it contains. We can easily learn the amount of this non-effective energy in any given reservoir system.

Let

$P =$ the difference between the high and low pressures. This is the potential pressure of the high-pressure reservoir.

$C =$ the pressure of the low-pressure reservoir, an amount that will here be assumed to be constant in order to simplify our procedure. This is therefore what has previously been designated as the constant back pressure. It is now directed against production from the high-pressure reservoir.

If we now say that $C$ is measured in terms of absolute pressure, we have

$$S = P + C \qquad (48)$$

where $S$ is the pressure within the high-pressure reservoir, expressed in terms of the absolute.

Furthermore, let

$V =$ the absolute volume of fluid within the high-pressure reservoir, according to the discussion in section 12.

Then we have

$$E_a = SV \qquad (49)$$

where

$E_a =$ the absolute mechanical energy within the reservoir, according to definition in section 15.

But by section 12 we can say that

$$V - Vo = vo \qquad (50)$$

where $V$ is the same absolute volume,

$Vo =$ the potential volume of fluid within the reservoir, and

$vo =$ the difference between the absolute and potential volumes.

By transposing terms in Equation 50 we obtain

$$V = Vo + vo \qquad (51)$$

On substituting the values of $S$ and $V$, as given in Equations 48 and 51, into Equation 49 we have

$$E_a = (P + C)\ (Vo + vo) \qquad (52)$$

This on expansion becomes

$$E_a = PVo + CVo + Pvo + Cvo \qquad (53)$$

The first term on the right represents potential energy, or effective energy in

the process of production from the high-pressure reservoir. The three last terms, taken together, represent the non-effective energy in the same reservoir. Of these three the first two, likewise taken together, are of particular interest to us, because of their relation to the proportional production of gas from a combination reservoir. The quantity of energy indicated by

$$CVo + Pvo$$

we shall call the *suppressed energy,* in view of the fact that, although it is withdrawn from the reservoir in the process of production, it is prevented from performing useful work in the process.[4] The third term, $Cvo$, is that energy which is retained by the reservoir in virtue of the constant back pressure.

52. *Joule's Law.*—The following law in thermodynamics, known as Joule's Law, has an important bearing upon the behavior of any reservoir of the closed type,[5] artificial or natural: *The intrinsic energy of a given mass of perfect gas depends upon its temperature only, and is independent of the volume which it occupies.*

An actual gas possesses intrinsic energy of two kinds:[6]

*a*) That due to the motion of its molecules. Its magnitude is reckoned as a function of the average velocity of all molecules, as they dart about, colliding with one another or with the walls of the containing vessel.

*b*) That due to the attractive and repulsive forces between molecules on account of their mass. Its magnitude is assumed to be a function of the average distance between the molecules.

By definition a perfect gas possesses molecules without mass; therefore it has no intrinsic energy of the second kind. In virtue of this ascribed property a perfect gas may comply not only with Boyle's Law, but with the present law as well, because no amount of pressure brought to bear upon such a gas will cause any forces which are due to the incompressibility of solid matter, or which are due to attraction or repulsion between masses, to come into action. At ordinary pressures the intrinsic energy possessed by an actual gas is due almost, but not quite, entirely to energy of the first kind. Apparently the molecules are sufficiently far apart so that the forces due to mass

[4] By "useful" I refer to the viewpoint of getting the fluid out of the reservoir. If we force the reservoir to deliver its fluid at an elevation higher than that of the orifice, we shall say that this energy of position is useless from the present viewpoint, admitting that from other viewpoints it may be useful. There is more to be said concerning this energy in subsequent sections.

[5] Here the source of energy is due to gas pressure. In reservoirs of the open type the facts stated by the law are of no importance, for in these the gas is merely an incident in the process of production.

[6] These are, of course, merely physical notions that are in accord with the kinetic theory of gases.

are very small, and generally negligible as compared with those causing or accompanying very rapid motion. For any given actual gas the preponderance of energy (*a*) over energy (*b*) may be increased by either raising the temperature of the gas or lowering the pressure to which it is subjected, and notably by doing both. The greater the preponderance the more nearly does the actual gas fulfill the two laws. On the other hand, high pressures and low temperatures cause a proportional increase in energy (*b*), and under these conditions the actual gas deviates widely from these laws. In fact, when these conditions are sufficiently extreme, the gas will break down completely and assume a liquid form.

The law was established experimentally by Joule with such an apparatus as shown in Figure 29. It consisted of two strong receivers, *A* and *B*, connected by a short tube fitted with a stopcock, all being immersed in a bath of water which was provided with a sensitive thermometer. Air in *A* was compressed to 22 atmospheres, while from *B* the air was exhausted. The water was constantly stirred, the temperature was noted, and the stopcock was then opened, allowing the air to rush from *A* to *B*. After equilibrium was established, the temperature was again read. No change was detected.

Under the conditions of the experiment no work external to the system composed of the two receivers was performed by the gas. Since the water remained at the same temperature, no heat passed from the air to the water, or vice versa. Obviously the intrinsic energy of the air was the same before and after the expansion. If the increase of volume had required the expenditure of internal work in forcing the molecules apart, that work would necessarily have been performed at the expense of the intrinsic energy due to the motion of the molecules, and as a result, the temperature would have been lowered. Inasmuch as the temperature remained constant, it is to be inferred that no such internal work was performed. Air has, therefore, no appreciable intrinsic energy due to forces between the molecular masses, at least none within the range of pressures and at the temperature of the experiment. The test was repeated on various other permanent gases with like result.

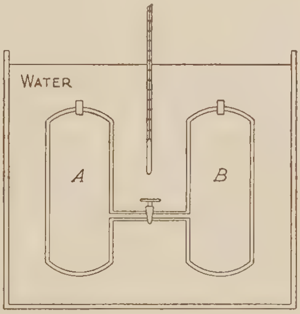

FIG. 29

The experiment was subsequently modified, and the two receivers were placed in separate vessels of water. Then the water surrounding *A* showed a lowering in temperature, while that surrounding *B* was raised by exactly the same amount. The arrangement which provided an entirely internal system in the previous case was in this one altered so as to furnish two systems, one internal and the other external. External work which *A* performed upon *B* cooled *A* and warmed *B* in equal amounts, and for this reason the total effect, when the receivers were in the same bath, was nil.

In the first experiment a definite amount of gas occupied a given space at a given pressure, and it was allowed to expand so as to occupy double the space at one-half the pressure, thus fulfilling the simple relation expressed by Boyle's Law, namely $pv = k$, where it is now evident that *the constant k actually represents units of intrinsic energy, unaltered when the gas expands without performing external work.*[7] The performance of external work does, as noted, lower the temperature of the gas, and thus, when testing Boyle's Law by experiment, if the gas happens to perform external work upon its expansion, a quantity of heat equivalent to the work performed must be added to the system in order that the temperature may be maintained constant, as the conditions of the law require. Conversely, if work is performed upon the gas for the purpose of compressing it, then an equivalent amount of heat must be withdrawn from the system in order that the temperature may be maintained constant.

It is important to note in connection with these experiments that it would be improper to speak of work which the gas performs upon itself. No such work is performed. *To be sure, immediately after flow begins, molecules of gas in A perform work upon molecules of the same kind of gas in B, but the molecules as they pass the orifice at the stopcock lose their identity as active agents, and become passive loads.* Although they are physically the same molecules before and after passing the orifice, they certainly differ in a dynamical sense. Obviously it is immaterial whether the receivers are in the same bath or in separate baths in this regard.

The foregoing experiments with the apparatus described possessed a weak point in that water was used to indicate a change of temperature in the gas. The gas might have suffered a slight change without revealing itself in the water, since the difference between the specific heats, or heat capacities, of the two fluids is extreme.[8] The best that could be said was that no heating or cooling effect of any considerable magnitude occurred. With the aid of Sir William Thomson means were devised so as to measure the temperature of the gas directly. The test with the apparatus which they employed has since been termed the "porous plug experiment." This experiment showed that the law is nearly true, but that for no actual gas is it perfectly so. The gases which are known to be more nearly perfect comply more closely with the law, and this fact has given rise to the assertion that the perfect gas would comply exactly with the law.

[7] Therefore in the reverse sense the expansion of gas, as a fact alone in itself, does not signify the performance of external work. *The drop in temperature accompanying expansion signifies the performance of external work.* It is essential that we fully appreciate the meaning of Joule's Law, as illustrated both by the present apparatus and by the porous-plug apparatus of the next section.

[8] The specific heat of a substance is defined as the quantity of heat in calories required to raise one gram of the substance one degree Centigrade.

53. *The porous plug experiment.*—The porous plug experiment may be performed with the apparatus shown in part in Figure 30. To avoid the production of eddy currents in the gas at the locus of expansion, a plug of cotton is held there by two metal disks pierced with many holes, as shown at $A$. This plug should be inclosed in a non-metallic cylinder $B$, which in its turn can be surrounded by cotton in the receptacle $C$, in order to avoid loss or gain of heat in the neighborhood of the plug. Gas is conveyed to the plug by means of a metal tube $D$ which forms the end of a long spiral tube which is submerged in a bath of water. A sensitive thermometer $T_1$ is placed so that its bulb is immediately above the plug, and so that the thermometer might be read conveniently the tube $E$ which leads the gas away should be of glass. $E$ is connected with a chamber in which a vacuum is maintained continuously; thus no external work is performed by the gas upon its expansion. The thermometer $T_2$ is immersed in the bath as shown. The gas to be tested may be compressed by a pump, and thereafter be forced to flow through the spiral tube, while the bath is constantly stirred and its temperature is maintained constant. The compression of the gas will cause its temperature to rise in accordance with the external work done upon it by the piston.

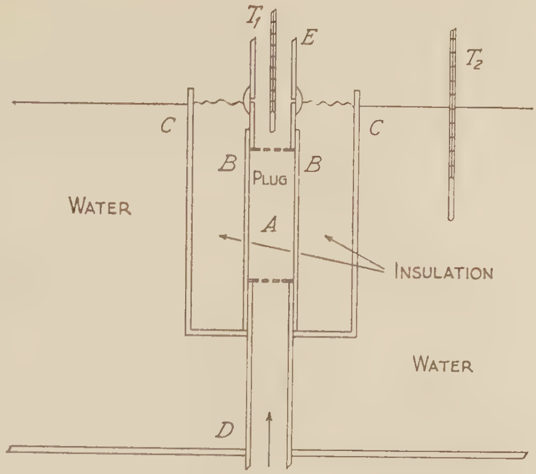

FIG. 30

All our well-known gases, with the notable exceptions of hydrogen and helium, lower their temperatures upon expansion, even though this expansion takes place without the performance of external work. The two exceptional gases actually show a slight rise in temperature when the experiment is made, as with other gases, at ordinary room temperatures. At lower temperatures, however, these behave as the other gases, and thus cool upon expansion. This thermal effect encountered in actual gases is called the "Joule-Thomson effect," and it is taken advantage of in the processes of liquefying them.

54. *Expansion within natural reservoirs.*—The situation in regard to natural reservoirs may be closely approximated with an apparatus as shown in Figure 31 (p. 96). This consists of a long metal tube $AB$, properly insulated in some manner so that no heat may pass through its walls. It is completely filled with a porous medium, say evenly packed sand, and it has an orifice at $O$. We

shall imagine that the tube is now filled with compressed gas at an absolute pressure of $S$ pounds per square inch, and for the present, in order that there will be no external work performed upon its expansion, we shall say that the orifice communicates with a vacuum chamber, as before. The porous sand acts in the manner of the cotton plug above. Let us first suppose that the gas is perfect. Now we may say that even the gas which travels the most extreme path on leaving the section $A$, entering the section $B$, arriving in front of $O$, and thereafter being produced into the vacuum chamber, does so without the loss of intrinsic energy, and without a change in temperature. A given mass of gas at $A$ decreases in pressure as it passes through $B$ toward $O$, but its volume increases at the same time, in accordance with the relation $pv = k$.

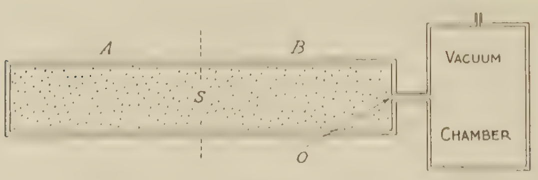

FIG. 31

Next let us suppose that the gas is a mixture taken directly from a producing well. We know that the so-called permanent constituents are somewhat imperfect, and that the vapors present are decidedly so. We shall now consider the heat phenomena under these conditions.

Heat is produced by friction. The molecules of gas collide more frequently, and they come into contact with the solid material of the porous medium. For any given state of the reservoir with respect to the number of gas molecules present, the operation is taking place at a higher temperature, and therefore the pressure $S$ is higher, than would otherwise be the case. We therefore observe that at least *some of the heat produced at the expense of mechanical energy is not lost as effective energy for production from this reservoir. Only that which remains with the solid materials of the apparatus after equilibrium is established, and that which escapes with the produced molecules, is lost as effective energy.*

The gas lowers in temperature through the Joule-Thomson effect. *The rise in temperature due to friction may be either partially or entirely offset, and even surpassed, so that the net effect is a reduction in temperature.* If we wish to measure this net effect with the apparatus described, we would naturally place the bulb of the thermometer at the locus of the greatest expansion; that is, immediately within the orifice $O$.

If our apparatus were supplied with a continuous inflow of the same kind of gas at the end opposite to that of the orifice, the net thermal effect, whether a heating or cooling, would be constant so long as $S$ is constant, but in the arrangement as shown, $S$ must decline according to production; consequently the net thermal effect is not constant.

By the Joule-Thomson effect sensible heat becomes latent heat. Both types of heat energy are invested in the molecules of gas, but whereas the former pertains to the speed of the molecules, the latter pertains to the

attractive and repulsive forces between the molecules.[9] In the form of latent heat the system loses effective energy, in that it is carried out of the reservoir by the molecules.

Now we arrive at the subject of insulation. Our apparatus is provided with walls which might be assumed to be perfectly insulated, but of course they are not exactly so. It would not be difficult, however, to provide walls which are more perfect non-conductors of heat than are the formations which adjoin productive natural reservoirs. Well, suppose the apparatus had been constructed with walls which were perfect conductors of heat, and that in contact with these walls there were an enormous heat reservoir capable of either furnishing or absorbing a necessary amount of heat for maintaining the apparatus at a constant temperature, such a heat reservoir being of sufficient size to experience no change of temperature in the process. Now any heat changes within the apparatus are not accompanied by changes in temperature, and thus the loss in effective energy is exactly compensated by the proper heat transfer through the walls. Again it is not possible to equip the apparatus so perfectly, but it would not be difficult to provide walls which are more perfect conductors of heat, and an exterior heat reservoir more efficient, than are those accompanying productive natural reservoirs.

The walls which adjoin productive natural reservoirs are imperfect conductors of heat; that is, they range somewhere between perfect conductors and perfect non-conductors. The thermodynamist tells us that if our artificial and natural reservoirs were provided with walls which are perfect conductors, and that if heat transference through them were provided for, then the expansion of gas within them would be *isothermal*. But if they were provided with walls which are perfect non-conductors, he tells us that the expansion of the gas within them would be *adiabatic*. And in either case the gas need not be the sole fluid present; it may be accompanied by a liquid, say oil or water, without affecting the nature of the expansion, in so far as the liquid is inert with respect to the expansion of the gas. Our actual natural reservoirs function somewhere between isothermal and adiabatic expansions.[10]

**55.** *Performance of external work.*—The heat phenomena which accompany the performance of external work produce effects that must be added to those of the preceding articles.

Whenever gas performs work upon an external load by expansion, it does so at the expense of its intrinsic energy. It accordingly must lose some of its heat, and thereby lower its temperature, for according to Joule's Law the change in volume and pressure alone does not affect its intrinsic energy. We know by experience that the temperature does lower upon the performance of such external work. This can be no less true for natural reservoirs than for

[9] At least we infer this to be the case in accordance with the kinetic theory of gases.

[10] The actual condition of expansion depends upon the rate at which the well produces.

artificial ones. If we allow a steam engine to run idle, the expanding steam loses little of its heat, but as soon as a load is attached, the steam loses considerable of its heat. The temperature change is little or great under the respective conditions.

We are perhaps more familiar with loads for steam engines than with loads for reservoirs. What might the nature of these loads be? The load consists of the back pressure exerted against production at the orifice.[11] If a tank produces into the atmosphere, the pressure of the atmosphere is a load for the tank to work against, and if the tank produces into a line which has a pressure greater than that of the atmosphere, the sum of the line pressure and the atmospheric pressure is a load for the tank to work against. With natural reservoirs the load may either be that offered by a gas-line or that offered by the weight of a column of liquid in the well. In each case the load of the atmosphere must be added to the measured amounts.[12] This type of load has been termed the "constant back pressure." In addition to this there is the load offered by the "frictional back pressure," or external friction head.

If our apparatus of Figure 31 were caused to produce against any pressure whatever, the external work might be performed isothermally, adiabatically, or intermediately between the two, depending upon the conductivity of the walls and the presence of the necessary exterior supply of heat. Figure 32 shows the pressure volume relations for the two extreme, perfect expansions. In the general equation of the hyperbola $x^n y = k$, $n$ is equal to unity for isothermal expansion, and it is arbitrarily taken at 1.5 for adiabatic expansion, as this is not an unreasonable number.[13] The actual expansion encountered in our natural reservoirs should then be represented by a curve which lies between these two. For convenience we may adopt the expression "polytropic expansion" to designate these intermediate stages.[14]

The question arises as to how much heat is lost by a gas in the performance of external work. Theoretically, though not actually, it is the same amount for all gases. The amount of heat required to produce a rise of temperature of one degree Centigrade in one gram-molecule of gas maintained

[11] See § 51.

[12] The load is a quantity that is intensive in its nature; that is, it is one measured in pounds per square inch. It is to be measured from absolute zero pressure; therefore the pressure of the atmosphere is to be added to the line pressure as recorded by the gauge, or to the weight of the column of liquid bearing on the orifice. (In the latter case it will be remembered that we shall consider the orifice of the natural reservoir to be at the bottom of the well. See § 20.)

[13] As a matter of fact $n$ in adiabatic expansion has a value equal to the ratio between the specific heat at constant pressure and the specific heat at constant volume. It is likewise equal to the ratio between quantities $C_p$ and $C_v$ as described in the next paragraph.

[14] The expression "polytropic expansion" is ordinarily used to designate all states of expansion in general, including isothermal and adiabatic expansions. I shall continue to use the expression in its restricted sense.

at a constant volume is called the *heat capacity at constant volume.*[15] In this case the amount of heat absorbed is equal to the increase in intrinsic energy, for when a system is heated at constant volume no external work is performed by it. On the other hand, the amount of heat required to produce the same rise of temperature in the same quantity of gas at a constant pressure is called the *heat capacity at constant pressure.* Here external work is performed by the gas. We may consider the gas to be heated at constant volume, thus providing an increase in pressure, then allow expansion against an external force to take place until the original pressure is restored, meanwhile adding heat to maintain the newly acquired temperature. Obviously the latter amount of heat is greater than the former, and if we may designate them $C_v$ and $C_p$, respectively, then it can be proved that $C_p - C_v = R$, where $R = 1.985$ calories, as given in section 35. In other words, if we assume this number to be correct in the actual case, every 22,390 cubic centimeters of gas within a reservoir may perform 84,700 gram-centimeters of external work and thereby lower its temperature by one degree Centigrade. These numbers, taken from the section referred to, may be reduced to foot-pounds and one degree Fahrenheit, if desired.[16]

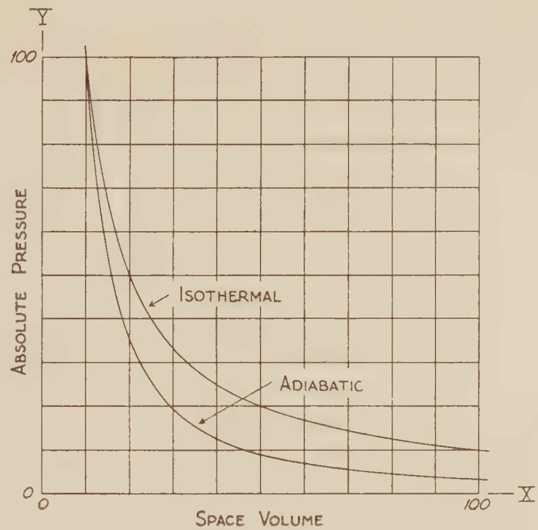


FIG. 32

The perfect conditions prescribed for isothermal expansion above provide for an instantaneous compensation of this heat expended in external work, and those prescribed for adiabatic expansion provide for the complete absence of compensation in all time. Given a sufficient time in polytropic expansion, there is bound to be a complete compensation. The sufficient time in any case depends upon the conductivity of the formations adjoining the productive reservoir, and also upon the previous rate of production from the reservoir and the intensity of the external load, inasmuch as these determine the

[15] "Specific heat" refers to a gram-mass of gas, while "heat capacity" refers to the gram-molecule of gas.

[16] For the conversion of units we have

$$777.5 \text{ foot-pounds} = 1 \text{ B.T.U.} = 252 \text{ calories}$$

The British Thermal Unit is the amount of heat required to raise one pound-mass of water one degree Fahrenheit.

temperature status of the reservoir at a given instant. It has so happened in practice that gas wells have been closed and abandoned because of the low pressure which they had attained. Upon subsequently returning to them their owners observed a favorably high pressure, and they drew the conclusion that the gas in the reservoir had in the meantime become replenished. In truth there had been no replenishment of gas, but merely a replenishment of heat within the reservoir.

Polytropic expansion has its effect upon the potential pressure-volume curve of Figure 14, and also upon all other primary function curves of performance. Under ordinary conditions of production the effect is slight, causing the curves to drop a little below their normal positions in ideal performance. *This expansion does not affect the eventual potential volume to be received from the reservoir, for the complete compensation in heat in sufficient time guarantees a volume as of isothermal expansion.*

The external work performed by a reservoir is non-effective from the point of view of production. We all know from practical experience that the best results are obtained from the well when the external load is reduced to the economic minimum. Thus we see pumps or lifts which take the load off the bottom of the well installed in our oil fields, and often the former are fitted so as to remove a part of the atmospheric pressure at the same time; that is, a vacuum is applied to the well.

**56.** *Adventitious events.*—All results of experiments with reservoirs are at least partially obscured by adventitious events. Every conjecture as to generalizations in performance, when based upon empirical data, is liable to error. This is particularly true with regard to natural reservoirs, which for the most part are inaccessible. Laboratory reservoirs herein present fewer difficulties, for they are subject to special precautions, the necessity or advantage of which becomes evident by direct vision of events during the experiment. Laboratory apparatus is most appropriate for the determination of the simplest laws which lie at the foundation of the science of fluid mechanics. Mathematical expressions for these laws should be immediately set up in the form of equations, and the manipulation of these equations should accompany observations upon the more complicated or the more obscure performances. Only in this manner may we best understand the behavior of reservoirs.

The adventitious events which we are most liable to encounter in experimentation with reservoirs of any kind are the following:

*a) The effects of turbulent flow, eddy currents, and atomization.* These are most prevalent in the vicinity of the orifice, and obviously they all consume mechanical energy. Where the reservoir is filled with a porous medium we do not expect turbulent flow in its extent, for the medium prevents it, just as the cotton plug does in the porous plug experiment. We shall regard the effects of turbulent flow, eddy currents, and atomization as adventitious

events. They are not entirely to be avoided in any case. They can be minimized, however, by the proper regulation of the velocity of flow.

b) *The effects of polytropic expansion.* Any cooling effect that is not entirely offset by the heat of friction within the porous mass is to be regarded as an adventitious event.[17] Furthermore, because actual gases are imperfect, there is a maximum velocity which they may attain in flowing through a given orifice. This maximum velocity varies for different gases, and it depends upon their specific heats or heat capacities at constant volume and constant pressure. This we shall also regard as an adventitious event. It is evidently thermodynamical in its nature. By the kinetic theory of gases it can be shown that the maximum velocity is equal to the velocity of sound in the gas, under the conditions of temperature, density, and pressure which exists at the orifice. In gas wells this velocity is seldom, if ever, attained; consequently we may here ignore it. However, it cannot be ignored in artificial reservoirs of gas that might be used for experimenting upon production, inasmuch as the maximum velocity is easily attained under the conditions of high pressures and small orifices.

**57.** *Summary and conclusions.*—When we undertake the study of fluid mechanics we immediately encounter the phenomena of thermodynamics, both in experimentation and in theory. Outside of the laboratory of physical chemistry we ordinarily do not think of principles of thermodynamics as applying to matter in the liquid state,[18] but in any engineering field in which we are concerned with the expansion of matter in the gaseous state these principles are quite universally recognized as entering into the solution of our problems. Wherever a reservoir, artificial or natural, produces gas, either alone or in combination with liquid, expansion of the gas is taking place. Shall we therefore solve our problems on the basis of principles in thermodynamics in case gas is being produced?

We are to remember that all problems in thermodynamics are such as to require treatment in what has been called here the absolute phase. In this phase the gas is subject to the following conditions: (*a*) its mass-volume is constant, while (*b*) its space-volume is variable. But these are not the conditions under which production takes place.[19] Production is in potential phase, wherein the gas is subject to the following conditions: (*a*) its space-volume is constant, while (*b*) its mass-volume is variable. The laws of thermodynamics, as we have them today, are not directly applicable to the

[17] It is to be regarded so at the given rate of production from the orifice. As previously noted, at a sufficiently slow rate of production the heat loss within the reservoir is compensated by a transfer of heat through the walls of the container.

[18] In physical chemistry certain principles of thermodynamics are applied to liquids and to gases in the same manner. These involve states of chemical equilibria.

[19] See § 47.

performance of a gas or combination reservoir. The amount of energy residing within the reservoir proper is varying as long as fluid is escaping from the orifice. When equilibrium is finally established in the process of production, the energy has become lowered in value to a datum plane—zero. Joule's Law constitutes an important principle in thermodynamics. Because of the phase involved its interpretation in terms of reservoir performance must be made with care.

As a matter of fact the mechanical aspect of energy differs from the thermodynamical aspect in matters other than in phase alone. In Figure 33 we see the two in contrast. At the right we have the mechanical aspect in accord with the argument of section 51. Certain horizons are denoted by the symbols $K$, $G$, $N$, and $I$, and to these we shall have occasion to refer at a later time. The spaces between these horizons represent definite portions of energy, all assumed to be mechanical, residing within a reservoir. *It is immaterial whether the fluid is a liquid or a gas, or both in combination.* No particular vertical scale is intended in the drawing.

At the left we have the thermodynamical aspect. The individual forms of energy retain their identity so long as they do not suffer a complete conversion to other forms. The alignment with the right-hand side is an approximation, except for the horizons $K$ and $I$ which apply in the same way to both aspects.

The following is a brief description of the separate energies appearing on this side of the figure:

*a*) This is the same as $PVo$ on the right, except in the fact that its amount is less, as occasioned by the conversion of some of the mechanical energy into sensitive heat on account of friction.
*b*) The total sensitive heat resulting from conversion.
*c*) The latent heat of vaporization, including the Joule-Thomson effect, and so on. We can imagine that changes here will cause (*b*) to shift upward or downward in its position, while both alter their vertical measurements.
*d*) The mechanical energy to be retained by the reservoir in virtue of the constant back pressure. This back pressure, as indicated by the horizon $N$, is also to retain sensitive and latent heats within the reservoir.
*e*) The sensitive and latent heats measured between absolute zero temperature and the temperature of the reservoir under normal conditions.
*f*) The mechanical energy of position measured between the center of the earth and the datum plane of the orifice of the reservoir.
*g*) Any and all other forms of energy, known or unknown, that may be required in completing the column.

The grouping of the various items is, I believe, self-explanatory in the light of that which has been said before.

*The divisions of energy on the left pertain to gases of any degree of perfection or imperfection, as these are permitted to expand under conditions whereby external work is performed.* They do not pertain to liquids, either alone or in combination with gases.

We may say that the mechanical aspect of energy is the one of greater

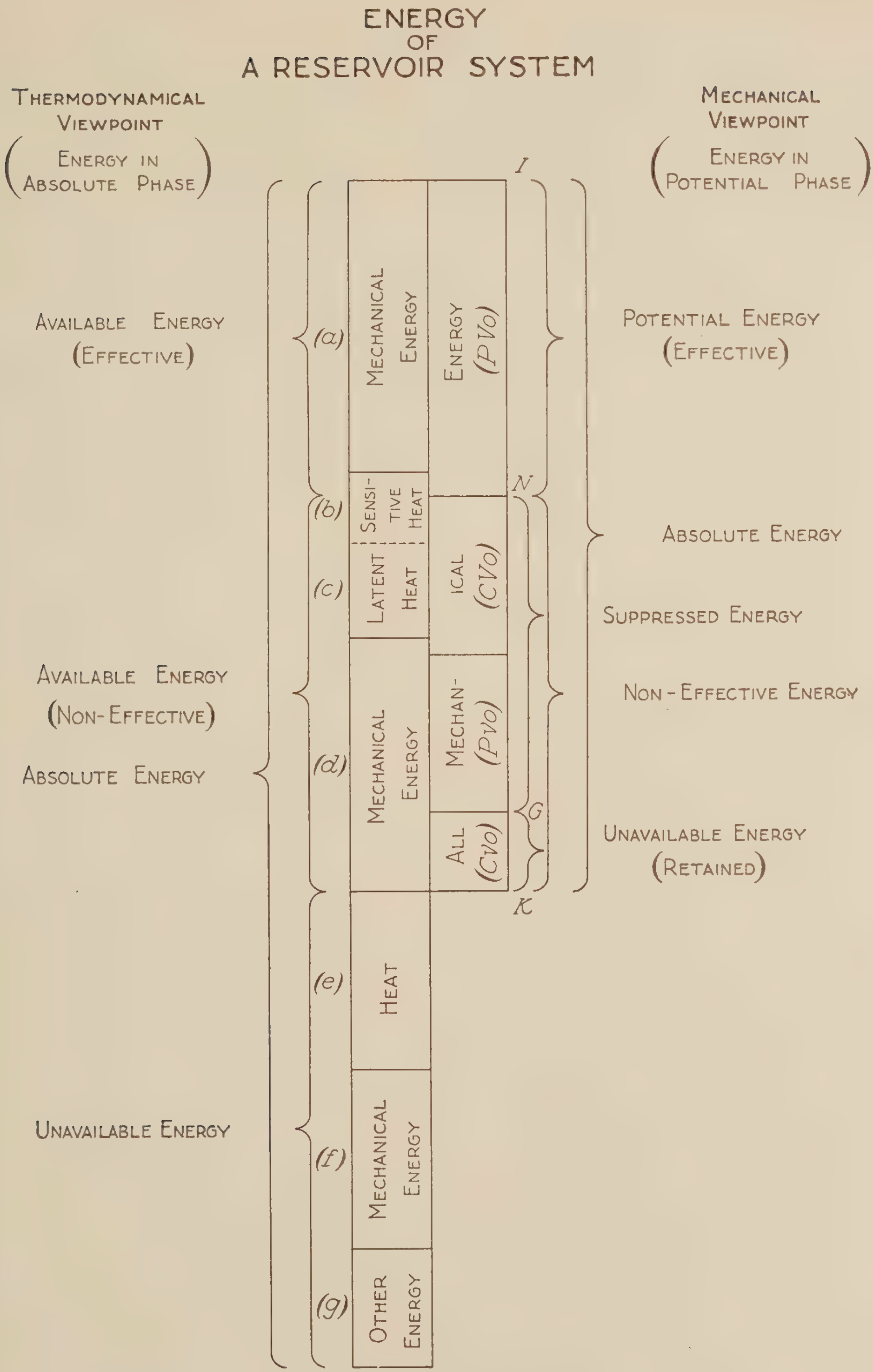
ENERGY
OF
A RESERVOIR SYSTEM
Thermodynamical Viewpoint
(Energy in Absolute Phase)
Mechanical Viewpoint
(Energy in Potential Phase)
Available Energy (Effective)
Available Energy (Non-Effective)
Absolute Energy
Unavailable Energy
(a)
(b)
(c)
(d)
(e)
(f)
(g)
Mechanical Energy
Sensi- tive Heat
Latent Heat
Mechanical Energy
Heat
Mechanical Energy
Other Energy
Energy (PVo)
ical (CVo)
Mechan- (PVo)
All (CVo)
I
N
G
K
Potential Energy (Effective)
Absolute Energy
Suppressed Energy
Non-Effective Energy
Unavailable Energy (Retained)

Fig. 33

importance to us. In this we would not infer that the thermodynamical aspect can be ignored completely, but rather that it is an aspect which must be given a subordinate place in any analysis of reservoir performance. *First it is essential that we thoroughly understand the mechanical principles of production in all our three controls;* and only thereafter should we attempt to apply thermodynamical principles to production. To accomplish our purpose we here deal with production as a problem in mechanics, recognizing the fact that thermodynamical effects are present. We see the relation between mechanics and thermodynamics; we admit without hesitation the effects of the imperfections of the fluids, the effects of polytropic expansion, and the effects of adventitious events upon the production of the fluids from reservoirs. In these chapters we have arranged for a temporary disposal of thermodynamical principles. We can perhaps proceed knowingly, fully aware of the fact that, after the principles of mechanics have been established, the day of reckoning with the principles of thermodynamics will be at hand.

# Part II. Reservoirs in Hydraulic Control

CHAPTER VIII

# Ideal Performance and Its Primary Functions

"From the exact and universal conformity to law of natural phenomena, a single observation of a condition that we may presume to be rigorously conformable to law suffices, it is true, at times to establish a rule with the highest degree of probability; just as, for example, we assume our knowledge of the skeleton of a prehistoric animal to be complete if we find only one complete skeleton of a single individual."—HERMANN VON HELMHOLTZ

**58.** *Introduction.*—All reservoirs, as stated in section 5, may be classified according to three mathematical systems of production curves without regard to several other possible bases of classification. These systems of curves define, and reveal to us, our three controls. If a reservoir possesses a hollow interior, that is, if its space is filled with fluid only—be it gas, liquid, or both—it can by no means be in Capillary Control, but must be in either Hydraulic or Volumetric Control. On the other hand, if a reservoir has its interior space filled with a porous medium containing both gas and liquid, then it may be in any one of the three controls. And inasmuch as natural reservoirs are safely assumed to be so filled, we see that among these there are three possibilities. But whether a given reservoir is artificial or natural, we in fact only need access to its orifice to determine the control; its main bulk may or may not be visible and accessible to us.

Figure 34 (p. 108) illustrates the relative positions held by the pressure and velocity curves in the three controls. These two functions are sufficient for the determination. For the sake of simplicity we will say that pressure and velocity are expressed in potential units;[1] that time, for the present, is expressed either as time elapsed or as time remaining; that the performance of the reservoir is ideal; and that the scales chosen for plotting the two functions as ordinates cause the left-hand initial points for each pair of curves to coincide.[2] Now the straight horizontal line *A* indicates Hydraulic Control. The line *B* is the pressure-time curve for the other two controls; *B* and *C*

[1] That is, in values of potential pressure and potential velocity according to §§ 11 and 13.

[2] Either the two vertical scales for pressure and velocity may be adjusted so that the initial points coincide, or one vertical scale representing percentage values may be used. In the latter either initial or present values of the functions may be taken at 100 per cent.

together indicate Volumetric Control, while *B* and *D* together indicate Capillary Control.[3] In general these curves pertain to the following fluids of production: (*a*) gas, when the reservoir produces gas alone; (*b*) liquid, when the reservoir produces liquid alone; and (*c*) liquid, when the reservoir produces

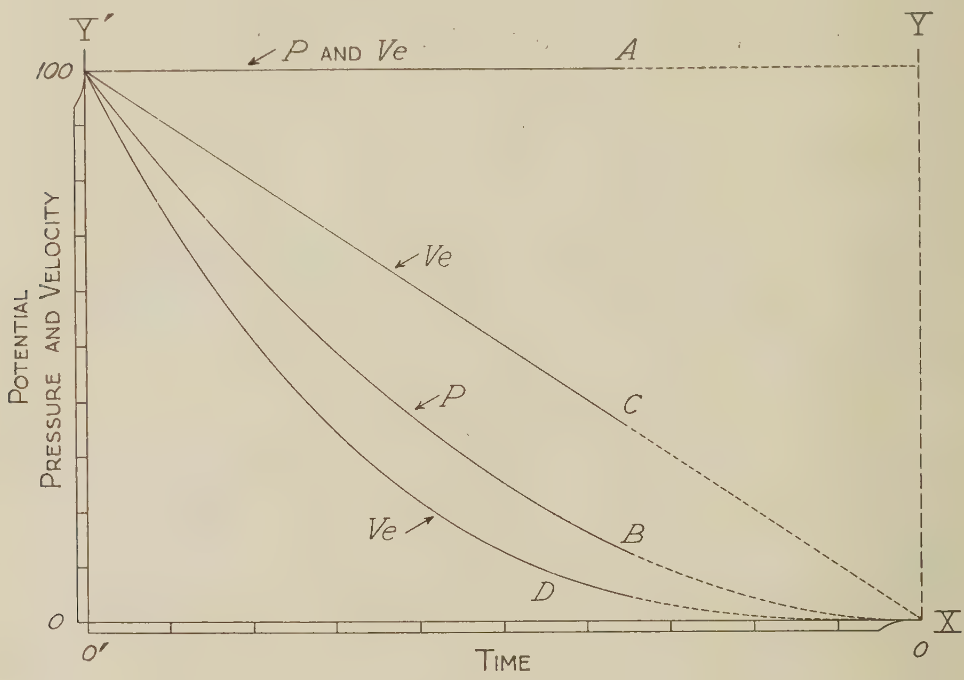


FIG. 34

both gas and liquid. The gas from the combination reservoirs of item (*c*) requires special attention, as stated in section 45.

To say that the curves of Figure 34 represent ideal performance means merely that the three cases in theoretic performance, as described in section 22, are not operative. If we know that one or more of these cases are operative, we must either adjust our data before plotting,[4] or resort to special tests which require only a short duration of time, such tests to be performed immediately at the orifice, or as close to it as may be convenient. These tests will be described later.

At any time that we are assured of the paths which are traveled by the two functions, we shall know at once how to interpret the performance of the reservoir as observed at the orifice, and how to describe and explain events which occur within the reservoir itself. This single set of observations suffices to establish a rule with, let us prefer to say, the highest degree of

[3] The relative positions occupied by the curves in Fig. 34 are well known in practice.

[4] This adjustment is for the purpose of placing them on a basis of identical conditions in production at the well, such conditions being exclusive of natural decline.

certainty. We shall now assume that pressure and velocity travel along paths of the nature of $A$, as shown.

**59.** *Type reservoirs in Hydraulic Control.*—For the purpose of analyzing fluid delivery from natural reservoirs it is convenient to select certain artificial reservoirs of like functions which may serve as models for experimentation and calculation. Provided the selection is properly made there are many advantages to be gained by so doing. Our experience makes us more familiar with the artificial reservoir; we may visualize the entire unit at one glance; we may alter the conditions of production in a variety of ways with greater precision; we may in fact place the entire reservoir within the laboratory, and perform many experiments which require careful observation throughout all parts of the reservoir. Such artificial reservoirs may be designated "type reservoirs," since they may be taken as the type of the class which they represent.

We may only know by comparison of functions whether the proper selection has been made as to type. The primary functions of performance must agree from the first. The secondary functions, which might only be directly viewed in the laboratory, will follow in due course, and these therefore will be more readily brought to our attention in the field.

As previously stated, two type reservoirs serve to illustrate Hydraulic Control. One holds gas, whereas the other holds liquid. These are as follows:

*a*) *The simple gas holder,* as shown in diagram in Figure 35. It consists of a steel bell mounted over a pit which contains water, the latter merely serving as a means of retaining the gas within the receptacle. The bell, as we all know, is provided with guides in its vertical movement, permitting the necessary adjustment in accordance with the amount of gas contained. *It is necessary and sufficient that the weight of the bell be supported by the compressed gas in order to maintain this control.* This bell exerts a constant pressure upon the gas unless its weight is deliberately increased or decreased by the alteration of any auxiliary load which may rest upon, or which may be attached to, it. The loss of weight suffered upon the immersion of the lower portion of the bell may be considered negligible, or, if the accuracy of an experiment warrants, it may be compensated by properly suspending a weight from ropes which pass over especially designed cams.

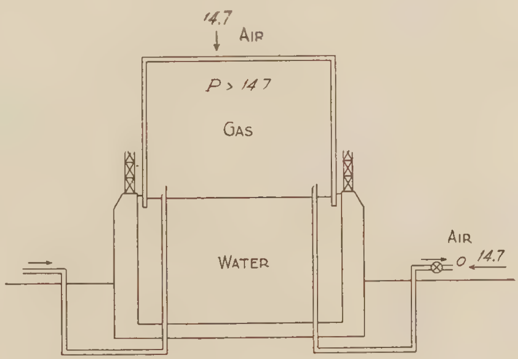


Fig. 35

*b*) *The solution tank,* as shown in Figure 36. For the sake of simplicity we will say that it is cylindrical, with a circular base and vertical sides.[5] The liquid must be maintained at a constant level for the purpose of providing a constant "closed-in" pressure upon the orifice; therefore *it is necessary and sufficient that the rate of inflow of liquid be constant, and equal to the rate of outflow, in order to maintain this control.*

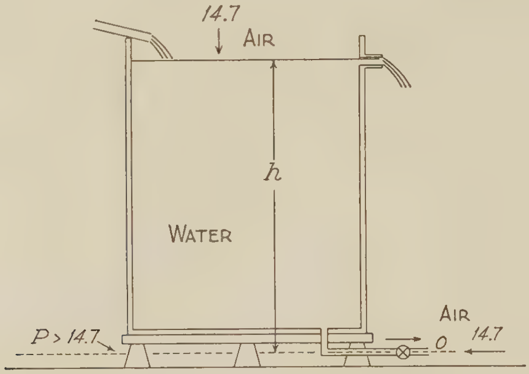


FIG. 36

These are the types. Now let us consider certain modifications in them. In Figure 37 (*a*) we have a piece of glass tube, bent in the form indicated, equipped with a stopcock, mounted on a simple stand, and filled with water, oil, and gas. Of course these fluids separate by gravity over the arch, in the manner shown. We may provide a continuous supply of water at *A*, sufficient to maintain a full tube, open the stopcock, and observe a reservoir producing gas in Hydraulic Control. The weight of the water and oil to the left of the gas acts in the same manner as the pressure exerted by the weight of the bell in the gas holder. Soon the oil arrives at the stopcock; then we observe a reservoir producing oil in Hydraulic Control. The weight of the liquids to the left now acts in the same manner as the weight of the liquid in the solution tank. Given sufficient time it is obvious that the water will arrive at the orifice, and no fluid other than it will be produced. Of course, we may have omitted the gas, the oil, or both, in setting up the apparatus. *The control is independent of the nature of the fluid within the reservoir.*

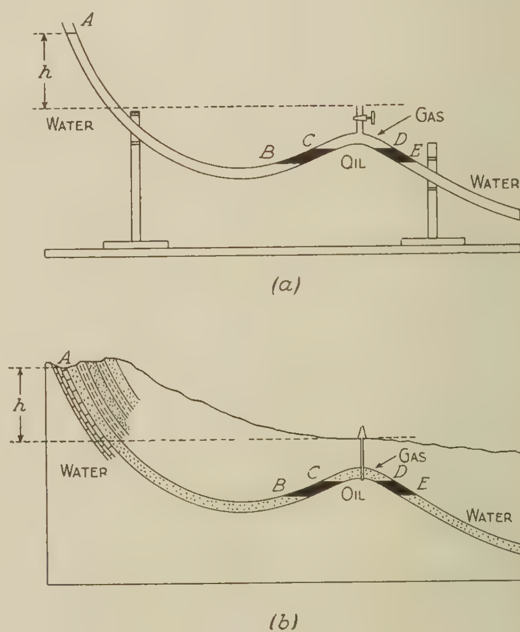


FIG. 37

[5] See § 40, first paragraph.

In Figure 37 (*b*) still greater modifications have been made. The "tube" has the same flexure as before, but where its cross-section was formerly circular, it is now indefinitely elongated in a direction perpendicular to the plane of the figure; thus it has become a cylindrical sheet with a horizontal axis of curvature.[6] The setting of the reservoir and the scale of its drawing are no longer the same. Furthermore, the interior of the reservoir is no longer hollow, but filled with a porous medium. The fluids are arranged in an identical manner, including a sufficient available supply of water at *A*. If we specify the condition that the pressure head of the liquid is sufficient to overpower any capillary action due to alternating bubbles of gas and globules of liquid within the reservoir—the action mentioned in section 29—what difference need we expect in the performance of the preceding artificial reservoir and the present natural one? As to the control, and therefore as to the laws of fluid delivery, we must expect none. While we may note differences in the values possessed by the seven primary functions of performance, we can be assured that the variation between them, in all their possible combinations by pairs, is identical in both reservoirs.[7] *The control is independent of the size and the setting of the reservoir, and it is provisionally independent of the absence or presence of a porous and permeable medium within the reservoir.*

60. *The ideal natural reservoir.*—Specifications for the ideal natural reservoir in general were given in section 22. To these we may now add the following ones which apply only to such a reservoir in Hydraulic Control:

*a*) Either the productive formation outcrops at the surface somewhere, it is in communication with a porous and permeable formation which does so, or it is possibly in communication with the surface along a fault or fault-zone. Under any of these circumstances surface water can enter the productive formation.

*b*) The column of water within the reservoir must be maintained at a constant level, in order that the pressure head at the orifice may be maintained at a constant value.[8]

*c*) The pressure head must be sufficient to overpower any capillary action due to bubbles of gas throughout the mass of liquid within the reservoir.

[6] This situation is provided for in the general description of an ideal natural reservoir in § 22.

[7] It is essential that we understand the laws of delivery to be invariable, while the numerical values of the functions depend upon physical conditions interior to, at, and exterior to the orifice of the reservoir. The latter therefore differ for various reservoirs, or for the same physical container serving as a reservoir under various conditions.

[8] For the sake of simplicity we shall assume that the column of water extends upward to the ground-surface. The excess of available water at this level can easily run off. This situation, however, is not an essential condition of Hydraulic Control. Under conditions readily imagined the height of the column can be maintained at lower levels.

*A natural reservoir in this control is of the open type;* that is, it is open to the atmosphere at some place other than possibly at the orifice. It is clear that this must be so; otherwise it would be impossible for the pressure head to be maintained at a constant value. *Surface water which enters the productive formation furnishes the constant pressure head, and this head is the sole source of energy within the reservoir.* Can the pressure exerted by the gas on the arch of either reservoir in Figure 37 exceed that due to the weight of the column of liquid upon it? No; the gas pressure is due only to the weight of the liquid.

So long as there exists a pocket of gas on the arches of the reservoirs in Figure 37, the actual pressure head measured at the horizon of the dotted line is equal to the vertical distance between *A* and *C*, for this distance alone determines the weight of the column of liquid. We would therefore experience a decrease in the pressure head on production, even granting that the column is maintained at *A*, because of the rise of *C*. When the gas pocket is fully depleted, the head is *h*, as shown. The depletion of the pocket of course does not mean the depletion of gas in the reservoir, since there yet remains the gas that is dissolved in the liquids by virtue of the pressure head.

If we were to restore the gas pocket in these systems after its depletion, it is clear that by pumping in the necessary gas at the orifice we would push a surface *C* downward, and thereby increase the head in accord with the measured vertical distance between *A* and *C*.

It is evident that in the processes of production from the gas pocket and its restoration the change in the pressure head is due solely to the fact that there is a downward slope to the reservoirs to the left of the orifices. If this portion were flat there would be no such change taking place. Furthermore, it is evident that in (*a*) the pushing of *C* downward is easy, for only a small amount of gas is required, whereas in (*b*) it is difficult on account of the enormous quantity of gas required.[9]

*We must admit that both of the reservoirs in Figure 37 will produce in the absence of gas either in a pocket or in solution, and that any gas which may be present possesses energy only in virtue of the weight of the column of liquid which bears upon it.*

It is not to be expected that ideally constant pressure heads are to be encountered in production from natural reservoirs in this control. The available supply of surface water is dependent upon the weather, and this, as we know, varies from month to month, and even from year to year, in almost every region. But aside from this matter of available water supply, there is little, if any, difference between the ideal and the actual reservoir in Hydraulic Control.

[9] The difficulty of increasing the pressure head is greater as the formations happen to be more nearly horizontal. Regardless of the "lay of the formation" in the vicinity of the orifice it is obvious that the gas pocket maintains the "spread" of the pool in the form of a ring about the structure.

**61.** *Pressure-time relations.*—The ideal well in Hydraulic Control is one which produces from an ideal reservoir in this control. Its performance is ideal when there are no alterations made in accord with theoretic performance, and the curves of production are ideal when performance itself is ideal. We are now to investigate the behavior of the functions of performance as illustrated by ideal curves.

If we plot potential pressure as ordinates and time as abscissas, the ideal conditions now defined give us without question the straight horizontal line $A$ of Figure 34. Its equation may be written as follows:

$$P = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

an expression which states the simple fact that the pressure has a constant value during time. The equation is of the algebraic form:

$$y = k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

Here we have an equation between two reservoir functions, $P$ and $T$, and another between two algebraic functions, $y$ and $x$, in which neither $T$ nor $x$ appears. Certainly these functions can be made to appear by writing the equations in their full form, namely,

$$P = KT^0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

and

$$y = kx^0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

The variables which are plotted as abscissas are now included in the expressions. They are given an exponent zero, and, of course, any numerical quantity raised to the zero power becomes equal to unity. Thus it is that $T$ and $x$ do not appear in the first equations.

The line $B$ in Figure 34 represents, as we know from the discussion in section 44, the pressure-time curve for Volumetric Control. We are to find later that it also represents the same relations in Capillary Control. Now the equation for this curve is

$$P = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

an expression of the general form

$$y = kx^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

denoting a parabola. The equations between pressure and time only differ as between the infinite and finite controls in having $T^0$ and $T^2$, respectively.[10] The former quantity retains a constant value, unity, in all investigations, whereas the latter continually varies in value in each investigation; and thus it follows that investigations in Hydraulic Control are of a simpler nature than those in the other two.

I believe we can say that the simplicity of the present control has frequently misled us in our conceptions of wells which decline naturally in accordance with production.

[10] See § 8.

It has been said that Nature attempts to establish an equilibrium within the reservoir of Hydraulic Control by causing flow at the orifice, but this equilibrium is not actually established within finite time. We may, however, regard it as being established upon the elapse of an infinite amount of time; that is, in Figure 34 the line $A$ meets the axis $X$ at an infinite distance to the right of the axis $Y'$. In other words, the true axis $Y$ for time remaining is located to the right at infinity, and here, where the axes $X$ and $Y$ intersect, is our true origin for the pressure-time curve. Now this is certainly an inconvenient place from which to measure time, for no scale of drawing whatever will bring it within the scope of our plotting paper, be this of any conceivable length. We meet with this difficulty not only as between pressure and time, but as between the other primary functions—volume, velocity, acceleration, energy, and power—and time, as well. Then how shall we handle our production curves in this control?

Let us adopt the simple expedient of placing our origin at the left, as at $Y'$, and reckon with time elapsed, which is to be measured from the instant when the orifice is first opened, or at any other subsequent and convenient instant. But we must remember that this is merely a mathematical expedient which is applicable only to this general class of reservoirs; we are not at all warranted in assuming that the same procedure is proper with the production curves of reservoirs in the finite controls, where the origin must be placed at the right, and where time must be expressed in terms of time remaining. In these controls equilibrium can actually be established within finite time, and the proper scale of drawing will place the point denoting this equilibrium within the reach of any piece of plotting paper which we may select.

The left-hand origin for curves seems to have misled us in the past; it has caused us to believe that it is the proper origin for the curves of all reservoirs. The origin for the rectangular hyperbola, as drawn in Figures 5, 9, 27, and 32, is certainly at the left, and consequently some have claimed that decline curves of production are hyperbolic. Again, the exponential curve, which has the general equation

$$y = ae^{-nx}$$

likewise has its origin at the left—as we ordinarily plot curves with the positive direction for abscissas reckoned from left to right—and it has received its allotment of support as the true equation for the curves of production. But what curves with a left-hand origin have not at some time been proposed as true, supposedly with analytical foundation, though wrongly so, and evidently with empirical foundation, as shown by plotted curves from actual data? So far as an empirical foundation is concerned, any given set of data may be shown to approximate the curve of any supposedly plausible equation which we may wish to assign to it. In passing curves through a series of points the eye is our sole guide, but the eye is mathematically incompetent in cases where the data are not associated with scientifically precise observations.

The general equation for the so-called "power-function curve" in co-ordinate geometry is

$$y = kx^n$$

in which the exponent $n$ may be either integral or fractional, positive or negative. When it is negative, the curve is hyperbolic and asymptotic to the axes; that is, the curve approaches the axes as either $x$ or $y$ becomes greater, but it does not intersect the axes within finite limits. Such were the values of $n$ in the equations for the curves of Figure 32, for although they were given in the forms $xy = k$, and $x^{1 \cdot 5}y = k$, they may as well have been given in the forms $y = kx^{-1}$, and $y = kx^{-1 \cdot 5}$. But when $n$ is positive, the curve is parabolic, and it intersects both axes at the vertex, as shown, for example, in Figure 23. When $n$ is zero, as we find it in the equation expressing the relations between pressure and time in Hydraulic Control, the curve intersects only the vertical axis. (We may regard zero either as a negative or as a positive number; that is, we may consider the straight horizontal line to belong to either the family of hyperbolas or the family of parabolas.) Thus we see that if $n$ is negative, the origin is at the left; if $n$ is positive, the origin is at the right; and if $n$ is zero, we may place it at either end to suit our convenience.

I believe these mathematical conceptions to be important, in so far as they have apparently been ignored in our theories concerning the nature of production curves.

62. *The pressure-time curve.*—When we talk about the pressure-time curve for ideal performance in Hydraulic Control, do we, or need we, refer to pressure as measured from a particular zero? We have already seen that we may be concerned with one of three logical zeros; namely, absolute zero, atmospheric zero, and the potential zero. In Figure 38 we have such a curve, drawn with respect to three horizontal axes which are determined by the zeros. Let us define five pressures which are represented by symbols:

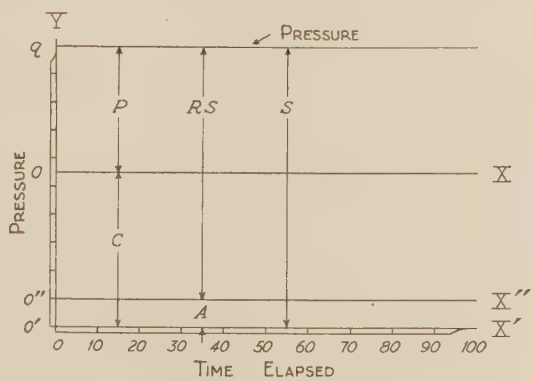


FIG. 38

$S$ = the value of the closed-in pressure when expressed in terms of the absolute. This we shall hereafter call the *static pressure* of the reservoir.[11]

[11] The term "static pressure" is not to be confused with the static pressure head of pitot tubes and flow-meters. (§§ 42 and 43.)

$A$ = the value of the atmospheric pressure, likewise expressed in terms of the absolute. It is assumed to be perfectly constant, in spite of the fact that we know it does vary at a given locality with the weather.

$RS$ = the value of the closed-in pressure as indicated by the gauge. This we shall hereafter call the *registered static pressure*.

$C$ = the value of the constant back pressure. It is expressed in terms of the absolute.

$P$ = the potential pressure as previously defined.[12]

Only one of the horizontal axes is absolutely fixed in position. This is $X'$. The mean position of $X''$ will be determined by the altitude of the locality above sea level, and the position of $X$ will be determined by the particular value of $C$.

If we let $F$ represent either $S$, $RS$, or $P$, then the equation of the pressure curve shown in the figure is simply $F = K$. Three values of $K$ are necessarily different, but the form of the equation itself is the same regardless of the significance of $F$. Evidently it is immaterial from our point of view which of the three pressures are actually plotted. Now these are the facts in Hydraulic Control, but because of them we must not assume the same conditions to exist in the finite controls. There we must plot the potential pressure, or at least accept the values of this pressure which alone satisfy the relation $P = KT^2$ in ideal and theoretic performance.

I believe we see here another source of error in our previous theories respecting the performance of oil and gas reservoirs. There was a time when all our information concerning the performance of reservoirs was to be received from the hydraulic engineer. He has always dealt largely, almost exclusively, with reservoirs in the present control; and consequently we have been led to adopt his principles from the beginning of oil and gas engineering. It is obvious that his principles are correct in their restricted field, and we should certainly abide by them so long as they prove to be appropriate, but we should not fail to substitute other principles when these are demanded by the mechanics of fluid delivery.

63. *The velocity-time curve.* Corresponding to our pressure-time curve in its relation to the three horizontal axes we have the velocity-time curve of Figure 39. In $Ve'$ is shown the value of the rate of production when the reservoir produces into a perfect vacuum, in $Ve''$ the value of the rate of production when the reservoir produces into the atmosphere, and in $Ve$ the actual rate of production realized in virtue of a potential pressure $P$, not equal to zero, and different from $S$ by an amount $C$. The three velocities constitute the *static velocity*, the *registered static velocity*, and the *potential velocity*, respectively.

[12] In accordance with the definition given to the potential pressure in § 11 it follows that the numerical value of $P = S - C$. (See also Equation 48, page 91.)

As we may write the equation between $Ve$ and $T$ thus, $Ve = K$, so might we let $V$ represent either $Ve'$, $Ve''$, or $Ve$, and write $V = K$. Thus we meet with the same circumstance in regard to the equation of the curve as before. In this control it is immaterial whether we reckon with one or another of these velocities in plotting our curve. In gas field practice the axis $X$, determined in position by the value of the line pressure plus atmospheric pressure, has in general been ignored. Wells are opened to the atmosphere, and values of the registered static velocity, ordinarily called the "open-flow rate," are determined.[13] The fact that the axis $X$ is subject to a greater fluctuation than the axis $X''$ may encourage us to continue with this practice. There is little danger of deceiving ourselves regarding the possible future performance of the reservoir if it is in the present control, while the danger is great if the reservoir is in either finite control.

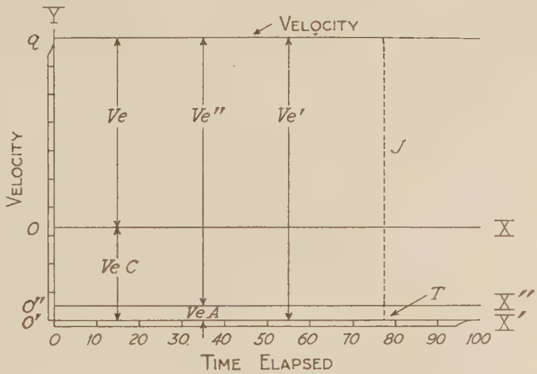


FIG. 39

Field measurements pertaining to liquid, as in the rate of production of oil, invariably refer to the proper axis $X$. A quantity of liquid is easily visualized and easily measured as to its amount; therefore we resort to its direct measurement as it flows from the well. Gas, on the other hand, fills every vessel in which it may be placed; consequently our measurements are made indirectly. While the open-flow rate has been adopted in gas field practice, no corresponding rate in regard to oil production has been thought of. We take the oil as it comes, and measure it accordingly. Perhaps the idea of an open-flow rate on oil wells, differing from the actual rate, sounds ridiculous. I believe it does, and furthermore, I believe we shall some day hold the same attitude toward the open-flow rate on gas wells, for one is no more rational in mechanics than the other. Are not the mechanics of liquids and gases identical?

It is to be admitted that we often do have an open-flow rate of production from our oil wells. When a pump removes the oil from the bottom of the

[13] It is a common experience to produce gas (or air) from artificial reservoirs directly into the atmosphere. Here the open-flow rate is the potential velocity of flow. However, as we know, production from a gas well takes place against a line pressure. When the well is opened to the atmosphere, it is done only for a time sufficient for the observation on its open-flow rate.

well as rapidly as it arrives, then the well is producing against the pressure of the atmosphere alone. The potential axis $X$ assumes a position coincident with the atmospheric axis $X''$. A gas well can be made to do likewise by means of a pump which is capable of removing the line pressure. Such a procedure is not good practice, we say; but this is clearly an opinion based upon economics rather than upon mechanics.

According to section 13 velocity is not actual fluid. It is a number which, when multiplied by time, gives a result actually signifying volume. In the language of mathematics we may say that

$$\frac{M \text{ barrels of oil}}{1 \text{ day}} \times N \text{ days} = MN \text{ barrels of oil}$$

and that

$$\frac{M \text{ cubic feet of gas}}{1 \text{ day}} \times N \text{ days} = MN \text{ cubic feet of gas}$$

Clearly the fraction on the left is velocity which is necessarily multiplied by time in order to produce the desired result. In these expressions the term "day" cancels with "days," just as in the multiplication of fractions in arithmetic any common factors may be cancelled out before the final result is obtained.

In geometry we may construct a rectangle whose altitude, drawn to scale, represents the velocity, and whose base, likewise drawn to scale, represents time, then the area of the rectangle represents actual volume of fluid. *Thus any area subtended by a velocity-time curve represents volume.* In Figure 39 the area bounded by the curve, the $X$ and $Y$ axes, and the line $J$, represents a *potential volume* actually produced by the reservoir in the period of time indicated by 0 and $T$. Again, the area bounded by the curve, the $X''$ and $Y$ axes, and the same line represents a *registered static volume,* or open-flow volume, which the reservoir would produce in the same interval of time, if it were to produce against the pressure of the atmosphere alone. Once more, the area bounded by the curve, the $X'$ and $Y$ axes, and the line, represents a *static volume,* or absolute volume, which the reservoir would produce in the same interval, if it were to produce into a perfect vacuum.[14]

It is obvious that such areas subtended by velocity-time curves are only rectangular when the curve itself is a straight horizontal line. Ideal performance in this control provides such a line, but ideal performances in the finite controls do not do so. At any time the areas are not rectangular we

[14] The term "static volume" is to be given preference hereafter over the term "absolute volume," in order to avoid the different circumstances encountered in reservoirs of the closed and open types. In § 12 we found that the air of the atmosphere above the open tank functions in the process of production. The static volume is always a definite quantity of fluid in each reservoir, irrespective of the nature of the fluid and the type of the reservoir.

cannot multiply bases by altitudes. We might either divide them into very small rectangles, each one representing a unit volume, and count or estimate their number, or we might conveniently use a planimeter for their direct measurement as a whole.

Whenever the velocity of production is constant the acceleration of production is zero. Naturally, any mass, be it a solid or a fluid, which moves with constant velocity, experiences no change in velocity. We may take the equation between velocity and time in Hydraulic Control, namely $Ve = K$, and obtain by differentiation with respect to time the following equation: $Ac =$ zero. This equation may be represented graphically by a straight horizontal line coincident with the horizontal axis, inasmuch as the vertical distance for all straight horizontal lines, represented in general by the constant $K$, simply becomes zero in the present case.

The function of acceleration in this control is not of interest except in comparison with the same function in the finite controls.

64. *The volume-time curve.*—The potential volume which a reservoir in Hydraulic Control is capable of producing is infinite in amount. Any conceivable reservoir has definite physical dimensions, it is true, but our conception of a reservoir is purely a mathematical one. Whether the physical container be large or small, so long as the rate of inflow is equal to the rate of outflow the container cannot become emptied of its contents within finite time. It must therefore contain, mathematically, an infinite volume.

Just as we cannot reckon with time remaining in this control, neither can we reckon with volume remaining, for no matter what finite volume might be produced, the volume yet remaining is infinite. We have adopted the expedient of employing time elapsed in order to avoid difficulty; likewise *let us adopt volume produced as our function of performance, and place the origin of the volume-time curve on the left.*

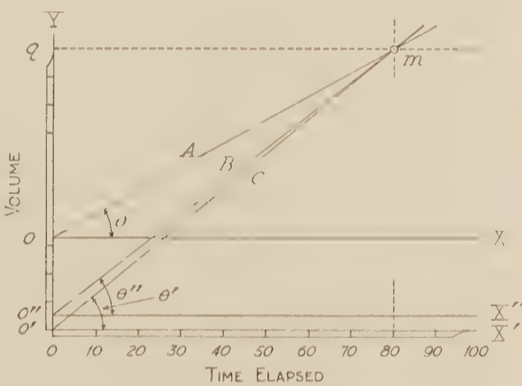


FIG. 40

Figure 40 illustrates ideal volume-time curves in accordance with the three horizontal axes. The line $A$ with its origin at potential zero represents the potential volume produced by the reservoir. By integrating the equation between velocity and time,

$$Ve = K \quad \dots\dots\dots\dots \quad (60)$$

with respect to time, we obtain the relation

$$Vo = KT \quad \dots\dots\dots\dots \quad (61)$$

There is in fact a constant of integration to be added in such mathematical processes, but this is omitted because its value is here equal to zero.[15] The equation is that of the inclined straight line of the general form, $y = kx$. The value of $K$ is actually equal to that of $Ve$ in Equation 60. This is to say, Equation 61 defines the volume as a quantity equal to the product of velocity and time. The present value of $k$ depends upon $K$ and the scales employed in plotting the curve, and the angle $\theta$ which $A$ makes with the axis $X$ in turn depends upon $k$—in fact $k$ is the slope of the line represented by the general equation.[16]

If the reservoir were permitted to produce against the pressure of the atmosphere alone, the volume curve would be the line $B$; and again, if the reservoir were permitted to produce into a perfect vacuum, the volume curve would be the line $C$. These curves have their respective origins at the atmospheric and absolute zeros. When the horizontal axes are properly spaced according to scale, the three curves meet at some point $m$.[17]

The geometric relation between velocity and volume is shown in Figure 41. From the preceding discussion concerning these functions it is clear that the following relation holds:

$$\frac{\text{Area } Oabc}{\text{Area } Odec} = \frac{\text{Ordinate } af}{\text{Ordinate } dg} \quad \ldots\ldots\ldots\ldots\ldots\ldots (62)$$

wherein all terms pertain to the same potential axes $X$. It is also possible to write the following relation:

$$\frac{\text{Area } Oabc}{\text{Area } Odec} = \frac{\text{Area } O''a''bc}{\text{Area } O''d''ec} \quad \ldots\ldots\ldots\ldots\ldots\ldots (63)$$

Evidently the right-hand members of these two equations are equal. To illustrate these relations let us take the case of a gas well in this control. Equation 62 shows that the areas subtended by the rate of production curve are

[15] Of course we cannot integrate Equation 60 without first converting it into the proper form of a differential equation. Thus

$$Ve = \frac{dVo}{dT} = K$$

Now the equation between the two terms on the right may be written in the following form:

$$dVo = KdT$$

which is an expression ready to be integrated. The constant of integration is zero to satisfy the condition that $Vo =$ zero, when $T =$ zero. (In simple processes of this nature we need not always give the details of the procedure. Hereafter only the more difficult cases of integration will be carried out in full.)

[16] This will be recognized as an elementary proposition in co-ordinate geometry.

[17] In Fig. 39 the rectangles of equal bases have areas proportional to their altitudes, and these areas are proportional to the ordinates of the volume-time curve. Furthermore, the three horizontal axes are the same for velocity and volume. As a consequence of these geometrical relations the lines meet at $m$.

proportional to the ordinates of the cumulative production curve. Equation 63 shows that the areas subtended by the rate of production curve are proportional to the areas subtended by the open-flow curve. Now an equation between the right-hand members of these two would show that the areas subtended by the open-flow curve are proportional to the ordinates of the cumulative production curve. While this relation holds true, we must not overlook the fact that it is so only on condition that the performance is ideal. Equation 62, however, holds true regardless of performance; that is, it holds in ideal, theoretic, and actual performance. It is the relation to which we should confine our attention in practice.

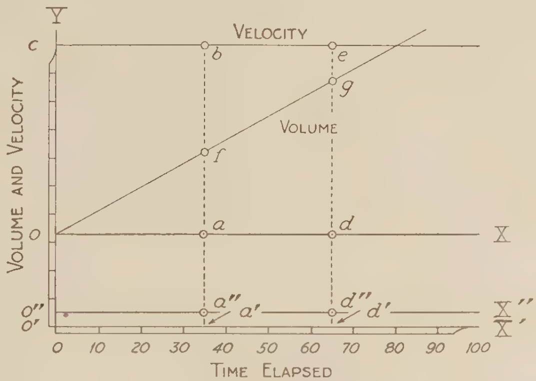


FIG. 41

65. *Energy-time relations.*—As stated in section 15, energy is the product of pressure and volume. We have the two following equations from above: $P = K_1$, and $Vo = K_2T$, in which for the moment the constants $K$ are given subscripts. By multiplying these two equations[18] we have

$$E = PVo = K_1K_2T$$

or simply the equation $E = KT$, where $K$ is now the product of the two former $K$'s. Like the volume-time relation this energy-time relation is represented graphically by the straight inclined line, and corresponding to Figure 40 we may construct Figure 42. The line $A$ represents *potential energy*, the line $B$ *registered static energy*, and the line $C$ *static energy*, or absolute energy. These are to be described in the next section.

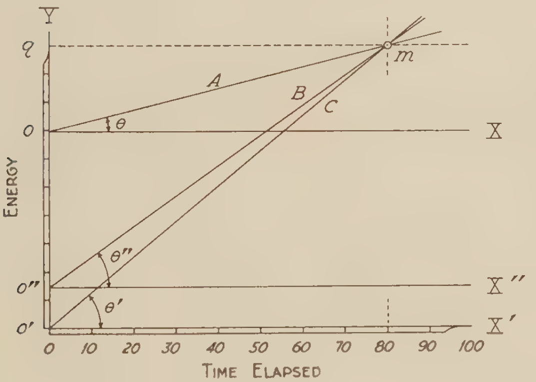


FIG. 42

When a reservoir in this control produces liquid only, or gas only, production is cumulative in accordance with the volume-time curve, but when

[18] If we write $E = KPVo$ the constant $K$ is unity for this control, and it may therefore be omitted in the quantitative equation between these functions. (See footnote 10, § 15, p. 24.)

a reservoir produces both fluids in combination, production of the liquid is cumulative in accord with the volume-time curve, while production of gas is cumulative in accord with the energy-time curve. *These circumstances are not dependent upon the source of the energy possessed by the resrvoir; they hold whether the weight of a column of liquid or the pressure of gas is the force which causes performance, and therefore they hold whether the reservoir is of the open or the closed type.*

The same statement, with slight modification, may be made in connection with reservoirs in finite control. There we must say that production is cumulative in accordance with the inverted volume-time and energy-time curves, for we are then to deal with volume and energy remaining in the potential reservoir, rather than with these functions expressed in terms of quantities produced.

This important dynamic relation between gas, which accompanies the liquid on production from combination reservoirs, and energy, is a consequence of the relation between the solubility of the gas in the liquid and the pressure to which the fluids are subjected within the reservoir; that is to say, it is a consequence of the phenomenon defined by Henry's Law. To understand these relations we should preferably commence with the subject of *the proportional production of gas and liquid.*[19] Let us leave this investigation for consideration in the next chapter, and proceed with the sixth and last primary function of performance.

66. *Power-time relations.*—Power, as defined in section 16, is the rate of displacement of energy, and it is therefore related to energy just as velocity is related to volume. If we differentiate the equation $E = KT$ with respect to time, we obtain the relation $Po = K$. Or, since power is the product of pressure and velocity, the two equations, $P = K_1$ and $Ve = K_2$, by multiplication give $Po = PVe = K_1K_2$, or simply $Po = K$. The curve for this relation between power and time is, of course, the straight horizontal line. Drawn with respect to the three axes it appears as in Figure 43.[20] In $Po'$ is shown the rate of delivery of energy when the reservoir produces into a perfect vacuum, in $Po''$ the rate when the reservoir produces into the atmosphere, and in $Po$ the actual rate realized in virtue of a potential pressure $P$, not equal to zero, and different from $S$ by an amount $C$. These three quantities of power constitute the *static power,* the *registered static power,* and the *potential power,* respectively. In this control they all have the same equation with respect to their axes, differing merely in their values of the constant $K$. By virtue of the first method of obtaining the equation between power and time given above it is obvious that $K$ has the same value as $K$ in the equation between energy and time.

*Power is not actual energy. It is a number which, when multiplied by*

[19] Frequently this is called the "formational gas-oil ratio."

[20] The three horizontal axes are the same for power and energy.

*time, gives a result actually signifying energy.* In the same manner as with velocity we may say that

$$\frac{M \text{ units of energy}}{1 \text{ unit of time}} \times N \text{ units of time} = MN \text{ units of energy}$$

In geometry a rectangle whose altitude, drawn to scale, represents power, and whose base, likewise drawn to scale, represents time, possesses an area which represents actual energy. Thus in Figure 43 the area bounded by the curve, the $X$ and $Y$ axes, and the line $J$, represents potential energy delivered by the reservoir in the period of time indicated by 0 and $T$. Again, the area bounded by the curve, the $X''$ and $Y''$ axes, and the same line, represents registered static energy, or open-flow energy, which the reservoir would deliver in the same interval of time, if it were to produce against the pressure of the atmosphere alone. Once more, the area bounded by the curve, the $X'$ and $Y'$ axes, and the line, represents static energy, or absolute energy, which the reservoir would deliver in the same interval, if it were to produce into a perfect vacuum. The potential and absolute energies were described in section 51, and they are shown in the diagram of Figure 33.[21] A horizon for the designation of the registered static energy would occupy some position between $K$ and $N$, as determined by the pressure of the atmosphere and the atmospheric volume.

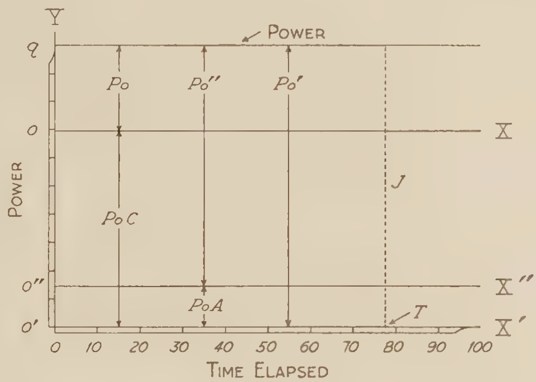


FIG. 43

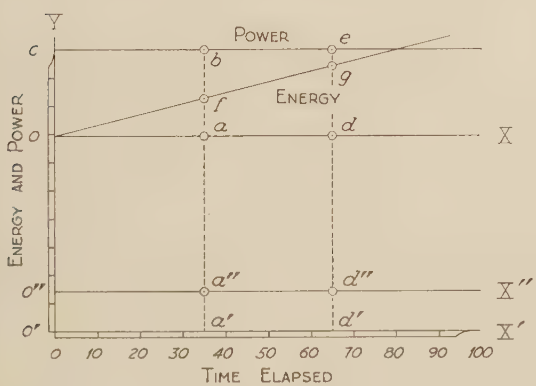


FIG. 44

To correspond with Figure 41 we have Figure 44 which shows the

[21] As with the case of volume, the term "static energy" is to be given preference over the term "absolute energy." Obviously the reasons are the same as before.

geometric relation between power and energy. As before, the following relations hold:

$$\frac{\text{Area } Oabc}{\text{Area } Odec} = \frac{\text{Ordinate } af}{\text{Ordinate } dg} \quad \ldots\ldots\ldots\ldots\ldots\ldots (64)$$

and

$$\frac{\text{Area } Oabc}{\text{Area } Odec} = \frac{\text{Area } O''a''bc}{\text{Area } O''d''ec} \quad \ldots\ldots\ldots\ldots\ldots\ldots (65)$$

The interpretation of these corresponds with the previous case. Equation 64 holds true regardless of the nature of performance.

67. *Summary of the fundamental relations.*—We have analyzed the six fundamental primary function curves for Hydraulic Control. At whatever instant we begin our reckoning in time—whether it be the initial instant of production or any subsequent convenient instant—life, or time remaining, is mathematically infinite. We shall say that our curves belong to the parabolic family, of the general equation, $y = kx^n$, wherein the exponent $n$ is either zero or unity, and the constant $k$ is either a positive number or zero. To meet these conditions we place the origin of the curves at the left, and reckon time in terms of time elapsed.

The following table gives the six relations as we have found them in the preceding articles:

HYDRAULIC CONTROL

FUNDAMENTAL PRIMARY FUNCTION RELATIONS

| | |
|---|---|
| Pressure-Time | $P = K$ |
| Velocity-Time | $Ve = K$ |
| Volume-Time | $Vo = KT$ |
| Acceleration-Time | $Ac =$ zero |
| Energy-Time | $E = KT$ |
| Power-Time | $Po = K$ |

Inasmuch as these symbols in the right-hand column are employed for representing only the potential functions, as stated in section 18, the equations as written refer only to the potential axis $X$. The corresponding curves, when placed upon the same plat, appear as in Figure 45. The constants $K$ in the pressure-time and velocity-time equations have values which depend upon the units employed in measuring the three functions involved; and upon these $K$'s the others depend, either by multiplication, integration, or differentiation. In Figures 38 and 39 arbitrary values for the constants are assumed, and the remaining ones take their values in accord with them. In combining all the separate curves as in Figure 45 their respective potential axes $X$ may be superimposed, although they in fact show the following positions with respect to the axes $X'$: ($a$) a position for pressure; ($b$) a position for velocity, one which also serves for volume and acceleration; and

(*c*) a position for energy, one which also serves for power. Such positions are due to the independence and dependence of the respective $K$'s. The propriety of superimposing the axes will be established in a later section.[22]

Time enters into the six fundamental equations in two ways: as a func-

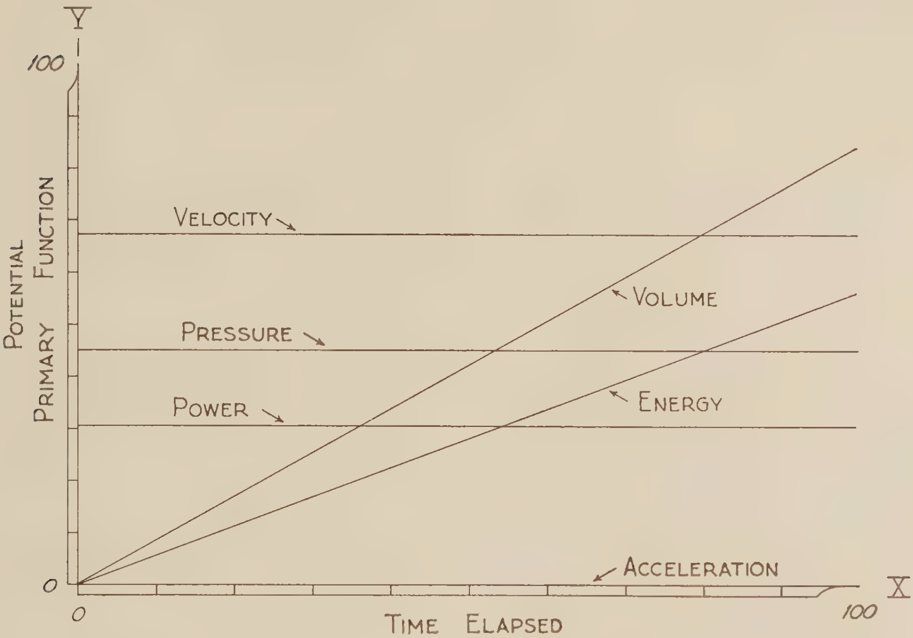


Fig. 45

tion $T$ of performance, and as a matter of definition. The mutual relations between pressure, volume, and energy are independent of time in both ways, as explained in section 41, although by virtue of our system of mechanics each function may be expressed separately in mathematical relation with $T$. Velocity, acceleration, and power involve time in their definitions, and in addition even each of these functions may be expressed separately in mathematical relation with $T$. When the function $T$ possesses the exponent one, the symbol must appear in the equation, but when it possesses the exponent zero, the symbol may or may not appear, as we choose, since any variable raised to the zero power is equal to unity. We are permitted to say, then, that the six primary functions of performance in Hydraulic Control can be expressed mathematically as functions of $T$, the seventh one.

[22] See § 109.

CHAPTER IX

# Ideal Performance and Its Primary Functions *(Continued)*

"A mathematical science is any body of propositions which is capable of an abstract formulation and arrangement in such a way that every proposition of the set after a certain one is a formal logical sequence of some or all such preceding propositions."— JOHN WESLEY YOUNG

**68.** *A mathematically deficient control.*—If our experiences with reservoirs were confined to ideal performance in Hydraulic Control, our knowledge of production in general would be extremely limited, and possibly in part erroneous. Time, of itself, neither causes nor is accompanied by changes in the values of pressure, velocity, acceleration, and power. These functions, when plotted with time as abscissas, are shown to travel straight horizontal paths throughout all time, without end. And due to this fact alone it would appear immaterial whether one or another of the three horizontal axes is used as a basis of measuring the ordinates, since the paths of these functions possess the same mathematical characteristics in all cases. Now I would ask if it is not true that this monotony in graphic representations can very easily mislead us in our conceptions concerning reservoir functions in general. Might we not too readily assume, for example, that the three axes for static, registered static, and potential pressures can be used indiscriminately in the analysis of all reservoirs? I think so; yet if we were to investigate no further than ideal performance in this control it seems to me that we should recognize the distinct appropriateness of the potential axis alone, if we give due consideration to the significance of volume in its relation to velocity, and energy in its relation to power. It is inconceivable that we should ever in practice base our curves for volume and energy, especially those for volume, upon any but their potential axes.

We might say that Hydraulic Control is mathematically deficient, in that the straight horizontal and inclined lines, to which the primary function curves are confined, fail to reach a point of equilibrium; and consequently we in turn fail to make proper discriminations in our analyses of reservoir functions.

Our reasons for first investigating performance in this control are doubly founded. Historically these reservoirs are supposedly best understood, for hydraulic engineers have dealt with them through centuries, from the time of ancient aqueducts, through the age of medieval water-power systems, to the

modern water-supply systems of cities and power-plants. All these were, and are, constructed upon ideas which we now associate with Hydraulic Control, wherein pressure and velocity maintain continuous and constant values. In truth do we not today place economic values upon our water wells in accordance with their continuous flow? There was a time indeed when we believed that all oil and gas wells should behave as these water wells. We know now that although many of them do so, more of them behave quite otherwise.

Again, the mathematical deficiency of this control provides, curiously enough, a simplicity which is not to be found in the finite controls. Four of the primary functions and all of the secondary functions here maintain a constant value, or a constant state, throughout time. When definite alterations are imposed upon the conditions of production, these reservoirs respond in accordance with them, only to thereafter proceed in the same manner as before.[1] The various functions are arbitrarily given new values at the time of the alteration, and these new values hold continuously, or at least until the time of a further alteration.

With these reasons in mind we should be willing to consider the precise mathematical details of performance in this control. While we treat a simple situation, it may appear at times that our nice discriminations only tend to make the subject more complicated. Nevertheless, I believe that niceties in mathematical interpretations cannot be established to better advantage than in the present analysis. I expect that eventually their content will be recognized as possessing practical value in the production of oil and gas. Through them we learn to avoid the application of principles which are appropriate only in Hydraulic Control to reservoirs in Volumetric and Capillary controls, and, conversely, to avoid the application of principles which are appropriate only in Volumetric and Capillary controls to reservoirs in Hydraulic Control. Methods in office and field engineering which prove to be exceptionally successful in one locality appear exceedingly attractive to us, and we are thereby frequently induced to adopt them universally, regardless of their applicability only to a particular control.

**69.** *The derived primary functions.*—We may take any two of the six fundamental primary function relations as listed in section 67 and algebraically eliminate the function $T$ from their equations. In this manner we obtain the derived primary function relations according to the list in section 9. Now a curious situation arises—one which we encounter only in Hydraulic Control. We desire an equation and its curve which shows the relation between two functions other than, and independent of, the function $T$. Obviously at least one of the functions must vary in value. How can we have a curve showing the variation between two quantities, neither of which varies? It is

[1] That is, the primary functions behave according to the same laws of delivery as before, and the secondary functions assume a constant state which does not, in its characteristics, differ in any way from the earlier state.

absurd, of course.[2] But if one or both of the quantities undergo variation in a natural process, such a curve is not only possible but perfectly plausible and logical, in that it possesses a significance which may be definitely interpreted in terms of the process.

A glance at the listed equations in section 67 shows only two functions which vary in the process of production. These are volume and energy, the two functions of performance which possess extensive properties in distinction from the other functions of pressure, velocity, acceleration, and power, which possess only intensive properties.[3] That is to say, volume and energy appear as actual commodities received from the reservoir, whereas pressure must be multiplied by area to give force, which is an extensive property, velocity by time to give volume, and power by time to give energy.[4] Inasmuch as we ordinarily construct our curves with abscissas varying, whether ordinates do or do not vary, we shall say that in the present case only volume and energy may serve as abscissas. As a consequence only nine of the fifteen derived primary function curves previously listed may be constructed in this control.

By virtue of the fact that six remaining derived relations cannot be represented graphically because of the absurdity involved, it follows that these relations cannot be expressed by analytical equations. Curves and equations are merely two modes of expressing the same thing. Let us tabulate the fifteen relations, giving the equations where possible:

DERIVED PRIMARY FUNCTION RELATIONS

| | |
|---|---|
| Pressure-Volume | $P = K$ |
| Velocity-Pressure | ...... |
| Acceleration-Pressure | ...... |
| Pressure-Energy | $P = K$ |
| Pressure-Power | ...... |
| Velocity-Volume | $Ve = K$ |
| Acceleration-Volume | $Ac =$ zero |
| Volume-Energy | $Vo = KE$ |
| Power-Volume | $Po = K$ |
| Acceleration-Velocity | ...... |
| Velocity-Energy | $Ve = K$ |
| Velocity-Power | ...... |
| Acceleration-Energy | $Ac =$ zero |
| Acceleration-Power | ...... |
| Power-Energy | $Po = K$ |

[2] The curve would possess only one point—that point determined by the values of the two non-variable functions. (See § 70.)

[3] The intensity of acceleration is zero in the present control.

[4] In the finite controls we shall add that acceleration must be multiplied by the square of time to give volume. In multiplying velocity by time, and power by time, a proper constant in accordance with the control (here unity) must be included, in order to express volume and energy quantitatively. The same is true of acceleration and the square of time in the finite controls.

It is observed that eight equations represent straight horizontal lines. Symbols for the second functions in the couplets may be placed in the equations, provided they be given the exponent zero.[5] For acceleration the constant $K$ is zero, and the curve coincides with the horizontal axis. One equation represents a straight inclined line. *As indicated by the symbols we are considering only potential functions, measured from the potential axis, and therefore expressed in potential units.* In designating couplets it is advisable to name first the function which is to be considered the dependent one, that is, the one to be represented by ordinates, and to name secondly the independent function, or the one to be represented by abscissas. Ordinarily we have our choice of order, as for example, we might prefer energy-volume to volume-energy in certain investigations. As noted previously, straight horizontal lines are to be preferred to straight vertical ones; consequently such a couplet as pressure-volume is more desirable than volume-pressure.

**70.** *Pressure-volume and velocity-pressure.*—Two of the derived relations above are of particular interest because of their economic and analytic importance. These are the relations between pressure and volume, and the relations between velocity and pressure. For the first couplet the equation is, as we have seen, $P = K$, or $P = KVo^0$ in its complete form. The corresponding curve is shown in Figure 46. Both the equation and the curve simply indicate the fact that for any value of cumulative production the pressure remains constant. Obviously it is immaterial whether the fluid produced is liquid or gas. Suppose the fluid is gas; then what of the relations between pressure and volume as expressed by Boyle's Law, namely $pv = k$? Clearly this hyperbolic relation between functions in absolute phase is a thing apart from our considerations in mechanics, where we deal in potential phase. To be sure, the gas is obeying Boyle's Law, perfectly or imperfectly, throughout the internal and external systems of the reservoir. That is one phenomenon. The gas is being produced according to laws of mechanics. This is another phenomenon. Contemporaneous phenomena must not be confused, else our understanding of reservoir performance will remain obscure.

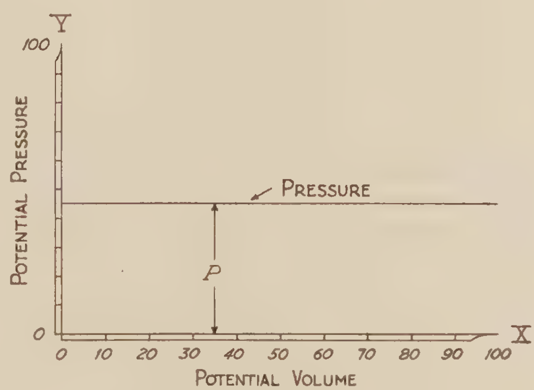


FIG. 46

[5] See § 61.

No curve or equation can be used for stating relations between velocity and pressure in this control, for the reason, as given above, that neither of these functions varies in value during ideal performance. This fact is significant, for it calls our attention to the exact nature of our laws of fluid delivery. These laws are purely analytical expressions which are founded upon ideal performance, and, as stated in section 8, they are equally correct in verbal, algebraic, and graphical forms. They always imply, as previously noted, a variation in at least one of the quantities involved. When these laws are expressed in algebraic form, they are not to be confused with expressions which we can describe as "single-valued equations."

Between velocity and pressure we have the equation of Torricelli's Theorem, as given in section 42. Now the algebraic expression $v = \sqrt{2gh}$ permits us to determine the lineal velocity of fluid issuing from an orifice, regardless of whether the fluid is liquid or gas. For ideal performance in Hydraulic Control it is a single-valued equation, in that there is a single value of $v$ to be computed by the substitution of a single-valued quantity $h$. This equation, if we rewrite it in the symbols of the potential functions of performance,[6] becomes $Ve = KP^{1/2}$. It is evident that this is a single-valued equation for ideal performance in Hydraulic Control, where a single value of $Ve$ expresses the velocity of flow in mass units.[7]

In addition to single-valued equations between velocity and pressure there are also like equations between pressure and power, and between velocity and power. For the same reason these are not expressions of the laws of delivery in Hydraulic Control. As single-valued equations all three expressions hold only in virtue of the laws of delivery in Volumetric Control.[8]

Single-valued relations in Hydraulic Control are verified by experiment. In verifying them we should not confine our experiments to a given reservoir system in ideal performance. Rather we should make definitely known alterations in the conditions of production and observe a series of results. In other words, single-valued relations should be founded upon theoretic performance instead of ideal performance. Now it is an interesting fact, and an important one, that the laws of theoretic performance in Hydraulic and Volumetric controls are identical, and in reality our single-valued equations serve equally well in both. From a practical point of view it is better to perform our experiments in theoretic performance, Hydraulic Control, because in this there are no natural changes taking place in accordance with production in the time required for the experiments.

[6] See § 76 for the transformation of the equation of Torricelli's Theorem into this form.

[7] The equation $Ve = KP^{1/2}$ is not a single-valued equation in Volumetric Control. It is there the expression of a law of delivery. Both functions vary in the process of production. (See § 113.)

[8] In Volumetric Control all the primary functions of performance, except acceleration, vary in the process of production.

The contrast between single-valued equations, on the one hand, and the laws of delivery as expressed by the derived primary function relations, on the other, aids us in making a distinction between Hydraulic and Volumetric controls. And if we see the justification for making a distinction between these controls, I believe we shall understand more readily the peculiarities of Capillary Control. It so happens that single-valued relations in Hydraulic Control are expressed by the same equations as the corresponding laws of delivery in Volumetric Control,[9] but it does not follow that any of these expressions are proper as laws of delivery in Capillary Control.

71. *A pressure diagram.*—Figure 47 is designed with the idea of presenting a complete conception of all the various pressures which function within

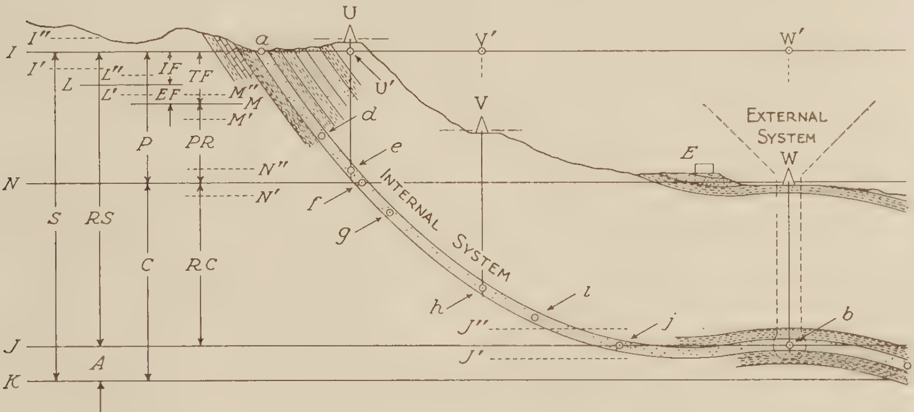


Fig. 47

a reservoir of Hydraulic Control. A natural reservoir that complies in general with the description of the ideal given in section 60 is chosen for this purpose, because it is, while most complicated, the most amenable to a complete analysis, and at the same time the form most closely allied with our present interests. As noted in section 59 there exists no difference between the performance of this reservoir and the performance of one which may be easily set up in the laboratory, provided that in the presence of the porous medium, in case both liquid and gas are present, the pressure is sufficiently great to overpower the Jamin action briefly described in section 29. We are to assume that this action is either absent or, if present, subordinate to pressure.

The scheme in Figure 47 is purely diagrammatic. Any notion of exact relative values for pressures is sacrificed in the attempt to express clearly the general relations which exist between them. The pressures, which are here represented by distances between the horizontal lines, may be taken in terms as

[9] As we shall find later, the laws for ideal performance in Volumetric Control specify the laws for theoretic performance in Hydraulic and Volumetric controls.

desired; for example, in pounds per square inch, or simply as pressure heads in feet of fluid. While each individual reservoir of the classes (*a*) to (*f*) in section 5 will display its own numerical and relative values of pressures, unless indeed some might happen to be either physically or mathematically identical, we may be assured that our analysis need take no account of the fact, so long as the reservoir remains in Hydraulic Control. Furthermore, it is evident that a definite order of lines from bottom to top has been selected. This order we shall find to be a convenient one, although there exist no analytical reasons why any other order, should we have a preference for it, cannot be made to serve. We might invert the entire series of lines, or merely an extreme or middle portion, leaving the remainder as shown.

The diagram shows the productive formation *abc* outcropping at the surface. It lies between two impermeable strata and forms a cylindrical surface with a horizontal axis. Laterally, perpendicularly to the plane of the drawing, its extent is undefined. In accordance with section 20 an orifice at *b*, provided by a single well *W*, separates the internal and external systems of the reservoir.

Four principal lines, *K, J, N*, and *I*, define the five pressures, *S, A, RS, C*, and *P*, of section 62, and also a sixth pressure *RC*, the *registered constant back pressure,* equal to the difference between *C* and *A*. The subordinate lines *M* and *L*, with *N* and *I*, define four more pressures of interest to us. These are as follows:

*IF* = the *internal friction head*
*EF* = the *external friction head*
*TF* = the *total friction head,* the sum of the internal and external friction heads
*PR* = the *residual pressure,* the difference between the potential pressure and the total friction head

In all we have ten pressures, each one bearing a definite relation to the others. Let us consider them separately in connection with the diagram.

The line *K*, representing absolute zero pressure, is fixed and immovable. At the distance *S*, representing the static pressure, is the line *I*. If *W* produces liquid, the casing from *b* to *W* can be imagined as extended upward sufficiently to accommodate a column *bW′*, where *W′* is on a level with the point *a*, the latter marking the position of the free surface of liquid in the formation. If *W* produces either liquid or gas, or both, a pressure gauge attached to the casing, with the well closed in, being located on the line *N* will indicate a pressure equivalent to the distance *WW′*; that is, equivalent to *P*. Liquid is maintained at the level of *a* by sufficient rainfall in the catchment area of the vicinity.

The line *J* is the atmospheric line. It is located from *K* at a distance which is determined by the pressure of the atmosphere. For ideal performance we are to assume that barometric pressures at *a* and *W* are constant in value, and preferably, as shown in the figure, that these pressures at *a* and *W′* are equal.

The distance between $J$ and $I$ represents the registered static pressure, the pressure which would be indicated by an aneroid gauge if it were placed at $b$. The constant back pressure is represented by the distance between the lines $K$ and $N$. It includes the registered constant back pressure and the atmospheric pressure, where the first of these is equal to the weight of the column $bW$, if the well produces liquid, or to the line pressure, if the well produces gas under the ordinary economic conditions. Both of these, the weight of the column and the line pressure, are quantities indicated directly by gauges when these are properly connected with the system. Obviously the weight of the column may be computed from the depth of $b$ and the density of the liquid, if the various pressures are to be expressed in pounds per square inch; otherwise the depth itself may be considered as the pressure head, provided due allowance is made for a possible difference between the density of the liquid in the column and presumably the density of water which is most conveniently assumed as a basis of measuring pressure heads. Like the registered static and atmospheric pressures, the registered constant back pressure is assumed to have a constant, that is, unaltered, value in ideal performance.

The potential pressure, as noted above, can be directly measured at $W$ by gauge readings. It is obviously equal to either $S - C$ or $RS - RC$, although in general it is preferably measured in accordance with the former of these expressions for reasons which were briefly stated in section 11. *The potential pressure is that pressure which determines the fact that there shall be flow at the orifice.*

In measuring these six pressures we need not concern ourselves in the least with the location, extent, thickness, the "lay" or structure, and the texture of the formation. We may, or may not, have knowledge of these features, which are purely geological. Though the existence of a reservoir such as here pictured depends upon geological features, with its existence granted our problem becomes immediately one in mechanics alone.

*Of the six pressures, but one, the potential pressure, lies entirely above the line* N *in our diagram;* the other five lie entirely or partially below it. Now it is convenient to subdivide this potential pressure into the subordinate pressures listed above. Let us consider these.

Flow at $b$ is associated with the movement of all the fluid between $a$ and $b$, and there is necessarily a frictional force in this internal system which is defined in extent by $a$ and $b$, one which tends to retard the movement of the fluid, and one whose amount in any case is real and definite, and capable of representation by a distance $IF$ between lines $L$ and $I$. Again, flow at $b$ is associated with the movement of the fluid in the casing and in any flow-line which might be attached at $W$. There is necessarily a frictional force in this external system which, we shall say, includes $b$ itself and all the line exterior to it, a force which also tends to retard the movement of the fluid, one whose amount in any case is likewise real and definite, and capable of representation by a distance $EF$ between lines $M$ and $L$. Together the two friction heads

amount to $TF$, the total friction head. We are not to concern ourselves with the numerical values of these friction heads; it is sufficient to acknowledge their existence and understand their analytical effects upon production in the present control.[10]

The difference between the potential pressure and the total friction head is the residual pressure, represented by the distance between the lines $N$ and $M$. *The residual pressure determines the velocity of flow at* b *under the conditions of production.* It does not determine the fact that there shall be flow at $b$, for this is the province of the potential pressure alone. Inasmuch as we are not to concern ourselves with the numerical value of the total friction head, we are likewise not to concern ourselves with the numerical value of the residual pressure.

Now with the lines $K$, $J$, $N$, $M$, $L$, and $I$ remaining unaltered in position during flow, our ideal natural reservoir produces in ideal performance according to the fundamental primary function curves of Figure 45. This reservoir may produce liquid alone, gas alone, or liquid and gas in combination. In case of the latter we must proceed farther with our investigation.

72. *The ideal combination reservoir.*—Figure 47 represents a finite portion of a mathematically infinite reservoir system which includes, in addition, the sun, the seas, and the atmosphere with its clouds and rain. It is not within the scope of mechanics to consider the origin of the heat of the sun, the origin of the water in the seas, and the causes of clouds and rains. Neither is it within the scope of mechanics to consider the origin of the fluids—oil, gas, and water, either singly or in combination—which happen to be encountered within our natural reservoirs. These things all exist, and the science of mechanics deals with them as they exist. Now our reservoir is to contain liquid and gas. Certainly a portion, and possibly all, of the liquid is water, either comparatively free of dissolved solid matter or considerably impregnated with it. In virtue of the favorable structural fold in the formation at depth oil may have accumulated and formed a pool in the vicinity of the well, located as at $W$. If the liquid is entirely water, the gas which we specify to be present in solution might be such as carbon dioxide and hydrogen sulphide. Where oil is present we usually find the chemically associated hydrocarbon gases and vapors with it.

The reservoir is to produce the two classes of fluid; it is to be, as previously stated, one which may be designated conveniently a *combination reservoir*. If it is to be ideal in the sense that its performance may be ideal with respect to both fluids, it is essential that the following specifications be exactly fulfilled:

*a*) More important than before is the fact that the texture of the formation must be homogeneous in all directions.

[10] See footnote 3, § 48, page 81.

*b*) With respect to the fluids certain homogeneous conditions must exist. Either there shall be but one liquid with homogeneous physical properties throughout the reservoir, or there shall be more than one, provided the same proportional amount of each is to be found underlying every horizontal area of unit dimension throughout the lateral extent of the reservoir.[11] Furthermore, there shall be but one gas, or but one homogeneous mixture of gases, properly distributed in the dissolved state throughout the mass of the liquid, and subject to pass into the free state upon the release of pressure. In so far as the supply of the liquid is continually replenished during production, so must the supply of gas be likewise continually replenished.[12]

The ideal well which is to reveal the ideal performance of this reservoir must penetrate the formation at a central location with respect to either a moderate structural fold of a reasonable, large, horizontal extent, or a plane section of the formation of like extent. The fold may be anticlinal or synclinal, symmetrical or asymmetrical, and the plane section may be horizontal or moderately inclined.

The foregoing conditions regarding the nature, the distribution, and the replenishment of the fluids, while necessary for ideal performance throughout finite time, are extreme. Let us say that, in so far as the reservoir of Figure 47 is concerned, the perfectly homogeneous nature and distribution of fluids is certainly true with respect to the areal extent of the fold which surrounds the well.[13] In this manner we may ignore the necessity of gas replenishment; we restrict uninterrupted ideal performance to a finite period of time, at the end of which it is possible, and probable, that the production of gas will diminish and finally reach zero. In such an event the reservoir is no longer a combination one; it proceeds thereafter in ideal performance for the delivery of liquid alone for a new period of time. The restriction to a consideration of the first period of time satisfies analytical requirements for an interval which is so frequently defined by our economic interest in combination reservoirs of this description.

73. *Proportional production of gas.*—The performance of this ideal combination reservoir now fulfills the curves of Figure 45. All six curves pertain to the production of liquid, whereas we shall find that but four of them pertain to the production of gas which accompanies the liquid. These are the pressure-time, acceleration-time, energy-time, and power-time curves. As stated in section 59, the gas in this reservoir of the open type possesses no energy which is intrinsically its own; all that it possesses is due solely to the pressure of the column of liquid which bears its weight upon it. The pres-

[11] It is not necessary here to define the area of unit dimension.

[12] The availability of this gas for replenishment is to be assumed in the case of the ideal reservoir.

[13] That is, we will assume the existence of a well-defined pool on the structure.

ence of the gas is not at all necessary for the production of liquid. Nevertheless, according to the discussion in section 65, there is a dynamic relation between the gas so produced with liquid and the energy curve, regardless of the fact that the reservoir is of the open type. But we must not on this account be misled with respect to the rôle played by the gas. Its office is merely that of an agent in all combination reservoirs which are open to the atmosphere at some point other than possibly at the orifice, an agent in the sense that in so far as it receives its energy from liquid, it may either temporarily or locally perform work for the liquid in the process of production, provided only that circumstances permit it to do so.[14]

The ideal well in our reservoir is given a central location with regard to the geological structure in its vicinity in order that there shall be no temporary or permanent accumulation in, or production from, a near-by space which may define a gas pocket. All that, or only that, proportional amount of gas which normally dissolves in the liquid at the static pressure of the reservoir is produced with the liquid.

The amount of gas dissolved in the liquid, and therefore the amount of gas produced with the liquid under the ideal conditions specified, is dependent upon the static pressure of the reservoir in accordance with Henry's Law.[15] Let us compare Figures 16 and 38. The curve *ab* of the former corresponds to the pressure curve of the latter. A variable potential pressure $P$ is now a constant $P$; $p_1$ and $p_2$, now equal, are designated by the symbol $S$; and $C$, referring to the axes $X$ and $X'$, remains the same. The amounts of gas, $v_1$ and $v_2$, are equal because of a constant $S$. These amounts, without subscripts, appear in Equation 8 (p. 63) as follows:

$$v = k\ (P + C)$$

wherein, as just noted, $v$ is the gas dissolved and the gas produced. The gas $v$ would be measured, say, in cubic feet per barrel of oil; it would refer to the gas as separated from the oil at absolute zero pressure, and thereafter measured at atmospheric pressure.

What proportion of the gas $v$ is liberated from solution by the time the fluids reach the point $b$ in Figure 47? Only that which is designated as $v'$ in Figure 16; that is, according to section 39, $v' = kP$. The amount $v''$ still re-

[14] For example, a combination reservoir may possess a chamber immediately at, or anterior to, the orifice, the chamber itself being supplied continuously by means of a small opening. Now flow may be permitted at intervals, and the gas within the chamber will temporarily act as the agent of production. This action takes place in properly constructed artificial reservoirs, and occasionally we find it in natural reservoirs.

[15] Obviously the statement holds only for a given reservoir with its fluids specified in quality and quantity. In general we must say that the amount of gas produced with the liquid under specified conditions is dependent upon ($a$) the static pressure of the reservoir, ($b$) the solubility of the gas in the liquid at standard temperature and pressure, and ($c$) the degree of saturation of the liquid with the gas.

mains in solution, where $v'' = kC$. Upon passing $b$ the fluids ascend the casing, experiencing a further release of pressure. As a consequence more gas assumes the free state. In amount this is $v''' = k(RC)$. The fluids are now subject only to the pressure of the atmosphere. There is still in solution the amount of gas, $v'''' = kA$.

The number of cubic feet of gas per barrel of oil in solution at a pressure $S$ is determinable by a laboratory test upon the given fluids issuing from the reservoir. In any case we may say that this number is 100 per cent. Thus we are permitted to write the following:

$$v = S\% = 100\%$$

$$v' = P\%$$

$$v'' = C\%$$

$$v''' = RC\%$$

and

$$v'''' = A\%$$

In addition we have

$$v - v'''' = S\% - A\% = RS\%$$

Since the various pressures for any given reservoir are known, or at least are determinable, the corresponding proportional amounts as expressed in percentages are determinablè. It is obvious that measurements of the proportional production of gas with liquid are dependent upon these quantities.

**74.** *Gas-time relations.*—Let us establish the relation between the rate at which gas is produced and the rate at which energy is displaced; that is, the relation between *gas velocity* and power.

*The proportional production of gas, multiplied by the velocity for the liquid, equals the gas velocity.* For example, $M$ cubic feet of gas per barrel of oil, times $N$ barrels of oil per day, equals $MN$ cubic feet of gas per day. The same thing may be expressed mathematically thus:

$$\frac{M \text{ cubic feet of gas}}{1 \text{ barrel of oil}} \times \frac{N \text{ barrels of oil}}{1 \text{ day}}$$

$$= \frac{MN \text{ cubic feet of gas}}{1 \text{ day}} \quad \text{.......................(66)}$$

For convenience suppose we abbreviate this equation in the following manner:

$$v \times ve = \text{gas velocity} \quad \text{...................(67)}$$

Now by Equation 3 (p. 62), $v = kp$. Therefore we may write

$$v/k = p \quad \text{........................(68)}$$

Furthermore, according to section 16,

$$p \times ve = \text{power} \quad \ldots\ldots (69)$$

and into this equation we may substitute the value of $p$ from Equation 68; thus

$$v/k \times ve = \text{power} \quad \ldots\ldots (70)$$

It therefore follows that

$$v \times ve = k \times \text{power} \quad \ldots\ldots (71)$$

Now from Equations 67 and 71 we may know that

$$\text{gas velocity} = k \times \text{power} \quad \ldots\ldots (72)$$

That is to say, *the rate at which gas is produced is directly proportional to the rate at which energy is displaced.* The amount of gas produced per day by a combination reservoir is a measure of the amount of energy delivered per day. As a consequence, if we understand power-time and energy-time relations, we are in a position to understand gas velocity and *gas volume,* the latter being the cumulative production of gas from the combination reservoir. This is our justification for the inclusion of power and energy in the primary functions of performance.

All terms in the seven equations above represent their respective quantities in general form; no distinction need be made in them concerning the exact nature of pressure, velocity, and power, for these may equally well be such as we now accept as primary functions of performance as measured from a potential horizontal axis, or otherwise. These seven relations, like those of the preceding section, hold for combination reservoirs in the three controls in precisely the same manner, for they depend only upon Henry's Law and the relations between functions of performance which we represent by ordinates in our curves. However, as soon as we enter into the consideration of gas-time relations, the question of control arises, since the functions of performance vary differently with respect to time in the controls. Where the functions possess values according to different paths with time represented by abscissas in our curves, we may expect gas-time relations likewise to travel different paths.

Each control will require, then, a separate investigation concerning gas-time relations. Here, of course, we shall confine our attention to Hydraulic Control, and in pursuing this subject we shall ignore the simplicity of this control that results from its mathematical deficiency. We shall proceed as if it were as complicated as the finite controls, in order to establish the particular method of investigation. This is done with the hope that we may be able to approach our later investigations with better understanding.

From the right-hand column of Figure 33 we learn of certain quantities with which we must deal. There we find the absolute energy of the system divided into the following four portions:

$$PVo, \quad CVo, \quad Pvo, \quad \text{and} \quad Cvo.$$

Corresponding to each of these divisions there are certain portions of power, as rates of displacement of energy, which enter into our analysis of production from the system. Let us consider four equations, as follows:

$$Ve = K_1 \quad \text{(73)}$$

$$P = K_2 \quad \text{(74)}$$

$$C = K_3 \quad \text{(75)}$$

and

$$ve = K_4 \quad \text{(76)}$$

The first three are in accord with the laws of delivery for this control. Values of the functions are indicated by subscripted $K$'s.[16] Now the fourth equation needs an explanation.

In Figure 47 it is easy to see that if the inflow of water is entirely shut off at $a$, the reservoir system eventually arrives at a state of equilibrium. Liquid remains at rest between $f$ and $b$ on account of the constant back pressure $C$ due to the weight of liquid between $W$ and $b$. We might say that all liquid between $f$ and $W$ constitutes a quantity $vo$ corresponding to $C$. This quantity $vo$ is retained by the reservoir. May we not recognize $vo$ even in the event that the rate of inflow at $a$ is equal to the rate of production from $W$? Certainly the presence of this quantity of liquid is independent of the existence of the situation at $a$. And may we not proceed farther in conceiving of a quantity $ve$ which in the life of the reservoir results in an accumulated quantity of liquid $vo$? We may say that $ve$ represents a conceptual rate of retention, or rate of accumulation, properly due to the existence of $C$.[17] The quantity $ve$ is of a constant value; consequently we may write Equation 76.

Now corresponding to the four portions of energy there are the following portions of power:

$$PVe, \quad CVe, \quad Pve, \quad \text{and} \quad Cve.$$

To be exact we should say that the first three of these quantities represent rates of displacement of energy, while the fourth represents a rate of retention or a rate of accumulation of energy within the system.

For the purpose of our analysis we will take $Ve + ve = 100\%$, and $P + C = 100\%$. By means of these we are to apportion the percentage values of the four portions of power. Furthermore, by means of these, we are to

[16] In Equation 75 we shall use the constant $K_3$ for the value of the constant $C$, as a matter of symmetry among all equations pertaining to the production of gas and liquid in combination.

[17] This quantity $ve$ is not to be confused in any way with the quantity $ve$ of Equations 69, 70, and 71. I believe the idea of this conceptual rate of retention is to be more readily grasped in the study of reservoirs in Volumetric Control. As we are to deal with $ve$ again in § 116, I would suggest a return to the present section for a further consideration of its significance in Hydraulic Control.

apportion the percentage values of the quantities of gas involved in the process of production. This is possible in virtue of the fact that power and gas velocity are related in the manner shown by Equations 71 and 72 (see above).[18]

We are now prepared to consider the production of gas, as an accompaniment to the production of liquid from a combination reservoir. In following a policy that is adopted merely as a matter of convenience, wherever no ambiguity might arise, we shall omit the sign for per cent in our equations.

The first gas-time equations to consider are those between the proportional production of gas to liquid and time. These show how the gas-oil ratio, for example, behaves in the process of production.[19]

If we define a quantity of gas corresponding to the potential pressure as *potential gas,* we may write the following expression directly from Equation 74:

$$G_{Pp} = K_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (77)$$

an equation which shows that the gas in the free state immediately at the orifice is constant with time. But there is also gas still in solution immediately at the orifice. From Equation 75 we may write the following:

$$G_{Cp} = K_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (78)$$

an equation which shows that the gas in the dissolved state immediately at the orifice is constant with time. This equation also shows the proportional gas to liquid as both are retained by the reservoir in virtue of the constant back pressure.

The total proportional production of gas is equal to the sum of Equations 77 and 78:

$$G_{Sp} = K_2 + K_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (79)$$

This is the quantity of gas per unit volume of liquid, per unit of time, as produced into a perfect vacuum, the fluids separated, and thereafter measured at atmospheric pressure.

Where production takes place against a constant back pressure $C$, greater than the pressure of the atmosphere $A$, yet where final delivery actually places the fluids under the pressure of the atmosphere, and the gas is separated from the liquid and measured at this pressure, the quantity is

$$G_{RSp} = K_2 + K_5 \quad \ldots\ldots\ldots\ldots\ldots\ldots (80)$$

$K_5$ being less than $K_3$ by an amount equal to $A$ in terms of $S$, all quantities being expressed, as usual, in percentage values.

[18] The constants $k$ appearing in the equations vanish in percentage equations.

[19] In this chapter we are to ignore the phenomenon known as the *by-passing of the gas.* This subject will be treated in chapter xiv.

The equations for the proportional production of gas to liquid depend, fundamentally, upon two quantities, $P$ and $C$. The equations between gas velocity and time, and those between gas volume and time, involve not only $P$ and $C$ but also the quantities $Ve$ and $ve$ for the liquid. Now these gas velocity and gas volume equations are to be derived from power-time and energy-time equations, respectively. The total power—the total rate of displacement and accumulation of energy—involved in the process of production is

$$Po_a = PVe + CVe + Pve + Cve \quad \ldots\ldots (81)$$

where the terms on the right are those given above. By means of Equations 73 to 76, inclusive, we may write

$$Po_a = K_1K_2 + K_1K_3 + K_2K_4 + K_3K_4 \quad \ldots\ldots (82)$$

The integration of Equation 81 with respect to time gives

$$E_a = PVo + CVo + Pvo + Cvo \quad \ldots\ldots (83)$$

wherein we revert to the energy equation previously given in section 51. Again, the integration of Equation 82 with respect to time gives

$$E_a = K_1K_2T + K_1K_3T + K_2K_4T + K_3K_4T \quad \ldots\ldots (84)$$

It is from these four equations that all gas velocity and gas volume equations are derived.

With the first right-hand terms of Equations 81 and 82 we find that

$$Po = PVe \quad \ldots\ldots (85)$$

which is the relation between the potential functions of power, pressure, and velocity, as already known. Now we may write

$$G_{Ve} = K_1K_2 \quad \ldots\ldots (86)$$

This is the relation between the rate of production of *potential gas* and time.

Likewise, with the first right-hand terms of Equations 83 and 84 we find that

$$E = PVo \quad \ldots\ldots (87)$$

which is the relation between the potential functions of energy, pressure, and volume, also as already known. Again we may write

$$G_{Vo} = K_1K_2T \quad \ldots\ldots (88)$$

This is the relation between the cumulative production of potential gas and time.

The second and third terms on the right in Equations 81 and 82 represent, when taken together, *suppressed power,* because of the relation between these quantities and their correspondents[20] in Equations 83 and 84. Then we have

$$Po' = CVe + Pve \quad \ldots\ldots (89)$$

[20] See § 51, and Fig. 33, page 103.

and in correspondence with this,

$$G'_{Ve} = K_1K_3 + K_2K_4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (90)$$

the latter refers to *suppressed gas.* It is produced with the potential gas, but the power and energy to which it pertains are themselves suppressed, since they are prevented from performing useful work in the process of production.[21]

The second and third terms on the right in Equations 83 and 84 represent, when taken together, *suppressed energy.* Then we have

$$E' = CVo + Pvo \quad \ldots\ldots\ldots\ldots\ldots\ldots (91)$$

and in correspondence with this,

$$G'_{Vo} = K_1K_3T + K_2K_4T \quad \ldots\ldots\ldots\ldots\ldots\ldots (92)$$

Whereas in Equation 90 we have the rate at which the suppressed gas is produced, in Equation 92 we have the cumulative amount of this gas.

All equations referring to suppressed power, suppressed energy, and suppressed gas pertain to production of gas and liquid against a constant back pressure, the fluids thereafter separated in a perfect vacuum, and measured at atmospheric pressure. Where the fluids are separated at atmospheric pressure, and measured accordingly, due allowance must be made for the gas remaining in solution at the pressure $A$.[22]

As a certain amount of energy remains within the reservoir in virtue of the constant back pressure, so also a certain amount of gas remains there. In accordance with the last terms of Equations 83 and 84 these are

$$E'' = Cvo \quad \ldots\ldots\ldots\ldots\ldots\ldots (93)$$

and

$$G''_{Vo} = K_3K_4T \quad \ldots\ldots\ldots\ldots\ldots\ldots (94)$$

The corresponding rate equations may be written from the last terms in Equations 81 and 82. This energy and gas is not available under the given conditions of production.[23]

[21] See footnote 4, § 51, page 92.

[22] Equation 80 displays such an allowance.

[23] This can be made effective to the extent of a reduction in the constant back pressure. The reduction sets up new values of $P$ and $Vo$, each including amounts previously non-effective. As a matter of fact we are to conclude from the discussion in § 116 that for any interval of time short of infinite time the rate $ve$ is zero; that is, $K_4$ is zero. $Po''$ and $G''_{Ve}$, corresponding to $E''$ and $G''_{Vo}$, are therefore zero, and consequently $E''$ and $G''_{Vo}$ are themselves zero for any interval of time short of infinite time. (See footnote 5, § 116, p. 291.) In pursuing this subject we have ignored the simplicity of this control that results from its mathematical deficiency, and we have proceeded as if it were as complicated as the finite controls, in order to establish the particular method of investigation. (See text above.)

For Hydraulic Control all the equations between power and gas velocity on the one hand and time on the other may be reduced to

$$\text{power} = k$$

and

$$\text{gas velocity} = k$$

Similarly, all the equations between energy and gas volume on the one hand and time on the other may be reduced to

$$\text{energy} = kt$$

and

$$\text{gas volume} = kt$$

Regardless of the number of terms on the right-hand side of the equation, each term contains the function $t$ with the same exponent, either zero or unity. We shall find that for the finite controls not only does the function $t$ appear with exponents other than these, but also that in all equations having more than one term on the right $t$ possesses different exponents in each term. This means that where we need not discriminate between potential and suppressed functions in Hydraulic Control, we must do so in Volumetric and Capillary controls.

CHAPTER X

# Theoretic Performance

"Mechanics is a twin sister of geometry; both sciences are applications of pure mathematics; the propositions of both, as to their certainty, stand on the same level; we have just as much right to ascribe absolute certainty to mechanical theorems as to geometrical."—GUSTAV KIRCHOFF

75. *Introduction.*—We have hitherto established the laws for the delivery of fluid from reservoirs in Hydraulic Control. These laws are based upon a comprehensive ideal; not only is the reservoir, as a physical container, perfectly ideal, but also are the imposed conditions upon its performance. No act originating either with Nature or with us disturbs the mathematically defined paths of the primary functions. Let us now consider theoretic performance. With the same ideal reservoir, be it such as shown in Figure 35, Figure 36, or Figure 47, we shall deliberately make alterations in the conditions for production, as provided for earlier in section 22. These alterations shall be definitely known, in that they shall be accurately defined by their effects upon the behavior of the primary functions of performance, particularly by their effects upon pressure-time and velocity-time relations. These effects are determined, of course, by observations upon performance, before and after the alterations are made.

It appears that all possible alterations fall into three groups which are conveniently designated as Cases 1, 2, and 3. The *first case* is concerned with any alteration that involves a manipulation of the orifice; that is, a change in the size or physical condition of the orifice. The bottom of a well may become partially closed with rock material—the well "sands up," or the formation "caves in"—and any paraffine or moisture happening to be present may freeze and assume a solid form. Conversely, if any of these circumstances already exist, the operator may clean the well. In any event it is evident that the velocity of production is affected in one way or the other. Such alterations as these are due to accidents which are largely beyond our control. There are, in addition, alterations that are entirely within our control. We may change the size of the tubing within the casing, or change the length and size of the line at the surface; we may install in the line such mechanical devices as valves and flow-nipples, and make changes within them from time to time.

These alterations likewise affect the velocity of production.[1] We shall consider all alterations of these two kinds to be analytically equivalent.

The *second case* is concerned with alterations in the value of the static pressure of the reservoir. We are to include here only such changes which may be described as "erratic," in so far as they are not in any way associated with time as a function of performance; that is, they are not changes which are due to a decline in accordance with production. In Figure 35 the weight of the bell-top may be altered by means of auxiliary loads which may either rest upon it or be attached to it in any manner whatever. In Figures 36 and 47 the level of the free surface of the liquid may be altered. In one we may raise or lower the spillway which guarantees a constant head, and in the other we may assume that seasonal changes in the weather may allow the head to fluctuate, and thus to possess different values from time to time. Alterations of this nature constitute changes in the value of the *real static pressure*. There are, in addition, alterations which cause an apparent change in the value of the static pressure, and these are of particular importance with such reservoirs as that of Figure 47. If in addition to $W$ there were one or more wells producing from the same reservoir, the closing-in of $W$ alone would register only an *apparent static pressure,* less than the real static pressure by an amount which depends upon the proximity and the number of the additional wells that are flowing at the time. Our attention is called to the fact that, from the point of view of the reservoir at large, we are not required to say that each well is a separate orifice, unless we choose to do so. It is perfectly permissible to group all closely adjoining wells together, and say that they constitute one orifice, conveniently termed a *multiple orifice.* It is as though a metal plate containing many holes were placed at the openings of the artificial tanks of the earlier figures. In such a circumstance it is obvious that we may consider the separate holes in such a plate as orifices, or all of them together as one orifice.

In the study of the primary functions in theoretic performance it is convenient to consider each well as a separate orifice. The principles are, of course, easily applied thereafter to a group of closely adjoining wells. Furthermore, in confining our attention here to the individual well we can treat real and apparent alterations in the static pressure together. For the single well there is no difference in the effects of these two alterations. The conception of a multiple orifice is of particular advantage in the study of the secondary functions in this performance.

The *third case* is concerned with alterations in the value of the constant back pressure. In Figure 35 the line pressure may be altered; in Figure 36 a flow-line which places delivery at some position other than at the dotted line may be attached to the tank; and in Figure 47 the well, instead of delivering

[1] Obviously all these alterations pertain to natural reservoirs. All have their analogous alterations in artificial reservoirs.

at the derrick floor, may be caused to deliver directly into a tank $E$, or it may be caused to deliver at some level between $b$ and $W$ by the removal of the upper part of the column with a pump. (In order that delivery may take place directly at $b$ it is necessary that the capacity of the pump be sufficient to remove the liquid as rapidly as it arrives at this point. At a less capacity the liquid will stand accordingly in the well.)

In citing these examples of the three cases I have not attempted to include all possible situations, nor all degrees of the ones mentioned. In consideration of these, however, we are presumably prepared to proceed. Whereas in ideal performance each primary function travels along a continuous, uninterrupted path, or "sweep," toward a state of equilibrium which is never attained in this control, in theoretic performance the sweep is interrupted at each and every alteration made in conformity with the cases. Time, or life, let us say, is divided into *periods* whose lengths are determined by the alterations. The performance during each individual period of time is yet ideal, under the prescribed conditions of production, although as between one period and its predecessor or successor, the performance is theoretic.

76. *Case 1. Alterations in external friction.*—Upon alterations in accordance with Case 1 the major lines, $K$, $J$, $N$, and $I$, in Figure 47, are not affected in their positions in any way. The values of the functions to which all distances between these lines refer remain unchanged. While the value of $P$ remains the same, its subdivisions alter their values; consequently the lines between $N$ and $I$ move to new positions. Briefly stated, the situation is as follows:[2]

| | |
|---|---|
| Static Pressure | Unaltered |
| Constant Back Pressure | Unaltered |
| Potential Pressure | Unaltered |
| External Friction Head | Altered |
| Internal Friction Head | Altered |
| Total Friction Head | Altered |
| Residual Pressure | Altered |

An alteration in the size or condition of the orifice first affects the value of the external friction head; a partially closed orifice causes the line $M$ to shift in the direction of $M'$, for the external friction head is increased; and, conversely, an enlarged orifice causes $M$ to shift in the direction of $M''$, since the external friction head is decreased. Under both these circumstances the line $L$ shifts in the direction opposite to that for $M$, since an increase in the external friction is accompanied by a decrease in the internal friction, and vice versa. Quantitatively the distance moved by $L$ is always less than that moved by $M$; therefore, as $EF$ increases or decreases, $TF$ also increases

[2] The ten pressures indicated in Fig. 47 are co-dimensional in pounds per square inch or in feet of liquid. Of the ten only seven are required to give a complete description of the alteration. If desired, however, the other three may be added to the list.

or decreases, respectively. In other words, there is a concordant relation between alterations in the external and total friction heads, and a discordant relation between both of these and the internal friction head. To say that the relations are concordant or discordant merely refers to the fact that the heads alter in the same or opposite directions, respectively; no inference is made with respect to comparative values of the heads. Now to continue, an increase or decrease in $TF$ means a decrease or increase in $PR$, respectively, inasmuch as their sum must remain constant. We may claim not only that $TF$ and $PR$ are discordant in their relations, but also that there is a *quantitative exchange in units* between them; what one gains, the other loses, and vice versa. As previously stated, $PR$ determines the velocity of production; if it is decreased in value, a slower rate of flow from the orifice is the result, and vice versa.

It is not necessary that we discriminate between causes and effects in this shifting of $M$ and $L$; consequently for our purposes we may with equal justification argue either forward or backward with respect to the various pressure heads included between the lines $N$ and $I$. That is to say, whereas the above discussion begins with the external friction head and ends with the residual pressure, it might as well begin with the residual pressure and end with the external friction head. The essential point to be brought out by either argument is the same, namely, that we understand clearly the relation between the velocity of production and the pressure heads, and that we know how our lines shift in accord with the alterations in Case 1.

As an extreme situation let us consider the well at $W$ completely closed. Now $EF = P$, $IF =$ zero, $TF = EF = P$, and $PR =$ zero; the line $L$ coincides with $I$, and the line $M$ coincides with $N$. There is no flow at $W$. While a valve at the well is gradually changed from an open position to a closed one, it is possible to visualize the gradual shifting of the lines to their limiting positions. The minimum velocity of flow from a reservoir is determined by external friction. Obviously, the minimum is easily established at zero.

The maximum velocity of flow from a reservoir is established by internal friction. With the well at $W$ it is easy to reach a point in the opening of the valve where further opening has no effect upon the rate of flow from the reservoir. The friction head on the upstream side of this valve, that is, the internal friction head for the valve itself considered as an orifice, determines the maximum velocity for the fluid. We may conceive the locus of this type of phenomenon to be at the bottom of the well, where we advantageously consider the orifice of a natural reservoir to be, and say that it now appears that this orifice can reach a point in its opening—by drilling additional wells in the vicinity—where further opening has no effect upon the rate of flow from the reservoir at large. This is indeed a true state of affairs as we shall learn in our study of the secondary functions of performance.

Although the internal friction head with hollow artificial reservoirs is generally so small in amount as to be safely ignored, with natural reservoirs the situation is quite different. In these the porous medium provides an

internal friction head of considerable analytical importance, for the production of oil and gas is, in certain respects, largely influenced by it. In a given reservoir this head, of course, depends upon the lineal velocity of the fluid in the reservoir. Since the latter depends upon the mass-velocity of production at the orifice, it is evident that the internal friction head indirectly depends upon the velocity of production. We can say that *when there is no flow at the orifice, there is no internal friction, for no fluid is in movement, and when flow at the orifice is a maximum, internal friction is also a maximum.* There exists a concordant relation between alterations in the internal friction head and velocity.

The residual pressure, as previously stated, is that pressure which determines the velocity of production. When this pressure is zero, velocity is zero, and when it is at a maximum, velocity is likewise at a maximum. Now the relation between these two functions may be taken from experience. It is none other than the relation expressed by Torricelli's Theorem. We can write it thus:

$$Ve = K(PR)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (95)$$

wherein a mass-velocity $Ve$ replaces a lineal velocity $v$, $K$ replaces the square root of $2g$, and $PR$, representing the residual pressure,[3] replaces $h$.

According to section 71 we are not to concern ourselves with the numerical values of the friction heads and the residual pressure. To understand their analytical effects upon production we may conveniently deal with them in their percentage values. Any set of conditions for production from a given reservoir may be adopted by us as a standard for the purpose of making subsequent comparisons, and for these conditions we can arbitrarily say that each of the various heads, and likewise the velocity, possesses a value of 100 per cent. *It is not necessary that the most favorable conditions be taken as the standard, for quantities greater than 100 per cent are handled with the same facility as those less than this amount.* Presumably most of us would be inclined to adopt what we consider as "normal conditions" for production, and base comparisons upon them. Having once selected our standard we can thereafter follow the variations of the functions. For example, from the equation above we may write the following expression in the form of ratios:

$$\frac{Ve_1}{Ve_2} = \frac{(PR)_1^{1/2}}{(PR)_2^{1/2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

and for the quantities $Ve_2$ and $(PR)_2$ we may substitute values of 100 per cent, drop the remaining subscripts, and reduce the expression to

$$Ve = 10\,(PR)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (97)$$

[3] In Equation 25 (p. 67) the pressure we now describe as $PR$ is assumed to be the same as $h$, but in acknowledgment of the fact that $PR$ differs from $h$ in practice certain constants $c$ and $c'$ are included in Equations 26 and 27 (p. 68).

Whenever the percentage change in velocity is known, the corresponding percentage change in the residual pressure may be computed from this equation. Perhaps the relation is more conveniently expressed in the following form:

$$PR = \frac{1}{100} Ve^2 \qquad (98)$$

If we wish to say that under the adopted standard conditions the internal friction head and the residual pressure both have values of 100 per cent, then we shall find that alterations in accordance with this case cause these two functions to change in the same proportion. This is to say that

$$IF = K(PR) \qquad (99)$$

When we study reservoirs in Volumetric Control we shall see that the relation as expressed by this equation is correct; therefore let us accept it for the present without question. Now by expressing the functions in percentages the constant $K$ may be omitted;[4] thus,

$$IF = PR \qquad (100)$$

Between this and Equation 98 we may derive the following relation:

$$IF = \frac{1}{100} Ve^2 \qquad (101)$$

The internal friction head varies as the square of the velocity of production. By the adoption of standard conditions we are able to determine the percentage variation between these functions.

Hereafter, when the relations between two functions may be expressed in the form of Equation 100, let us say that there is a *harmonious percentage variation* between the functions, and when expressed in the form of Equation 101 that there is a *parabolic percentage variation* between the functions.

77. *Case 1 (continued)*.—The potential pressure $P$ cannot be included in the discussion of variations, since its value remains constant for alterations which fall within this case. For example, the following equation is improper: $Ve = KP^{1/2}$. The fact that flow-meters may be calibrated, or that the constants in their empirical equations may be determined by actual tests with reservoirs in Hydraulic Control, upon which are imposed alterations in accordance with this case, is dependent upon Equation 95 (p. 148). It is not concerned in any way with this improper equation.

Obviously the potential pressure-time curve for Case 1 is the uninterrupted, straight horizontal line as in ideal performance. The residual pressure-time curve would, upon arbitrarily assumed alterations, appear as shown in

[4] That is, $IF\,\% = PR\,\%$. We are to hold in mind the fact that Equations 97 to 101, inclusive, are in percentage values, and not in unit values. The convenience afforded by the omission of the per cent sign will be appreciated when we study the finite controls.

Figure 48. Ordinates are conveniently expressed in percentages based upon standard conditions, and abscissas are measured in units of time elapsed, with the origin at the left in the usual manner. The line $A$ represents the path of the function at standard conditions, 100 per cent, during a first period of time. Owing to the first alteration we have, say, the line $B$ during a second period; and owing to a second alteration, the line $C$ during a third period; and so on. Only for the sake of convenience have the lines subsequent to $A$ been provided at positions less than 100 per cent.

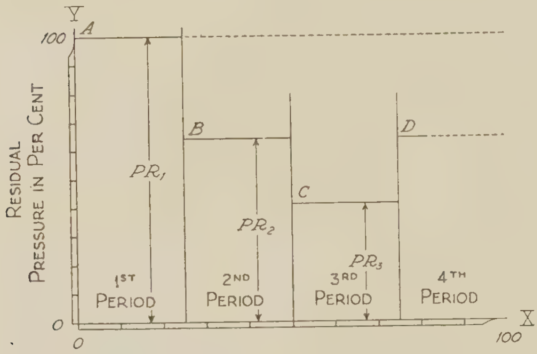


FIG. 48

As a matter of fact the successive values for $PR$ are assumed as follows; and corresponding to them, in accordance with Equation 98 (p. 149), are the values for $Ve$, likewise expressed in percentages which are based upon standard conditions:

| Line | $PR$ | $Ve$ |
|---|---|---|
| $A$ | 100.00% | 100.00% |
| $B$ | 64.00% | 80.00% |
| $C$ | 40.96% | 64.00% |

The velocity-time curve appears as in Figure 49, where the periods of time correspond to those of the preceding figure.

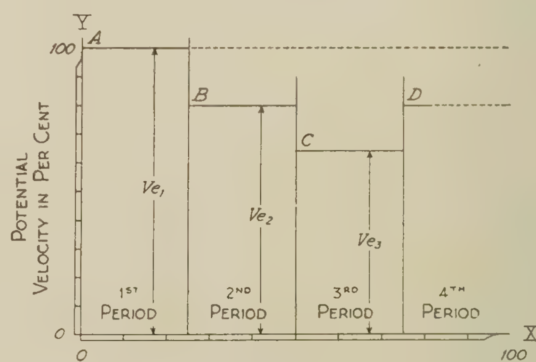


FIG. 49

To illustrate the analytical principles which are involved in Case 1 let us suppose that we have a gas well producing from a reservoir in this control. It is provided with a valve, on the upstream side of which is a pressure gauge. Presumably the well produces directly into the line which extends to the point or points of consumption. The line pressure is assumed to be constant. Now the gauge will register the external friction head, plus the constant back pressure, for any position of the valve. We should first take two observations, as follows:

*a*) The valve is fully open. This provides for the lowest possible value

registered by the gauge for the given well in its present condition and with its present equipment. External friction head is at its lowest, internal friction head is at its highest, and the residual pressure is at its highest. We have the greatest velocity possible when the valve is in this position. We shall say that the internal friction head is 100 per cent, the residual pressure is 100 per cent, and the sum of the two, at the same time, is also 100 per cent.[5] These are our standard conditions, say, for the basis of subsequent comparisons.

*b*) The valve is completely closed. The gauge now indicates the registered static pressure of the reservoir. But we have already seen that there is a further significance to be attached to this reading: The external friction head is at its highest, the internal friction head is zero, and the residual pressure is zero. We have zero velocity with the valve in this position.

These two readings are extreme, for the values of the internal friction head and the residual pressure change from 100 per cent to zero per cent. It is likwise true that the sum of these functions changes from 100 per cent to zero per cent. Now while all variations in the position of the valve provide readings which lie between these two, they also provide a harmonious percentage variation between the internal friction head and the residual pressure, and therefore they provide a harmonious percentage variation between either function and their sum.

*c*) Now for the third observation we may give the valve any desired position, and read the gauge.

Our procedure should thereafter be as follows:

1. Designate the entire range of pressure readings as shown by (*a*) and (*b*) by the letter $D$.

2. Subtract the reading for (*a*) from that for (*c*). Say that this is $F$, the added external friction head.

3. Divide $F$ by $D$, and multiply by 100 per cent. This is the relative value of the external friction head.

Now there is a quantitative exchange in units, and therefore in percentages, between the external friction head on the one hand, and the sum of the internal friction head and the residual pressure on the other, since the total sum of the three is the constant $P$. That is to say, what the external friction head gains or loses by any alteration, the sum of the internal friction

[5] In performing arithmetical operations upon percentage values it is well to keep in mind the following facts:

*a*) Percentage values are ratios. We can have as many bases of comparison (100 per cent values) in the same problem as we wish. Operations in addition and subtraction must conform to these bases.

*b*) One hundred per cent is unity, and all percentage values between zero and one hundred are equivalent to decimal parts of unity.

*c*) Operations in multiplication and division are conveniently made in equivalent decimal numbers and the results thereafter converted into percentage values. These operations must also conform to the bases of comparison.

head and the residual pressure loses or gains, respectively. Therefore to continue:

4. Subtract the value obtained in (3) from 100 per cent, and thereby obtain the relative value of the sum of the internal friction head and the residual pressure.

Now the value obtained by (4) is the same as the relative value for the residual pressure alone, because of the harmonious percentage variation between the internal friction head, the residual pressure, and their sum, as mentioned above. Consequently, if we consider the velocity for (*a*) to be 100 per cent, the velocity for (*c*) may be determined by Equation 97 (p. 148).

Perhaps a simple specific example will remove any difficulties in the mathematical operations, and allow us to visualize the underlying principles more clearly. Let us consider the following:

The gauge registers 20 pounds with the valve fully open, and the velocity of flow is measured at 500,000 cubic feet of gas per 24 hours. With the valve completely closed the registered static pressure is 270 pounds. What is the velocity of flow when the valve is set so that the gauge shows 110 pounds?

In accordance with (1):

| | |
|---|---|
| 270 | Valve completely closed |
| 20 | Valve fully open |
| 250 | *D*, the entire range of pressure |

In accordance with (2):

| | |
|---|---|
| 110 | Valve partially open |
| 20 | Valve fully open |
| 90 | *F*, the added external friction head |

Then by (3) and (4), $\frac{90}{250} \times 100\% = 36\%$, and $100\% - 36\% = 64\%$.

Finally, by the equation referred to, $Ve = 10(64)^{1/2}$, or $Ve = 80\%$. This is the relative value of the new velocity in comparison with the old one. The reduction to units is easily accomplished: $80\% \times 500{,}000 = 400{,}000$ cubic feet of gas per 24 hours. Thus we have the answer to the problem as stated.

It is observed to what little extent the values of the various pressure heads in units need be known. Only the sum of the external friction head and the constant back pressure for each observation appears in units. We have no idea respecting the numerical values of the static and potential pressures, nor of the internal friction head. Whatever value the constant back pressure may have, such value disappears by subtraction in the first two steps of the computation.

**78.** *Case 2. Alterations in static pressure.*—As stated in section 75, this case is concerned with alterations in the static pressure of the reservoir. These alterations may be either real or apparent, and they do not include

changes which are due to decline in accordance with production, inasmuch as these changes properly belong to ideal performance in Volumetric Control.

By alterations in conformity with this case the line $I$ in Figure 47 shifts in the direction of either $I'$ or $I''$ in response to a decrease or an increase, respectively, in the static pressure. The lines $K$, $J$, and $N$ are not affected in their positions, while the lines $M$ and $L$ shift in the same direction as $I$. Briefly stated, the situation is now as follows:

| | |
|---|---|
| Static Pressure | Altered |
| Constant Back Pressure | Unaltered |
| Potential Pressure | Altered |
| External Friction Head | Altered |
| Internal Friction Head | Altered |
| Total Friction Head | Altered |
| Residual Pressure | Altered |

It is observed that the situation differs from Case 1. Six of the pressure heads, instead of four, now alter their values.

As a matter of fact, alterations which fall within Case 2 have characteristics in common with ideal performance in Volumetric Control, for the two circumstances only differ from the point of view of time as a function of performance. Where in the latter these alterations are natural, in so far as they result from decline, in the present case they are erratic, in that they have no relation to time as a function. As in the preceding case it is necessary that some of the features of Volumetric Control be accepted for the present without question.

The static and potential pressures alter concordantly, and by the same amount in units of pressure. The potential pressure and its four subdivisions alter concordantly, by different amounts in units of pressure, but all five alter with harmonious percentage variation, as we shall learn from the principles of Volumetric Control. As a consequence we may say that the lines $M$, $L$, and $I$ not only shift in the same direction with respect to the line $N$, but also that each shifts the same percentage amount as measured from $N$. Thus we see that the distances between these four lines increase or decrease concordantly, as just stated.

It is of particular interest to note the harmonious percentage variation between the potential and residual pressures. This is strikingly different from the situation in Case 1. The equation

$$Ve = KP^{1/2} \quad \text{.........................} (102)$$

is now a proper one,[6] for

$$Ve = K(PR)^{1/2} \quad \text{.....................} (103)$$

as before, and now

$$PR = KP \quad \text{.........................} (104)$$

[6] Although it is still a single-valued equation, we are now concerned with a succession of single values. The meaning of the equation is restricted in this manner because of the fact that no variation due to natural decline is either involved or inferred.

We may indeed substitute $P$ for $PR$ in Equations 95 to 101, inclusive. Thus, for examples,

$$Ve = 10P^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (105)$$

and/or

$$P = \frac{1}{100} Ve^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (106)$$

These two equations, as we know, necessitate the expression of the functions in percentages.

In virtue of the alterations in the potential pressure we may construct a potential pressure-time curve for this case. Let us conveniently assume that the same percentage values of the residual pressure in Case 1 are likewise established by the proper alterations here. Then the residual pressure-time curve is again that of Figure 48, while the potential pressure-time curve is shown in Figure 50. By Equation 104 above, it is evident that the two curves are alike, for if the functions are expressed in percentages, we have simply

$$PR = P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (107)$$

Either Equation 105 or Equation 106 permits us to construct the corresponding velocity-time curve for this case. The prearranged agreement between the residual pressure-time curves now provides a velocity-time curve identical with that of Figure 49.

The analytical principles involved in this case are somewhat simpler than those of the first case. Now we need determine the value of the potential pressure before and after the alteration in the static pressure, and apply Equation 105. Let us consider the following example:

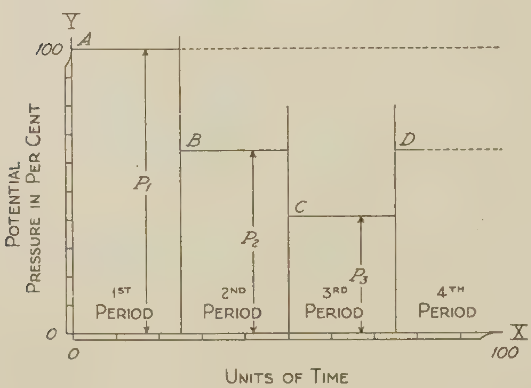


FIG. 50

A gas well indicated a registered static pressure of 270 pounds, and the velocity of flow, by measurement, was 500,000 cubic feet per 24 hours. The registered constant back pressure at the same time was 20 pounds. Now because either the height of the column of liquid which bears its weight upon the gas stands at a lower value, or closely adjoining wells are drilled in and allowed to flow continuously, the registered static pressure is 180 pounds. No alteration is made in the constant back pressure. What is the new velocity of flow?

Our procedure is as follows:

Compute the first potential pressure, and adopt this value, with its corresponding value of velocity, as 100 per cent standard conditions.

| | |
|---|---|
| 270 | First registered static pressure |
| 20 | Registered constant back pressure |
| 250 | First potential pressure |

Thus 250 pounds = 100%, and 500,000 cubic feet = 100%.

Compute the second potential pressure, and determine its relative percentage value:

| | |
|---|---|
| 180 | Second registered static pressure |
| 20 | Registered constant back pressure |
| 160 | Second potential pressure |

Thus

$$\frac{160}{250} \times 100\% = 64\%.$$

Substitute this value into Equation 105: $Ve = 10\ (64)^{1/2}$, or $Ve = 80\%$. This is the relative value of the new velocity in comparison with the old one. To reduce this to units: $80\% \times 500{,}000 = 400{,}000$ cubic feet of gas per 24 hours. Thus we have the answer to the problem as stated.

I have chosen the data for the two cases in such a manner that the resulting velocity is expressed by the same number. This has been done for the specific purpose of showing that it is immaterial, so far as velocity is concerned, whether the external friction head is increased by 90 pounds per square inch, or that the static pressure is decreased by the same amount. Thus in general it is to be noted that the analytical effects in the two cases differ in their nature, while it is possible that the numerical effects agree.

As an extreme alteration let us suppose that by some imaginative cause the value of the static pressure becomes diminished and equal to the value of the unaltered constant back pressure. The column of liquid does not extend above the point $f$ in Figure 47. It is clear that $P$, the pressure which determines that there shall be flow, is zero, and $PR$, the pressure which determines the velocity of flow, is likewise zero. Obviously there is no flow, for the reservoir is in equilibrium. Even the friction heads, since they alter in harmonious percentage variation with $P$ and $PR$, assume zero value. If we may imagine all intermediate positions for the free surface of the liquid in the column between the points $a$ and $f$, and reflect upon the corresponding analytical conditions which are determined by these positions, does it not become evident that the line $N$ in our present figure corresponds exactly with the potential axis $X$ in all our time curves? To be sure, horizontal distances in the one are replaced by units of time in the others; nevertheless $N$ and $X$ appear to possess analytical properties in common. So closely are they related that we may be certain in the assertion that the delivery of fluid from a reservoir takes place solely in accordance with the potential axis, except in so far as the proportional amount of gas which accompanies liquid from a combi-

nation reservoir reflects downward to some axis lying between the potential and absolute axes,[7] while the liquid itself remains in accordance with the potential axis.

We see the propriety of superimposing the potential axes for *P, Vo, Ve, Ac, E,* and *Po,* in Figure 45 (p. 125).[8]

79. *Case 3. Alterations in constant back pressure.*—This third case is concerned with alterations in the value of the constant back pressure, as stated in section 75. In Figure 47 the line $N$ shifts in the direction of either $N'$ or $N''$ in response to a decrease or an increase in the value of this pressure, respectively. The lines $K$ and $I$ are not affected in their positions, while $N$ may occupy any position between them. If for any reservoir the value of the constant back pressure is equal to that of the static pressure, $N$ coincides with $I$, and $P$ becomes zero. On the other hand, if delivery takes place at atmospheric pressure, $N$ coincides with $J$, and $P$ becomes equal to $RS$, while $C$ is identified as $A$. Under these circumstances of course $A$ may vary with the weather; consequently the line $J$ may shift toward either $J'$ or $J''$. As an opposite extreme to the situation wherein $N$ coincides with $I$, delivery into a perfect vacuum causes $N$ to coincide with $K$; $P$ becomes equal to $S$; and $C$ becomes zero. In the manner of the first two cases the situation may be briefly stated as follows:

| | |
|---|---|
| Static Pressure | Unaltered |
| Constant Back Pressure | Altered |
| Potential Pressure | Altered |
| External Friction Head | Altered |
| Internal Friction Head | Altered |
| Total Friction Head | Altered |
| Residual Pressure | Altered |

Again six of the pressure heads alter their values. The constant back pressure and the static pressure have exchanged situations in Cases 2 and 3.

As in the preceding case, alterations which fall within Case 3 have characteristics in common with ideal performance in Volumetric Control, some of the features of which we must anticipate once more.

Since the static pressure remains unaltered, the potential and constant back pressures alter discordantly, and there is a quantitative exchange in pressure units between them. The potential pressure and its four subdivisions alter concordantly, by different amounts in pressure units, but all five alter with harmonious percentage variation. Now as a consequence we may say that the lines $N$, $M$, and $L$ not only shift in the same direction with respect

[7] This axis corresponds to the horizon $G$ in the energy diagram of Fig. 33. (See § 57.)

[8] We have found three positions for the potential axes $X$ with respect to the axes $X'$ (see § 67). It is evident that if we superimpose the axes $X$, the axes $X'$ do not coincide unless proper vertical scales for the functions are chosen. It is not essential, however, that we cause them to coincide.

to the line $I$, but also that each shifts the same percentage amount as measured from $I$.

The harmonious percentage variation between the potential and residual pressures validates all the equations of section 78 in the present case. We may construct a potential pressure-time curve, and if we again conveniently assume the same percentage values of the residual pressure in Case 1, we have once more the curves of Figures 48 and 50. But clearly we have a different position for $N$ during each of the several different periods of time; therefore we must likewise have a different position for the axis $X$, as measured from the absolute axis $X'$, to correspond. Our figures should appear as shown in Figure 51, wherein three positions for $X$ are to be seen in their relation to $X'$. Only upon an agreement to ignore the individuality of each potential axis may the curves be made to appear as in Figures 48 and 50. In them all such axes are placed in alignment. Thus by shifting $O_2$, $O_3$, and so on, to a position in line with $O_1$, the figures become identical in appearance. This shifting is permissible provided we are fully aware of it; otherwise we should be easily led into serious errors respecting the future performance of the reservoir. How these errors may come about we shall see clearly in our study of decline curves in the finite controls.

If we confine our attention to the static pressure of the reservoir in this case, it is evident that its curve is the straight horizontal line $A$, $B$, $C$, and so on, in Figure 51, as measured by $P + C$ from the axis $X'$. A lone record of this pressure for a well shows us little, if anything, regarding the behavior of the well. As we know, delivery does not take place in accordance with the static pressure, but only in accordance with the potential and residual pressures.

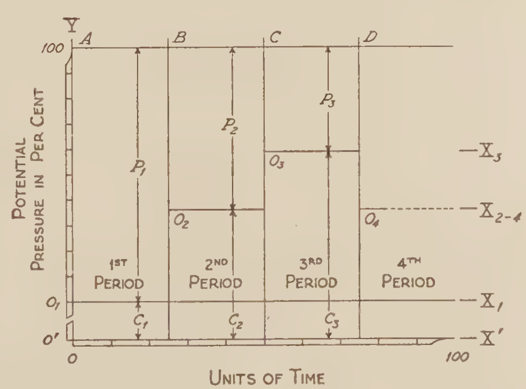


Fig. 51

While the periods of time have their individual potential axes for pressure, they certainly have them as well for velocity. Corresponding to Figure 51 we have Figure 52 (p. 158). When these potential axes are placed in alignment we once more have Figure 49.

The analytical principles involved in this case are somewhat more difficult than those of Case 2. This fact is occasioned by the shifting of the potential axis in conformity with alterations in the constant back pressure. The solution of a practical example, however, is obtained in quite the same manner as in the preceding case. Let us consider the following:

A gas well indicated a registered static pressure of 270 pounds, and the

velocity of flow, by measurement, was 500,000 cubic feet per 24 hours. The registered constant back pressure was 20 pounds. New conditions are such that the constant back pressure is increased to 110 pounds. What is the new velocity of flow?

We simply need to determine the potential pressure before and after the alteration in the constant back pressure, and apply Equation 105 (p. 154). The procedure is as follows:

Compute the first potential pressure, and adopt this value, with its corresponding value of velocity, as 100 per cent standard conditions.

| | |
|---|---|
| 270 | Static pressure |
| 20 | First registered constant back pressure |
| 250 | First potential pressure |

Thus 250 pounds = 100%, and 500,000 cubic feet = 100%.

Compute the second potential pressure, and determine its relative percentage value:

| | |
|---|---|
| 270 | Static pressure |
| 110 | Second registered constant back pressure |
| 160 | Second potential pressure |

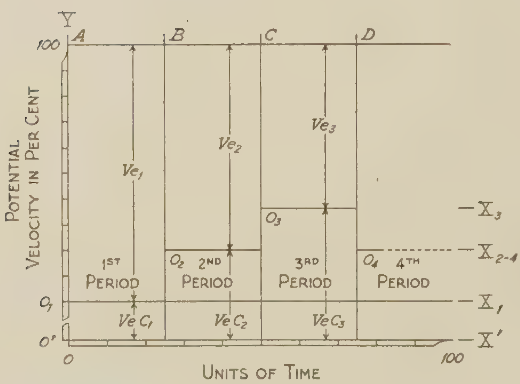


Fig. 52

Thus $\frac{160}{250} \times 100\% = 64\%$.

Substitute this value into Equation 105: $Ve = 10\,(64)^{1/2}$, or $Ve = 80\%$. This is the relative value of the new velocity in comparison with the old one. To reduce this to units: $80\% \times 500{,}000 = 400{,}000$ cubic feet of gas per 24 hours. Once more we have the answer to the problem stated.

I have chosen the data for this case in such a manner that the resulting velocity agrees with those for the preceding cases for the purpose of showing that it does not matter which of the three following alterations are made, so far as the velocity is concerned:

*a*) External friction head is increased by 90 pounds per square inch,

*b*) Static pressure is decreased by 90 pounds per square inch, and

*c*) Constant back pressure is increased by 90 pounds per square inch.

We may now include Case 3 in the general remarks concerning the fact that the analytical effects in the cases differ in their nature, while it is possible that the numerical effects agree. The latter circumstance is, however, no justification for a failure to discriminate between the three cases.

CHAPTER XI

# Theoretic Performance *(Continued)*

"For scientific purposes our mental representations of the facts of sensual experience must be submitted to conceptual formulation. Only thus may they be used for discovering by abstract mathematical rules unknown properties conceived to be dependent on certain initial properties having definite and assignable arithmetic values; or, for completing what has been only partly given."—ERNST MACH

**80.** *Dynamic relation between cases.*—The problems which were given in sections 77, 78, and 79, to illustrate the principles involved in the three cases of theoretic performance, applied specifically to a well that produced gas alone. Problems on an oil well, or on a water well, might have been chosen with equal propriety, and the method of calculation would not have differed in the least, since the mechanics of liquids and gases, as set forth in section 45, are the same. In calculating the effects of alterations upon the flow of gas we should not differentiate between, say, hydrocarbon gases and noncombustible gases and employ only the numbers which pertain to the former. Neither should we differentiate between oil and water that happen to issue together and employ only the numbers which pertain to the one of greater value per unit of volume. *Velocity refers to all liquid issuing from the well, just as it refers to all gas issuing from its well.* If we have once made the calculation with respect to all liquid, then further consideration of the proportional production of the constituent liquids is in order.[1] In the light of this it may seem strange that we must differentiate between the liquid and the gas issuing from a combination well. With two liquids, as with two gases, one is not attempting to separate itself from the other upon the release of pressure and thereupon assume a different form; mechanically, both constituents are acting harmoniously. We are already aware of the situation between a gas and a liquid. Velocity here refers to the liquid issuing from the well.

Now it appears that regardless of the analytical differences between the three cases there must be a relation between them; otherwise the problems as given would not have been reduced to the same result. In Figure 47 we

[1] For example, our rate of production curves should show oil plus water. This curve may then be used for computations which can thereafter be corrected in accordance with an auxiliary curve on the same plat, showing the separation between oil and water.

can certainly see this relation. It is purely a dynamic one, in that *PR*, the pressure which determines the velocity, can only change its value by an alteration in the distance between the lines *N* and *M*, and this may be brought about by any one of the three methods described.

So long as the alterations in the distance between *N* and *M* are due to an act which involves but one of the cases, the result is not difficult to analyze. But what if an act, or combination of acts, involve two of the three, or all three, cases simultaneously? Let us study these possibilities. We shall recognize them as frequent occurrences in the field.

If we consider the full list of ten pressures which appear in the pressure diagram, we may imagine a multitude of combinations in their alterations. Certainly some of these possible combinations are in themselves identical. It was for the purpose of avoiding identities that only seven of the ten were listed in the analyses of the individual cases, for clearly we need not include both static and registered static pressures, nor both constant back pressure and registered constant back pressure; and as for the atmospheric pressure, its value disappears by subtraction in the calculations, unless unbalanced by unequal barometric conditions at the points *a* and *W′*, or by an applied vacuum at the orifice. In such cases the unbalanced portion of this pressure must be included with the static pressure, or with the constant back pressure, or with both, depending upon circumstances.

After all, if we omit those combinations which result in a single one of the three cases, and those which exactly compensate each other, the multitude of combinations in the alterations of the seven pressures may be reduced to the following four situations: (*a*) a combination of Cases 1 and 2; (*b*) a combination of Cases 2 and 3; (*c*) a combination of Cases 1, 2, and 3; and (*d*) a combination of Cases 1 and 3. Obviously no other combinations are possible.

Analytically, the total effect of any combination is equal to the algebraic sum of the effects due to the separate cases which constitute the combination. Let us suppose a well to be equipped with a pressure gauge in a manner so that we may know at all times the pressure at the point *b* in Figure 47. A higher reading on this gauge indicates the following: (1) for Case 1 a decreased velocity, (2) for Case 2 an increased velocity, and (3) for Case 3 a decreased velocity. On the other hand, a lower reading indicates the opposite effect upon the velocity. Now if we were to calculate a new velocity from an old one by means of a comparison between the new and old pressures, it is evident that we meet with difficulties in employing the readings in combinations (*a*), (*b*), and (*c*), above, for they include contrary effects as stated in (1), (2), and (3). There is one combination, that of (*d*), where this difficulty does not arise. We may say, in fact, that *in three combinations there are discordant relations between cases, whereas in one there are concordant relations.*

In our study of individual cases we proceeded on the basis that it is possible and convenient for us to know definitely the amount of alteration, either

by noting the effect upon velocity and calculating the alteration in pressure, or by knowing in advance the alteration in pressure and calculating therefrom the effect upon velocity. It is clear that the various combinations of cases complicate our calculations, and furthermore, a rapid succession of alterations, whether due to one or more cases, causes the calculations to become inconvenient, if not impossible. However, in the flow-meter we have a simple and convenient device for handling all fluctuations in the conditions of production. These meters are capable of efficient operation, and their continuous pressure records serve as a reliable basis for calculations. The nature and frequency of alterations are immaterial, while the range of fluctuations is a matter of practical consideration. We see from the relation $Ve = K(PR)^{1/2}$, wherein $Ve$ is the rate of flow through the meter, $PR$ is the pressure during flow—the pressure which forces the fluid through the meter, and $K$ is a constant dependent upon the units in which $Ve$ and $PR$ are expressed, that the average velocity for a period of time, and therefore the volume of fluid which passes through the meter during the period, can be accurately determined only from the mean of the square roots of all instantaneous values of pressure during the period. This mean is never the same as the square root of the average pressure when the pressure fluctuates; the difference between them depends upon the range of fluctuations.[2] Those recording devices that furnish us with the average value of the pressure are simpler in construction than those which provide the value of the mean square root; consequently most meters have the simpler one. It is clear that the accuracy of these depends upon the range of the fluctuations.

The concordant relations between Cases 1 and 3 have special significance in connection with the operation of wells in the field. The external friction head and the constant back pressure affect the residual pressure in somewhat the same manner, inasmuch as these two back pressures act in unison at any single instant during the life of the reservoir. An alteration in either one affects the sum of the two in the same way and by the same amount as expressed in units of pressure. The effect on velocity at the instant is the same, regardless of whether the one or the other undergoes alteration. If all our reservoirs were in Hydraulic Control, the distinction between the two back pressures would be, for most purposes, unnecessary. Whether performance is ideal or theoretic, both may be said to behave as mathematical constants.[3] But because the majority of our reservoirs, particularly the natural ones, are in finite control, the distinction is quite essential, for, as

[2] To illustrate: The mean square root of 9, 36, and 144 is 7.00, while the square root of their average is 7.94. Again, the mean square root of 25, 36, and 100 is 7.00, while the square root of their average is 7.33. The results more nearly agrée where the range of numbers is smaller. In the limit, where the range is zero, the difference between the results is zero.

[3] In this control the potential volume is not altered in Case 1 or in Case 3. See § 81.

stated in section 21, in these one of the back pressures is acting as a mathematical variable while the other is acting as a mathematical constant.[4]

**81.** *Alterations in volume-time.*—We have seen how the pressure-time and velocity-time curves are constructed in percentages. The values possessed by the functions of pressure and velocity during the first period of time were, in the given curves, arbitrarily adopted as 100 per cent standard conditions for the purposes of subsequent comparisons. The line $A$, then, represented these standard conditions in each case, and the line $B$ represented the conditions under the assumed alterations. But had we chosen to do so, we might as well have selected the conditions for $B$ as standard. $A$ would then have represented conditions exceeding 100 per cent; but what of it? There is nothing at all illogical in such a situation, for standard conditions need not be maximum conditions in all events.[5] For whatever conditions selected as a standard the numerical results which pertain to two periods of time, as determined by a given alteration, are identically the same. For example, in velocity we had line $A = 500{,}000 = 100\%$, and line $B = 400{,}000 = 80\%$; but if we had chosen line $B = 400{,}000 = 100\%$, then line $A = 500{,}000 = 125\%$. This last percentage would have been derived from Equation 105 (p. 154) in the usual manner, for $P$ would have been 156.25 per cent in place of 64 per cent.

The first alteration as illustrated by the various time curves in the preceding chapter agree with the alterations specified in the three problems. Since these alterations were arbitrarily made to agree in their numerical results, the particular curves suit the problems. (It is true that subsequent alterations appear in the figures, but these have no definite relation to the problems as given; they merely serve to illustrate a more complete history of theoretic performance for a given reservoir.) Now from these figures we may construct the curves for the other primary functions of performance, as these are altered in agreement with pressure and velocity, and in these curves the first and second periods of time will continue to agree with the stated problems.

We found velocity to be constant, as of ideal performance, during each of the individual periods of time; therefore acceleration is zero during each of the periods. This function does not change in value; it always remains zero in this control, regardless of the nature of alterations in theoretic performance.

The volume-time curve is shown in Figure 53. The volume of fluid delivered during the first period of time is selected as the standard amount for subsequent comparisons; that is, the ordinate $ab$, which corresponds to the area subtended by the velocity curve in the first period of Figure 49, is

[4] In the finite controls the potential volume is not altered in Case 1, whereas it is altered in Case 3.

[5] See § 76.

100 per cent. This amount is of course indicated by the scale to the left of the point $b$. Now in accordance with the prescribed alterations the total per cent volume delivered by the end of the second period is indicated by the scale to the left of the point $c$. Clearly, if the rate of production for the first period defines cumulative production along a path $Ob$, an inclined straight line standing at an angle $\theta_1$ with the axis $X$, then the rate for the second period defines cumulative production along a path $bc$, a straight line standing at an angle $\theta_2$ with $bb'$ parallel to $X$. The tangents of the two angles bear the same relation to each other as do the values of the constants $K$ in the equations for the two volume-time curves; that is, the same relation as the velocities bear to each other—80 per cent, as provided in the problems. Subsequent angles likewise agree with the corresponding velocities in Figure 49.

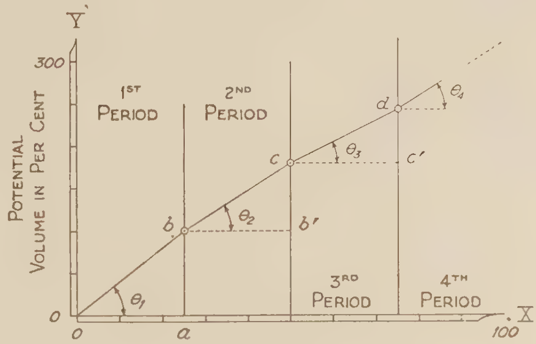


FIG. 53

To be exact once more in Case 3 our figure should possess the separate potential axes $X_1$, $X_2$, $X_3$, and so on, in correspondence with like axes in Figure 52, but the curves, if again shifted to superimpose these horizontal axes, would appear identical with those for Cases 1 and 2.

In so far as the external friction head and the constant back pressure have the same effect upon velocity, they must have the same effect upon volume.[6] In this control volume is infinite; neither of the back pressures alters this amount of fluid. But what is volume? It is the total liquid produced, if the reservoir produces either liquid alone or liquid and gas in combination; and it is the total gas produced, if the reservoir produces gas alone. These totals are purely mathematical quantities of fluid, without reference to the physical properties of the liquid and gas. If we were to specify a particular liquid, or a particular gas, it is clear that an infinite volume in production would require a replenishment of the very fluid itself at a rate equal to that of production; otherwise a time must come when the specified fluid is depleted, and in its place a fluid of a different kind is produced. With natural reservoirs we know well that water is the only fluid available for a continual replenishment. Oil and gas must become depleted within finite time. A pool of these fluids has finite limits, and these limits are certainly more restricted than those of the reservoir system depicted in Figure 47. Inevitably, within finite

[6] This statement would be incorrect if it were applied to reservoirs in finite control. See footnotes 3 and 4 above.

time, production in oil and gas must change to production in water. Meanwhile velocity-time and volume-time curves for infinite time have been established, and these shall proceed regardless of the change in the kind of fluid.[7]

The volume of oil and gas to be recovered from the pool, although finite in amount, is in no way affected by alterations in either of the back pressures. However, both do affect the velocity of recovery, and these in the same manner and degree. When the orifice of the reservoir is a multiple one, that is, when more than one well produces from the reservoir, *alterations in these back pressures affect the volume of oil and gas to be recovered from the individual wells.* A decrease in the velocity at one well gives the others an advantage, and vice versa.

**82.** *Alterations in energy-time.*—By multiplying potential pressure and potential volume we obtain potential energy. Ordinarily we multiply numbers which express units of the two functions to obtain units of energy. But here it suits our purpose to simply multiply quantities in their percentage values. Percentage values are relative values, and they may be handled in the same mathematical way as actual numbers, in spite of the fact that a reference to a standard condition is always inferred, if not expressed.

In Case 1 the potential pressure curve is not altered, although the potential volume curve is altered.[8] While the former function retains its value at 100 per cent, the latter assumes different percentage values in accordance with the alterations in velocity. Now if we multiply any quantity by 100 per cent its value is unchanged; consequently, for this case, the energy-time curve is identical with the volume-time curve. In Figure 53 we need only replace the word "volume" by "energy," and at any point on the axis $X$, corresponding to a definite instant in the life of the reservoir, the ordinate represents the potential energy which has left the reservoir in the interval between the zero point of reckoning time and the instant.

For Cases 2 and 3 the energy-time curve is shown in Figure 54. Both potential functions now alter their percentage values. The tangents of the angles bear the relation shown in the following column $E$:

| | $P$ | $\times$ | $Vo$ | $=$ | $E$ |
|---|---|---|---|---|---|
| Tan $\theta_1$ | 100.00% | | 100.00% | | 100.00% |
| Tan $\theta_2$ | 64.00% | | 80.00% | | 51.20% |
| Tan $\theta_3$ | 40.96% | | 64.00% | | 26.21% |

[7] Due allowance must be made for any change in the rate on account of a change in viscosity. Such a change may be treated as an alteration in Case 1. In this instance the case involves a change in the internal friction head, as well as a change in the external friction head. (Incidentally we see in this an inference of the rôle of viscosity in the process of production.)

[8] Of course a change in the volume-time curve in this case does not signify a change in the amount of fluid to be produced. It only defines a change in the amount to be produced in a specified period of time.

For Case 3 the potential axes $X_1$, $X_2$, $X_3$, and so on, are superimposed, thus causing the curves for the two cases to be identical.

It is observed that with respect to energy Cases 1 and 3 are quite unlike; that is, numerically equivalent alterations in the external friction head and the constant back pressure now give rise to a different set of conditions at the reservoir. These will be investigated in the succeeding section.

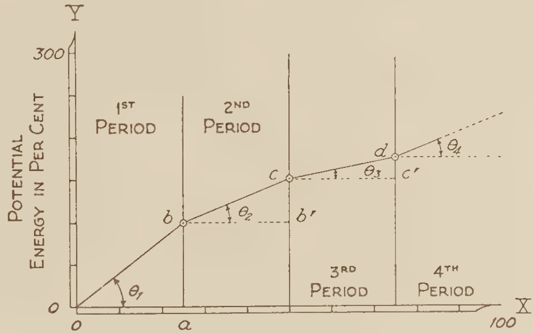

FIG. 54

**83.** *Alterations in power-time.*—Due to the fact that the potential pressure does not alter in Case 1, the power-time curve for this case is identical with the velocity-time curve of Figure 49. Power, as we know, is equal to the product of pressure and velocity; and again, if we multiply any quantity by 100 per cent, its value is unchanged. In the figure referred to we need only replace the word "velocity" by "power."

For Cases 2 and 3 the power-time curve is shown in Figure 55. The relation which we found to exist between the volume-time and velocity-time curves is now exactly duplicated by the relation between the energy-time and power-time curves. The tangents in the energy-time curve bear the same relation to each other as the constants $K$ in the energy-time equations, and this relation is the same as that which the various values of the power bear to each other. It follows, then, that the ordinates in Figure 55 possess the relation listed in column $E$ of the table in the preceding section. Once more the curves for the two cases are made identical by superimposing the various potential axes $X$ in Case 3.

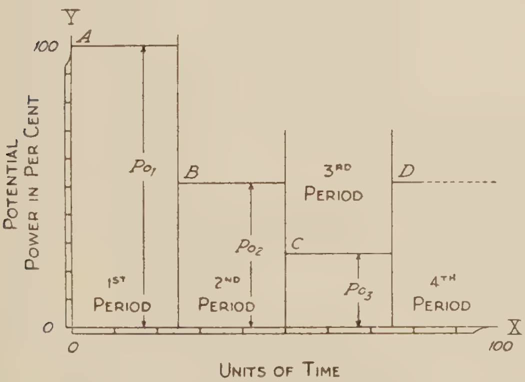

FIG. 55

The areas subtended by all power-time curves represent quantities of energy, and here we see a greater area subtended in Case 1 than in Case 3, a circumstance which agrees with the differences between the corresponding energy-time curves. A study of this feature should undoubtedly prove to be profitable; therefore let us undertake it.

In Figure 56 we have once more the type reservoirs in Hydraulic Control. Specific values are now given to some of the functions of performance. We will first confine our attention to Case 3, for which provision is made with both tanks, and subsequently consider Case 1.

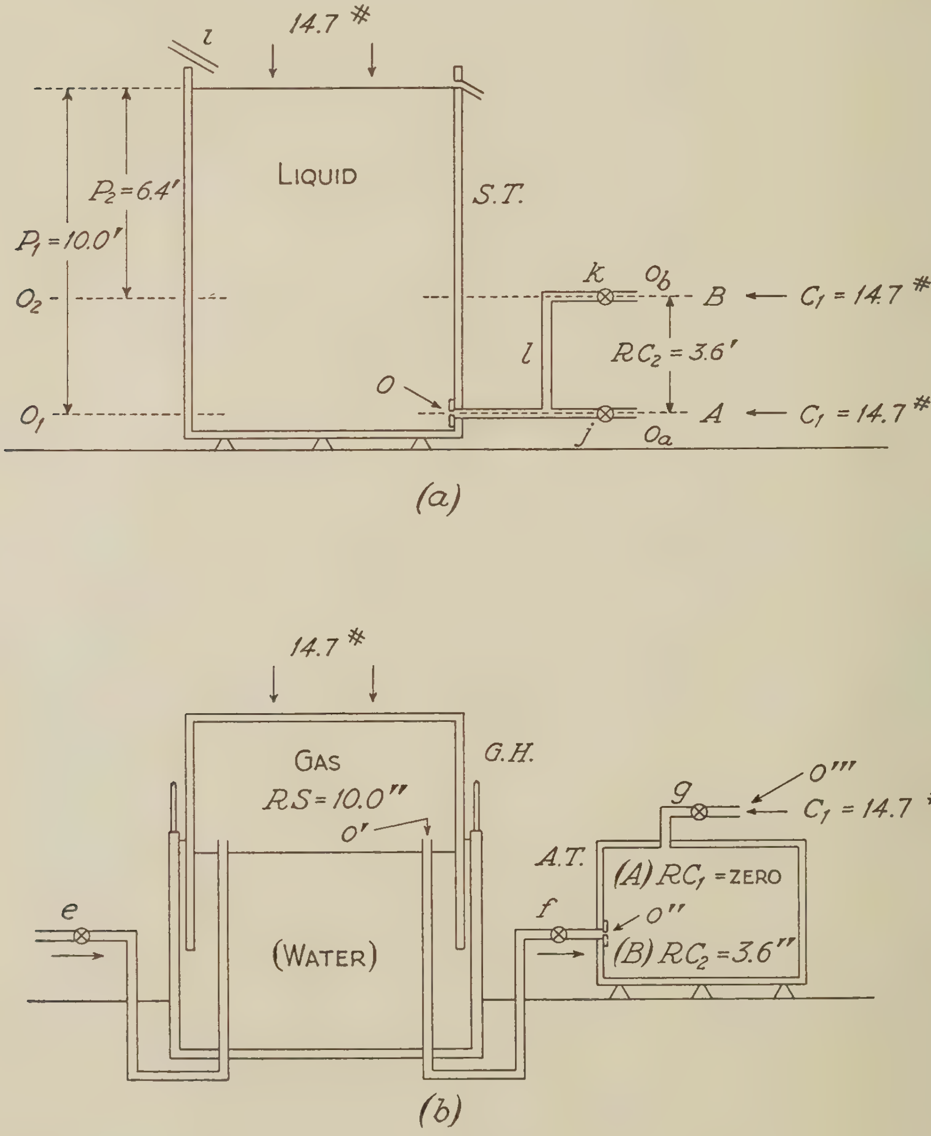


FIG. 56

The solution tank in (*a*) has its head of liquid maintained at a constant level. An orifice at *O* consists of a plate which contains a hole considerably smaller in diameter than the flow-line exterior to it, thereby providing a negligible external friction in the line. By means of the valves *j* and *k* the

constant back pressure may be altered in the manner, and to the extent, indicated. Now if we may express pressure heads both in feet of liquid and in pounds per square inch to maintain the identity of the heads, then, in accordance with the figure,

$$S = 10.0 \text{ ft.} + 14.7 \text{ lb.}$$
$$RS = 10.0 \text{ ft.}$$
$$C_1 = 14.7 \text{ lb.}$$
$$C_2 = 3.6 \text{ ft.} + 14.7 \text{ lb.}$$
$$RC_1 = \text{zero}$$
$$RC_2 = 3.6 \text{ ft.}$$
$$P_1 = 10.0 \text{ ft.}$$

and

$$P_2 = 6.4 \text{ ft.}$$

It is clear that with the potential pressure $P_2$ the liquid is delivered at a level $B$ higher than $A$. At this elevation the liquid has energy of position which it does not possess at the elevation of $A$; consequently work can yet be performed by it in attaining the lower position. $RC_2$ is in fact an elevation head as that denoted by $z$ in Bernoulli's equation, section 43. This elevation head is easily converted into a velocity head $v^2/2g$ by allowing the liquid to fall from $B$ to $A$, and it is conceivable that upon arrival at $A$ the velocity head may itself be converted into a pressure head $p/w$, if the liquid in falling is constrained to compress gas which is situated within a properly arranged container. But if the liquid is simply permitted to fall from the higher to the lower level without the performance of work that results in a storage of energy, and this energy thus made available for subsequent utilization, what becomes of its energy in the form of $Wz$ at $B$ and $Wv^2/2g$ at $A$? This mechanical energy is converted into heat, and we may safely presume that this heat, although temporarily held by the fallen mass of liquid, eventually becomes dissipated into the atmosphere.

In Figure 56 ($b$) we have a simple gas holder, such a one as might easily be set up in a laboratory, with continual supply through $e$ and flow through $f$. The bell is always supported entirely by the gas within it. For convenience we may place an auxiliary tank in the line between $f$ and $g$, wherein a constant back pressure of any reasonable amount, less than the pressure of the gas within the holder, can be imagined to exist. While this tank is not essential in most respects, it does permit us to accept the fact that there is a constant back pressure, without requiring of us an explanation of its source. That is to say, so far as we are concerned at present, we need not state whether this back pressure is due to line pressure or friction, or both, beyond the

confines of our apparatus. We are to interest ourselves only with the system as shown in the drawing; and this for our purposes is made complete by the inclusion of the auxiliary tank.

Inasmuch as the orifice of a natural reservoir is to be considered as located at the bottom of the well for reasons stated in section 20, an analogous position for the orifice of the solution tank was selected in preference to others; that is, $O$ was selected in preference to $O_a$ and $O_b$, Figure 56 ($a$). Perhaps to be consistent we should select in our gas system the position at $O'$ for the orifice in preference to $O''$ or $O'''$, Figure 56 ($b$). But we again desire a negligible external friction. Rather than provide a plate with a small hole at $O'$, let us consider $O''$ as the orifice. By doing this the friction in the line between $O'$ and $O''$ is internal instead of external. Now we need only consider alterations in the constant back pressure which is offered by the gas within the auxiliary tank. Now if we may express pressure heads both in inches of water and in pounds per square inch to maintain the identity of the heads, for two arbitrarily chosen values for the constant back pressure we may write

$$S = 10.0 \text{ in.} + 14.7 \text{ lb.}$$

$$RS = 10.0 \text{ in.}$$

$$C_1 = 14.7 \text{ lb.}$$

$$C_2 = 3.6 \text{ in.} + 14.7 \text{ lb.}$$

$$RC_1 = \text{zero}$$

$$RC_2 = 3.6 \text{ in.}$$

$$P_1 = 10.0 \text{ in.}$$

and

$$P_2 = 6.4 \text{ in.}$$

If we say that $RC_1$ defines a condition $A$, and $RC_2$ a condition $B$, then it is clear that with the potential pressure $P_2$ the gas is delivered at a pressure $B$ higher than $A$. At this pressure the gas has energy which it does not possess under the condition $A$; consequently work can yet be performed by it in attaining the lower pressure. $RC_2$ is in fact a pressure head comparable to that denoted by $p/w$ in Bernoulli's equation. It is easily converted into a velocity head $v^2/2g$ by allowing the gas to expand freely from $B$ to $A$, and it is conceivable that upon arrival at $A$ the velocity head may itself be converted into an elevation head $z$, if the gas in expanding is constrained to raise liquid which is situated within a properly arranged container. But if the gas is simply permitted to expand from the higher to the lower pressure without the performance of work that results in a storage of energy, what becomes of its energy in the form of $Wp/w$ at $B$ and $Wv^2/2g$ at $A$? As before, this mechanical energy is converted into heat to be eventually dissipated into the atmosphere.

We see that the two systems pictured in Figure 56 are mechanically identical with respect to the pressure heads and energy. This was, of course, to be expected, since, as stated in section 45, the mechanics of liquids and gases are the same. To continue with our study of energy no distinction between one system and the other will be required.

**84.** *Disposition of energy.*—When our interest in a reservoir is confined solely to the production of its fluid, all the mechanical energy possessed by the fluid is to be converted into heat.[9] This is the final disposition of the energy; it is to be dissipated into the atmosphere. In virtue of this fact we are today obtaining oil, gas, and water from our natural reservoirs. If our desire is to acquire possession of fluid, no better disposition can be made. Surely we do not consider placing power machinery in the flow-line which leads from a well simply to divert some of the mechanical energy into other channels, for to do so would hamper the delivery of fluid. We aim, or we should aim, to give the well the maximum amount of mechanical freedom. In other words, *we provide, or we should provide, the mechanical energy within a reservoir with its shortest path to dissipation into the atmosphere.* To accomplish this it is only necessary to reduce the external friction head and the constant back pressure to such minimum amounts as are consistent with the best economic and technologic practices.

While the mechanical freedom of a reservoir may be restricted in the same degree by either one of the back pressures, we must note a difference in the manner of restriction. Herein lies a distinction between Cases 1 and 3. With any given reservoir system, so long as there is flow, for any possible value of the external friction head the conversion of the mechanical energy of the reservoir into dissipated heat is direct, whereas for any possible value of the constant back pressure the conversion of a definite amount of the mechanical energy into dissipated heat becomes indirect, for the energy first assumes a mechanical form exterior to the reservoir.[10] In the two reservoir systems of Figure 56 we can imagine alterations in accordance with Cases 1 and 3, wherein the effects upon the velocity of production are numerically equivalent; nevertheless the distinction between the direct and indirect conversions of mechanical energy into heat remains intact. The external friction head involved in Case 1 continues to convert this energy directly into heat, while the constant back pressure involved in Case 3 continues to convert this energy indirectly into heat. The equivalence of new velocities in accordance with the two cases should not induce us to regard the cases themselves as equivalent.

The constant back pressure is a load against which a well, for example,

[9] Energy of position, or of pressure, is converted into energy of motion at the orifice, and this on the exterior side of the orifice resolves itself into molecular agitation that is, in fact, heat.

[10] Its amount is equal to the sum of the quantities $CVo + Pvo$ of §§ 51 and 57. This is suppressed energy.

working as a machine, must expend energy. Unless the fluid already is delivered in the presence of such a back pressure at a rate which pleases us, either with respect to our needs or with respect to our facilities for handling it, we aim to free the reservoir from the necessity of creating or maintaining a store of mechanical energy exterior to the well, at least to free it from this duty to an extent consistent with values and costs. To reduce the amount of constant back pressure is mechanically a simple problem in any event, for the proper sort of a pump or lift with the necessary capacity is all that is required. It is likewise true that any friction in the flow-line exterior to the orifice is a load against production. The same pump or lift will remove this. We do not only want the mechanical energy of the reservoir directly converted into heat that is subsequently subject to dissipation into the atmosphere, but we also want the conversion, so far as possible, to take place in a space which is not in direct communication with the orifice.[11]

In section 21 the external friction head was defined as the back pressure exerted against production in virtue of friction at the orifice and in any flow-line exterior to it. We may therefore differentiate between two parts of this head. Heretofore our considerations either included the two together, or referred solely to the friction in the flow-line. Let us investigate the nature of the friction offered by the orifice itself.

*For any given reservoir with its fluid, two holes, like or unlike with respect to size and physical condition, offer less external friction than either one alone offers. Furthermore, with two holes of the same physical condition, the larger one offers less external friction than the smaller one.* These facts we know from experience, for with two holes, or with a larger hole, the delivery of fluid from the reservoir takes place at a more rapid rate. Thus it it is in both artificial and natural reservoirs, where the holes may be either at or near the bottom of a solution tank, anywhere in a gas tank, or at the bottom of a well.

In so far as the internal friction within a hollow reservoir is a negligibly small quantity, we may say that for these reservoirs the orifice alone calls the viscosity of the fluid into play, and thus regulates the flow from them. *In natural reservoirs, however, wherein a porous medium exists, the internal friction is certainly not negligible; the orifice of these is not alone in regulating flow.* The internal friction and the external friction offered by the orifice act together in this.

If, instead of specifying a hole in a plate or the bottom of a well as a definite, localized orifice, we had agreed that either of these holes together with a part or all of the flow-line exterior to it were to constitute the orifice, then the foregoing statements in regard to two holes, or to large and small

[11] By keeping the exterior side of the orifice free from fluid in agitation we provide for a better conversion of energy of position, or of pressure, into energy of motion immediately at the orifice.

holes, would apply equally to it. Thus two wells, like or unlike with respect to the physical condition at the bottom, to the size and length of the casing or tubing, and to the equipment at the surface, offer less external friction than either one alone offers. (Here we are not differentiating between two parts of external friction. In some investigations it is well not to make the distinction between the parts, while in others it is advantageous to do so, since we see that one part may be removed so easily by a pump or lift. At any time a continuation of the separate parts becomes superfluous, we may simply combine them and proceed with the particular problem at hand.)

To say that the two wells offer less external friction than one of them alone offers may appear to be a contradiction of generally accepted principles in engineering. Surely two wells convert by friction a greater amount of mechanical energy into heat than either one alone does, just as two miles of pipe-line convert by friction a greater amount of mechanical energy into heat than either of its half portions. Are we inconsistent in our statement? No; we are here dealing with all friction heads as intensive factors, and not as extensive ones. In pipe-line calculations the friction head is taken as an extensive factor.

It is essential that we understand the intensive nature of all pressures represented in Figure 47. All are expressed and scaled in pounds per square inch, or in feet of liquid.[12] Of course we deal also with extensive factors, for some of these are indeed to be found among the seven functions of performance. Suppose we list and classify them, in order that we may appreciate the difference between the two sorts of factors:

| Function | Nature | Dimension |
|---|---|---|
| Pressure | Intensive | Pounds per square inch, etc. |
| Volume | Extensive | Cubic feet, barrels, etc. |
| Velocity | Intensive | Cubic feet per day, barrels per day, etc. |
| Acceleration | Intensive | Cubic feet per day per day, barrels per day per day, etc. |
| Energy | Extensive | Foot-pounds, etc.[13] |
| Power | Intensive | Foot-pounds per day, etc. |
| Time | Extensive | Days, etc. |

In stating an intensive factor the word "per," or its equivalent, must appear in the expression, whereas in stating an extensive factor no such word can appear. We have already seen how the multiplication of intensive factors by the proper quantities results in extensive factors.[14] In mathematical operations a series of values pertaining to any particular intensive factor can only be reduced to an average value by obtaining the corresponding amount of the extensive factor for each such value in the series.

[12] See footnote 2, § 76, page 146.

[13] See footnote 9, § 15, page 23.

[14] See §§ 63 and 66.

Now when an engineer demands a value for the friction head per mile of pipe-line—a value which is admittedly of an intensive nature—he does so for the purpose of immediately obtaining the total for all his miles of line. The latter is really the value in which he is interested, and it is clearly of an extensive nature. In this investigation we shall have no occasion to multiply, say, the potential pressure, or one of the friction heads, by area in order to obtain the extensive factor of force. We deal with all pressure heads in their intensities only, and therefore the foregoing statements regarding the external friction are proper.

**85.** *Gas-time relations in the cases.*—In order to establish principles of fundamental importance in our system of mechanics—principles which pertain equally and identically to each of the three controls, and likewise to ideal, theoretic, and actual performance—we have wandered somewhat from the straight highway. Let us return to the three cases in theoretic performance, and consider briefly their applications to combination reservoirs. Our study of the cases in the preceding chapter was concerned with the effects of alterations on the production of liquid alone and gas alone. Now we may add that the same principles hold with regard to the liquid, where both liquid and gas are produced, but not with regard to the gas accompanying this liquid.[15]

In section 74 we obtained equations showing the relation between the following:

$G_{Pp}$ and Time .................... [77]

$G_{Cp}$ and Time .................... [78]

$G_{Sp}$ and Time .................... [79]

where the quantities pertain to the *proportional amount of gas to liquid;*

$G_{Ve}$ and Time ...................... [86]

$G_{Vo}$ and Time .................... [88]

where the quantities pertain to the velocity and volume of *potential gas;*

$G'_{Ve}$ and Time .................... [90]

$G'_{Vo}$ and Time .................... [92]

where the quantities pertain to the velocity and volume of *suppressed gas;* and

$G''_{Ve}$ and Time[16]

$G''_{Vo}$ and Time ................... [94]

where the quantities pertain to the *gas retained by the reservoir.* Now the

[15] See §§ 45 and 73.

[16] This relation was given no consideration except in footnote 23, § 74, page 142.

question is: How are these gas-time equations affected, or not affected, in each of the three cases? Let us consider the cases separately.[17]

CASE 1. *The pressures* S, C, *and* P *are not altered.* The equations between the proportional production of gas and time—Equations 77, 78, and 79—are not affected in any way by alterations in accordance with this case.[18] Immediately at the orifice the potential gas, $P/S$ in the free state, and the amount $C/S$ in the dissolved state are the same before and after the alteration.

Because the value of $Ve$ for the liquid is altered, the remaining equations in the above list are affected. The constants $K_1$ and $K_4$ assume new, unique values in them. For no other reason are these equations altered.

CASE 2. *The pressures* S *and* P *are altered, while* C *is unaltered.* Will the proportional production of gas be affected? The answer depends upon circumstances within the reservoir system. The ideal combination reservoir described in section 72, with its one well given a central location with respect to the geological structure in its vicinity, provides for no such alteration in the ratio between the amounts of gas and liquid, if the gas is measured in units of volume in the usual manner. This is because the ideal conditions so specified prohibit a temporary or permanent accumulation in, or production from, a near-by space which may define a gas pocket.

While the proportional production of gas remains the same, as measured in units of volume, the percentage values $P/S$ and $C/S$ are obviously altered, inasmuch as $P$ and $S$ are altered, and these in different percentage amounts. There is an excess or deficiency of gas in solution immediately at the orifice, as held by $C$, according to a decrease or an increase in $S$, respectively. It is as if the degree of saturation of the liquid by the gas were altered.

Equations 77, 78, and 79, then, are altered because of percentage changes. The constants $K_2$ and $K_3$ assume new, unique values. All other equations, in so far as they depend upon new values of $K_1$, $K_2$, $K_3$, and $K_4$, are likewise altered.

CASE 3. *The pressure* S *is unaltered, while* P *and* C *are altered.* The percentages $P/S$ and $C/S$ are certainly altered, but the amount of gas for $S$ remains the same. An increase or decrease in $C$ causes a decrease or increase, respectively, in potential gas, and an increase or decrease in the suppressed gas and the gas retained by the reservoir. Equations 77 and 78 are altered in so far as the values of $K_2$ and $K_3$ are altered. Equation 79 remains the

[17] As in the preceding chapter we are to ignore for the present the possible by-passing of the gas.

[18] Herein lies a difference between this and the finite controls. In the latter we must say that the proportional production of gas is not affected, although the equation between this function and time is affected. The equation assumes a new value of the constant $K_2$. We must indeed be careful in applying statements made here with respect to the three cases in Hydraulic Control to the identical cases in Volumetric and Capillary controls.

same, since the sum of $K_2$ and $K_3$ is unaltered. The remaining equations alter because of the fact that the values of $K_1$, $K_2$, $K_3$, and $K_4$ are altered.

These are the simple principles involved in the production of gas from an ideal reservoir in theoretic performance. Performance, however, whether ideal, theoretic, or actual, in reality involves further principles that tend to complicate the problem. By-passing of the gas, the mutual effects of additional orifices in reservoirs of lateral extent, and the action of gas pockets in reservoirs possessing structure, all constitute features of importance. We are safe in assuming that no natural reservoir is free from them, and that in but few, if any, natural reservoirs can they be ignored. These features are to be studied in chapters xiii and xiv as proper subjects in the secondary functions of performance.

CHAPTER XII

# Secondary Functions of Performance

"All the difficulty of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of Nature, and then from these forces to demonstrate the other phenomena."—SIR ISAAC NEWTON

**86.** *Introduction.*—Events and conditions within artificial and natural reservoirs that are either producing or capable of producing fluid are to constitute the secondary functions of performance, as previously stated in section 24. A thorough investigation of all problems which actually arise in connection with these functions, in so far as they pertain particularly to reservoirs whose interior spaces are filled with porous mediums, would carry us beyond the scope of the present treatise. We can attempt to include only those mechanical principles upon which depends, it seems, not only our understanding of reservoir performance, but also our understanding of best oil and gas field practice.

From the beginning we have consistently, and persistently, pursued a method of procedure which is based primarily upon a convenient and an advantageous idealization of matter and circumstance. This is essential, we say, if our goal is to satisfy our determination to learn fundamental principles. Now to study events and conditions within the reservoir we shall continue to idealize—not too generously or recklessly, but sufficiently to fulfill the basic requirements. And inasmuch as the manner and the degree of idealization are matters which are purely arbitrary with us at all times, why should we not exercise the privilege of modifying the ideal whensoever we may wish to advance toward actuality? If our purpose were solely the acquisition of knowledge concerning a fictitious ideal, any subsequent modification which deviates from the ideal would appear incongruous. But let us not deceive ourselves with regard to the service rendered by the ideal; it is invented so that we may derive certain principles, or laws, in a perfect, geometrical form, and upon these, as a foundation, we are to erect our superstructures in accordance with the true nature of things. A knowledge of the true nature of things is, of course, that for which we strive, and we can say from experience that by the method of idealization the physical sciences are best developed, for thus is order most easily evolved from chaos.

The liquids and gases with which we deal are perfect to the extent that

they exactly obey the formal laws of physics. They possess real values of viscosity and surface tension, and these values are not subject to fluctuation on account of circumstances which are to be considered as normal within the reservoir during the process of production. We may assume that temperature changes either exist or do not exist within the reservoir. In the former we need only assume further that these changes have not the slightest effect upon the physical properties of the fluid or fluids present.[1] Again, in so far as the fluid experiences a *pressure drop* on its path toward the orifice,[2] both in ideal and theoretic performance, we shall assume that these changes likewise have no effect upon the physical properties of the fluid. In the event that both liquid and gas are present, the one is acknowledged to be dissolved in the other in direct proportion to the absolute pressure which is exerted upon them, at whatever position they may occupy on their path toward the orifice.[3]

We are to continue with the study of artificial tank reservoirs, for they will here serve to illustrate features of importance to us. Heretofore the tank constituted the reservoir proper, and any flow-line attached to it was considered as an accessory part of the more comprehensive system. The orifice was said to be in the immediate vicinity of the tank, and the flow-line was considered to be exterior to this orifice. Now, in order to investigate the secondary functions, it will be more convenient to consider the more comprehensive system itself, in which the tank is but a part. The orifice is to be located at an advantageous downstream point in the flow-line. In short, the flow-line is to be a portion of the internal system of the reservoir, and the events and conditions which we find in this line will be referred to the natural reservoir.

87. *The pressure gradients.*—The fluid within a reservoir capable of producing exerts a pressure of a definite intensity against its closed orifice. In return the closed orifice exerts a pressure of identically the same intensity against the fluid. This is in agreement with Newton's third law of motion: *To every action there is always an equal and contrary reaction; or, the mutual*

[1] Obviously this is an assumption contrary to known facts. If we find these changes to be of importance in the process of production, we can easily make the necessary, subsequent corrections for them. It is not safe to assume that these changes are of importance simply because laboratory tests show them to be measurable. Their effects may be excessively overbalanced by other physical phenomena within the reservoir. This is certainly the case in the combination reservoir, where Jamin action in its effects greatly overbalances the effects of changes in viscosity and surface tension due to alterations in the amount of gas dissolved per unit volume of liquid. In Part IV we take up this subject in further detail.

[2] The pressure drop is a decrease in pressure intensity due either to an increase in the lineal velocity of flow (according to the terms of Bernoulli's Theorem), or to a conversion of mechanical energy into heat along the path of flow.

[3] It is clear that in this we do not refer to the static pressure of the reservoir, but to the absolute pressure anywhere in the reservoir during the process of production.

*actions of any two bodies are always equal and oppositely directed.* The forces are perfectly balanced; consequently the fluid assumes a position of rest within the reservoir. Now if we agree that the pressure exerted by the fluid is the action, and that exerted by the closed orifice is the reaction, it is evident that directly upon opening the orifice the latter is reduced in amount; in fact it is reduced to a value equal to the sum of the two back pressures against production. What happens to the former? According to the law it is reduced to the same amount, thereby leaving a certain pressure intensity unbalanced. This unbalanced portion, diminished by any internal friction head, is the residual pressure. So long as the reservoir possesses a potential pressure, measurable, as we know, only with the orifice closed, such a residual pressure will exist upon the opening of the orifice.

The foregoing situations may be expressed in our usual symbols, as follows: $S$ is the value of the action and reaction when the orifice is closed; $C + EF$ is the value of the action and reaction when the orifice is opened; $S - (C + EF)$ is the unbalanced portion, yet to be diminished by $IF$; and $S - (C + EF) - IF = PR$. If we group these quantities in the following manner, $(S - C) - (EF + IF) = PR$, we have an expression which is equivalent to $P - TF = PR$, and in this we see an agreement between our present analysis and the earlier one wherein we obtained the definition of the residual pressure.

If for the first and only time we were here encountering the residual pressure, we might logically say that this pressure, as an action without a corresponding reaction, is the cause of flow through the orifice. I believe we should prefer to adhere to the idea as expressed in section 71, and consider the potential pressure as the one which determines the fact that there shall be flow at the orifice, while the residual pressure alone determines the velocity, or rate of flow.[4] I see no definite reason for temporarily abandoning this idea in the present instance, or in other instances to follow.

From experience we know of the existence of different pressure conditions within a given reservoir system when the orifice is closed and when it is open. Figure 57 (p. 178) represents a system composed of a solution tank, maintained in Hydraulic Control by the usual methods, and a single tube which extends outward horizontally, this tube being equipped with a series of standpipes in the manner shown. The tube and pipes are of uniform cross-section. We shall consider the orifice at $O_a$. It is clear that when this orifice is closed, the free surfaces of the liquid in the standpipes rest at *a, a, a,* on a horizontal line *A,* as determined by the elevation of the liquid in the tank. The line *A* in

[4] Both ideas must be recognized as conceptual ideas of reservoir performance. Where we have two or more conceptions of one event, these being equally permissible and consistent with the process at large, we are obliged to maintain them separately throughout a particular investigation; otherwise confusion and inconsistency in argument are certain to arise.

fact corresponds with the line $I$ in Figure 47. Through $O_a$ pass the lines $J$ and $N$, for these now coincide, and below is $K$, a line which represents absolute zero pressure in the usual manner. The potential pressure $P$ is represented by the distance between the lines $N$ and $I$, in agreement with the height of the liquid above the orifice when closed. Upon the opening of the orifice

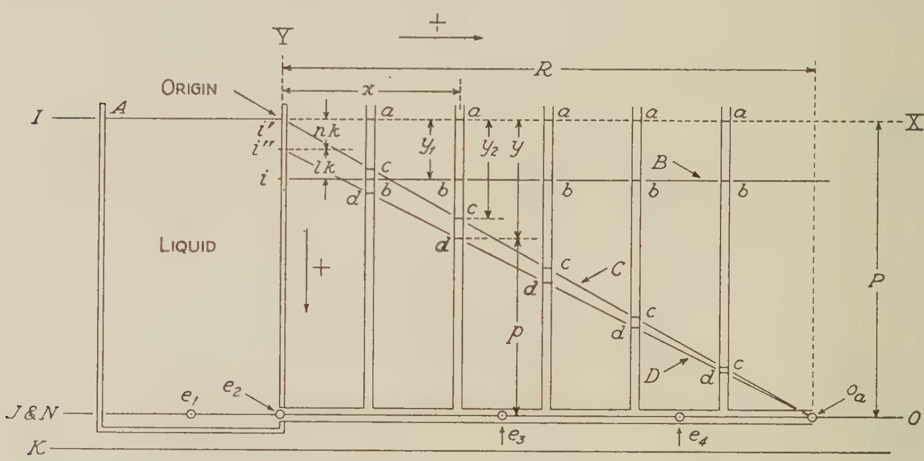


FIG. 57

the free surfaces within the standpipes drop, and remain at a fixed position so long as flow continues in accordance with ideal performance. We will now investigate the nature of the positions held by these free surfaces.

First, I propose that we call the horizontal line $A$ the *static pressure gradient*, to indicate thereby the position of the free surfaces when the liquid is at rest throughout the system. If we select axes $X$ and $Y$ as shown in the drawing, and say that $x$ represents horizontal distance, measured positively from left to right, while $y$ represents a pressure drop which may or may not exist, measured positively downward, then the equation of this static pressure gradient is simply

$$y = \text{zero} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (108)$$

The limits of $x$ are obviously determined by the extent of the reservoir system. Inasmuch as we are to concern ourselves at present with pressure drops which are due to flow from the orifice, we shall say that the limits of $x$ are $x =$ zero and $x = R$, the latter denoting a *radius of action* possessed by $O_a$. We shall assume that pressure drops due to flow at the orifice are confined to this distance $R$.

The tank, we shall say, is of sufficient lateral extent to insure a particle of liquid at $e_1$ at rest during flow from the orifice. This particle possesses mechanical energy in accordance with Bernoulli's Theorem as follows:

*Wz*, an amount due to its elevation above an arbitrary datum plane. If we employ a datum at the elevation of the line $N$, this quantity of energy is zero.

$W\frac{v^2}{2g}$, an amount due to its velocity head—now zero.

$W\frac{p}{w}$, an amount due to its pressure head in virtue of the weight of liquid which rests upon it.

How does the energy of a particle at $e_2$ differ from that at $e_1$? The particle is here in motion; a part of the pressure head is converted into a velocity head. There is a quantitative exchange in pressure units from $p/w$ to $v^2/2g$; just so much as the latter is increased from zero, the former is decreased, $z$ being the same for both particles. If we assume that there is absolutely no friction against flow within the horizontal tube, a condition which may be easily approximated in a test which is performed with a non-viscous liquid, a smooth tube, and a low velocity, then the static gradient simply lowers from $A$ to a position as shown by $B$. There is a pressure drop $y_1$ extending from $x =$ zero to $x = R$, one that is invariable throughout $R$. The free surfaces within the standpipes rest at $b$, $b$, $b$. The equation for the gradient is now

$$y_1 = k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (109)$$

wherein $k$ is equal to the velocity head $v^2/2g$. This velocity head is constant throughout the length of the tube; that is, particles at $e_3$ and $e_4$ are moving at the same rate as the one at $e_2$.

If now we desire to acknowledge friction within the tube, a circumstance which may be made necessary by the fact that the liquid possesses a viscosity which cannot be neglected, or by the fact that the velocity is great, we can determine the gradient resulting from this friction. Let us determine this gradient for the present system to the exclusion of the gradient $B$. To do this we may imagine the tank and the free surface of liquid within it sufficiently increased in height (in a manner not indicated in the drawing) so that a new value of the potential pressure equal to

$$P + \frac{v^2}{2g}$$

really exists. Now $B$ is raised to the position of $A$, the point $i$ moves to $i'$, and the $b$'s assume the position of the $a$'s. The pressure head due to the weight of so much of the liquid from $e_2$ to $i'$ is, with the orifice fully open, entirely consumed by friction. The gradient is now the inclined straight line $C$, and the free surfaces within the standpipes rest at $c$, $c$, $c$.

To obtain the equation of the line $C$ let us refer to Figure 58 (p. 180), where we have the same system as in the preceding figure. For convenience the origin of co-ordinates is transferred temporarily to the point $O_a$, the axes $X$ and $Y$ occupying the positions indicated. Positive directions for $x$ and $y$ are now

reversed.[5] The horizontal tube, being perfectly uniform throughout its length, offers a certain friction $f$ per unit of distance. We can represent this friction by the straight horizontal line $F$ at a distance $y = f$ above the axis $X$. The fact that $F$ is horizontal merely implies the same friction at all points in the tube, and this must be true, since the tube is in the same condition, and the

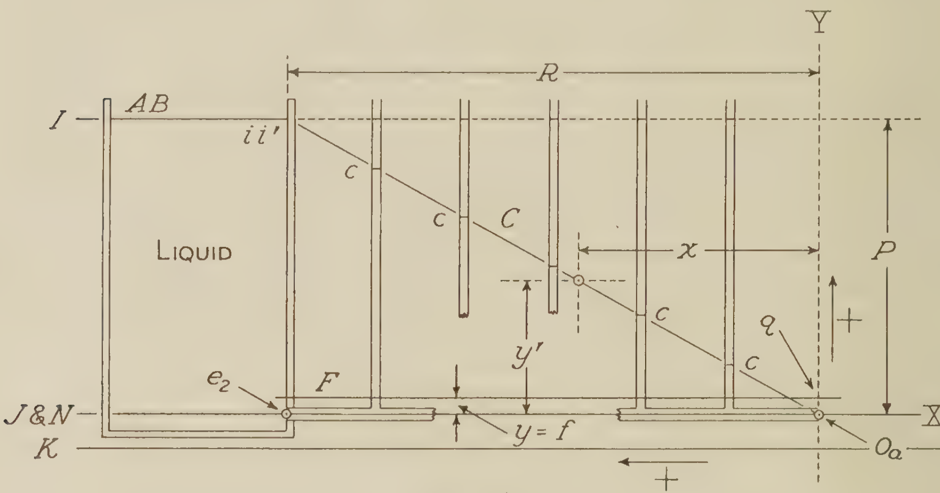


FIG. 58

velocity is the same, throughout $R$. If we reckon from $O_a$ to $e_2$, we can say that the effect of this friction is cumulative as against $P$ for each unit of distance advanced. And being thus cumulative we can determine its curve of accumulation by integrating the equation for $F$.[6] Thus

$$y = f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (110)$$

becomes by this process

$$y' = fx + \text{a constant}$$

the latter being the so-called constant of integration. When

$$x = \text{zero},\ y' = \text{zero};$$

therefore this constant is zero, and it may consequently be omitted in the expression. Again, when $x = R$, $y' = P$; that is, $P = fR$, from which we learn that the value of $f$ is $P/R$. Now the equation stands,

$$y' = \frac{P}{R}x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (111)$$

[5] The transference of co-ordinates is indeed not necessary. It does permit, perhaps, a clearer conception of the relation between mathematical operations and mechanical principles.

[6] In a simple system of this sort we might use arithmetical methods in place of those of calculus. The latter are used here in order to illustrate in advance the methods of analyzing the more complicated system wherein flow is radial. (See next section.)

This is the equation of the straight inclined line $C$. By transferring the origin back to its position in Figure 57 we have

$$y_2 = \frac{P}{R}x \qquad (112)$$

where $y_2$ is the ordinate measured downward from the original axis $X$, and $x$ is measured to the right from the original axis $Y$.

From the viewpoint of total friction head, $B$ and $C$ represent opposite extremes for the system with its static pressure gradient $A$. For $B$ the internal friction is zero, and the total friction is comprised only of external friction at $O_a$. On the other hand, for $C$ the total friction is comprised only of internal friction between $e_2$ and $O_a$. In actuality the point $i$ advances toward $i'$ as the internal friction is caused to increase by altering the interior condition of the tube in any manner that will provide such an increase. For all possible positions of this point, one of which is shown at $i''$, the constant $k$ of Equation 109 is divided into $nk$ and $lk$, where $n$ and $l$ are fractions whose sum is unity. The equation of the actual gradient $D$ becomes

$$y = nk + k''x \qquad (113)$$

where $k''$ is a constant to be determined. When $x = R$, $y = P$; therefore

$$k'' = \frac{P - nk}{R}$$

and the equation of the gradient is now

$$y = nk + \frac{P - nk}{R}x \qquad (114)$$

The quantity $nk$ is of course itself a constant. It is equal to the actual value of $v^2/2g$ for the system with any definite amount of internal friction. The line $D$ determines the position of the free surfaces of liquid within the standpipes at $d$, $d$, $d$. I propose that we call such a line the *kinetic pressure gradient*.

A gas system, corresponding to the liquid system of Figure 57, can be constructed. The solution tank may be replaced by a gas holder, and the standpipes may be reduced to nipples, to which manometers or pressure gauges should be attached. In such a system the static and kinetic pressure gradients are identical with those for liquid. Whereas for liquid we might have said that $k = KVe^2$, where $Ve$ is mass-velocity of flow through the tube, and $K$ is a constant depending upon the units of measurement and the size of the tube, we should necessarily express the value of $k$ in this manner when the fluid is a gas. During flow the lineal velocity of gas at $e_2$ is less than at $e_3$, $e_4$, and $O_a$, because the fluid is expanding as it moves toward the

orifice. Nevertheless it must be clear that $Ve$ in mass units is the same at all points along the tube.[7]

That the effects of friction for a given mass of gas are independent of the density of the gas, that is, the space occupied by the gas, was determined by Maxwell. The effects within an expanded gas are less per unit of space occupied, but a correspondingly greater number of units of space are occupied by the gas; consequently the net result appears the same. Maxwell's experiments remind us somewhat of Boyle's and Joule's laws.

**88.** *Tubular versus radial flow.*—The system depicted in Figure 57 provides, as we know, flow along a straight line through the horizontal tube. Let us consider a battery of such systems arranged about the orifice as a center, or perhaps preferably a complete unit as shown in Figure 59. In this system the solution tank is annular, and the horizontal tubes with their standpipes radiate from a central chamber at $O_a$. $R$ is here the radius of a circle, and the axis $Y$, heretofore a single line, is now a number of lines which lie on the surface of a cylinder of radius $R$. In so far as we may place a horizontal tube at any point on the inner circumference of the tank, we may, if we choose, say that the axis $Y$ is the surface of the cylinder. This conception of a vertical axis, although not a particularly essential one in the present system, is desirable in view of subsequent analyses; therefore let us adopt it here. Now the static pressure gradient is a horizontal plane, and the kinetic pressure gradient is the surface of a right circular cone $D$, with its axis passing vertically through the center at $O_a$.

This system and the simpler one in Figure 57 have an important feature in common: namely, flow takes place through horizontal tubes, as already stated. Let us choose to say that we have *tubular flow* in these systems, in order to make a distinction between this type of flow and a *radial flow* which is now to be described.

Figure 60 (p. 184) shows a plan and profile of a system which, in some respects, resembles the one in Figure 59. Instead of horizontal tubes, however, we now have two horizontal plates at a distance $j$ apart, the lower one being provided with a central orifice at $O_a$, while the upper one has standpipes as shown.[8] When the orifice is closed, the static pressure gradient, as before, is a horizontal plane, represented by the line $A$. The free surfaces of liquid in the standpipes rest on this line, as at $a$.

[7] This is in accordance with the well-known principle of continuity: *The mass of fluid entering any region is equal to the mass leaving in the same time, unless the density at the place is changing with time.* In "steady flow" the density is not changing at any and all individual points per se. (In fact the condition of non-changing density at individual points taken separately defines *steady flow,* the state to which Bernoulli's Theorem refers.)

[8] To simplify the drawing not all the standpipes are shown. We are to imagine the system perfectly symmetrical about its center.

Upon opening the orifice the liquid must converge as it travels from all points on any circumference surrounding the orifice; its lineal velocity must increase as the circumference becomes smaller. This was not the case in our

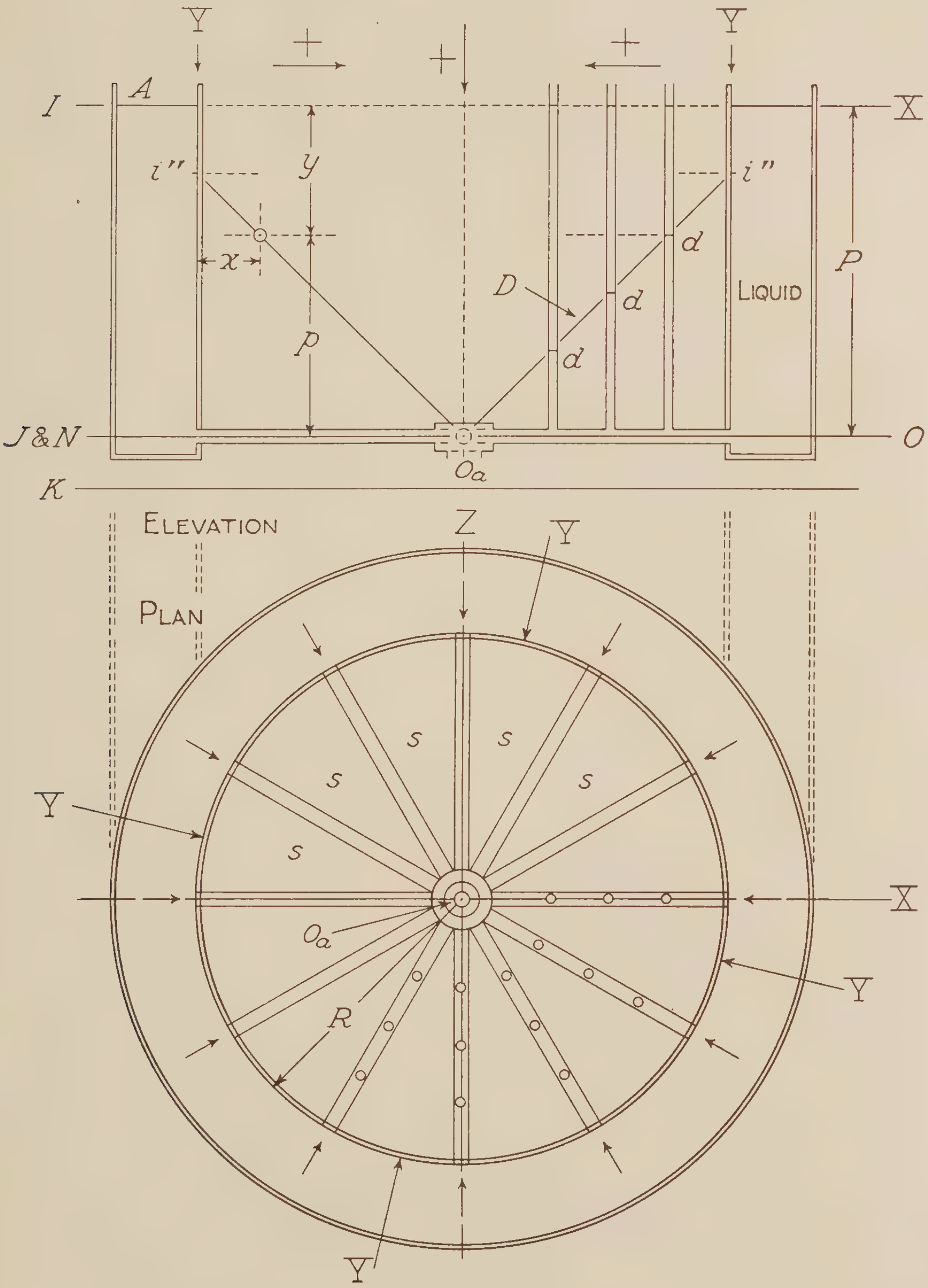


FIG. 59

other systems, for there the lineal velocity remained the same at all points along the tubes of uniform cross-section. We now have radial flow; the kinetic pressure gradient is represented neither by a straight inclined line,

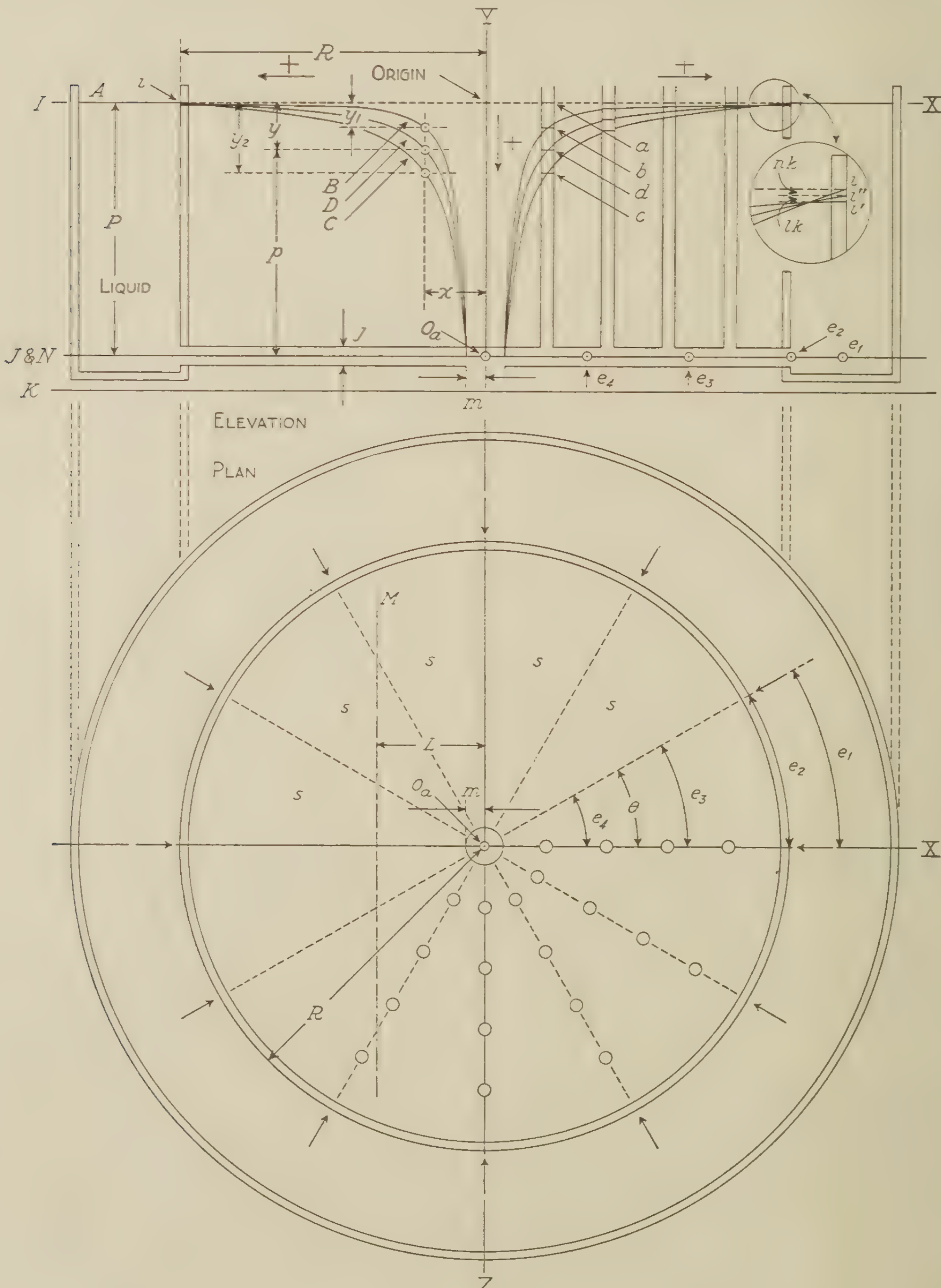


Fig. 60

nor by the surface of a right circular cone. Our problem is to determine the nature of this gradient.

For any horizontal sector subtending an angle $\theta$ at the center there is a definite amount of liquid passing its boundary at $e_2$, and this amount is the same which passes $e_3$ and $e_4$. While this system is specifically designed for liquid, we might as well make our arrangements for a corresponding gas system by expressing velocity of flow in mass units. Hereafter, then, let us refer to the fluid of the system, without discriminating between liquid and gas, except in so far as we need refer to the particular system illustrated in the present figure. We will say that $ve_2$ and $ve_3$ represent the velocities at $e_2$ and $e_3$, respectively, and consequently we may write

$$ve_2 e_2 j = ve_3 e_3 j \qquad (115)$$

where each member represents the definite amount of fluid passing the curved lines during each unit of time.[9] This equation may be reduced to

$$\frac{ve_2}{ve_3} = \frac{e_3}{e_2} \qquad (116)$$

*The mass-velocities at the lines vary inversely as the length of the lines, the angle subtended by both being the same.* But the length of the line varies directly with the distance $x$ from the center; that is,

$$\frac{e_3}{e_2} = \frac{x_3}{x_2} \qquad (117)$$

Now from Equations 116 and 117 we have

$$\frac{ve_2}{ve_3} = \frac{x_3}{x_2} \qquad (118)$$

This equation shows us that *the velocity varies inversely as the distance from the center of the orifice.* More generally we may say that

$$ve = \frac{K}{x} \qquad (119)$$

where $K$ is a constant the value of which for the given system depends only upon the units for expressing $ve$ and $x$.

Once more we will agree that the tank is of sufficient lateral extent to insure a particle of fluid at rest on the line $e_1$ during flow from the orifice. This particle possesses mechanical energy in accordance with Bernoulli's Theorem, like the one at $e_1$ in the case of tubular flow. And as between particles at $e_1$ and at $e_2$ or $e_3$ the same exchange in pressure units from $p/w$ to $v^2/2g$ occurs, $z$ again being the same for all particles. If we assume that there is absolutely no friction against flow between the plates, then the static gradient lowers from $A$ to some position to be determined. For every particle between $e_2$ and the orifice there is a definite pressure drop $y_1$ in accordance

[9] The equation simply states the principle of continuity.

with its particular velocity. The relation between the velocity and the pressure drop is, by Bernoulli's Theorem, and likewise by Torricelli's Theorem,

$$Ve = k'y_1^{1/2} \qquad (120)$$

where $k'$ includes $2g$ and another constant which is demanded by the circumstance that $Ve$ is a mass-velocity, and not a lineal velocity.

According to chapter v the velocities $ve$ and $Ve$ are always mathematically analogous; therefore we may combine Equations 119 and 120 and reduce the resulting expression to

$$y_1 = \frac{k}{x^2} \qquad (121)$$

This is the equation of the line $B$ in Figure 60. It is a "quadratic hyperbola," asymptotic to the $X$ and $Y$ axes in the manner shown. Where the spaces $s$, $s$, and so on, in Figure 59 are void, so far as the system itself is concerned, the corresponding spaces in Figure 60 are not so; and as a result we have this hyperbola in place of the straight horizontal line. At the edge of the orifice, where $x = m$, the radius of the orifice, the hyperbola is broken. The fluid here leaves the internal system and enters the external system of the reservoir; it is freed from one set of conditions and becomes subject to another set, the latter being defined by the nature of any flow-line exterior to the orifice. Remotely from the orifice the curve is again broken; now at $x = R$. The point $i'$ is slightly below the point $i$. Free surfaces of liquid in the standpipes rest as at $b$.

Since $y_1 = P$ when $x = m$, these values may be substituted into Equation 121. Thus we find that $k = Pm^2$. Now the equation stands,

$$y_1 = \frac{Pm^2}{x^2} \qquad (122)$$

The curve $B$ is due to velocity alone; friction between the plates is assumed to be zero. If we once more desire to acknowledge friction, we can determine the gradient resulting from it. Again we raise $B$ by increasing the height of the free surface of liquid in the tank sufficiently now to make $i'$ take the position occupied by $i$. The pressure head due to the weight of so much of the liquid from $e_2$ to $i'$ is, with the orifice fully open, entirely consumed by friction. The gradient is represented by the curve $C$. Its equation is determined in the same manner as in the case of tubular flow.

In Figure 61 we have a portion of the system shown in Figure 60. The origin of co-ordinates is transferred to the point $O_a$, the $X$ and $Y$ axes occupying the positions indicated. The positive direction for $x$ remains the same, while that for $y$ is reversed.[10] Now we know that the plates offer a friction which is a function of the velocity. We can say from Equation 119 that

$$Ve \text{ varies as } \frac{1}{x}$$

[10] Here again, the transference of co-ordinates is unnecessary. (See footnote 5, § 87, p. 180.)

and we know from our earlier investigations that $y$, the friction head at any point within a given reservoir system, is such that

$$y \text{ varies as } Ve^2$$

Therefore it is clear that

$$y \text{ varies as } \frac{1}{x^2}$$

or that

$$y = \frac{f}{x^2} \quad \text{.......................} (123)$$

where $f$ is a constant equal to the friction head at unit distance from $O_a$, since $y = f$ when $x = 1$. The curve $F$ for this equation is shown in the

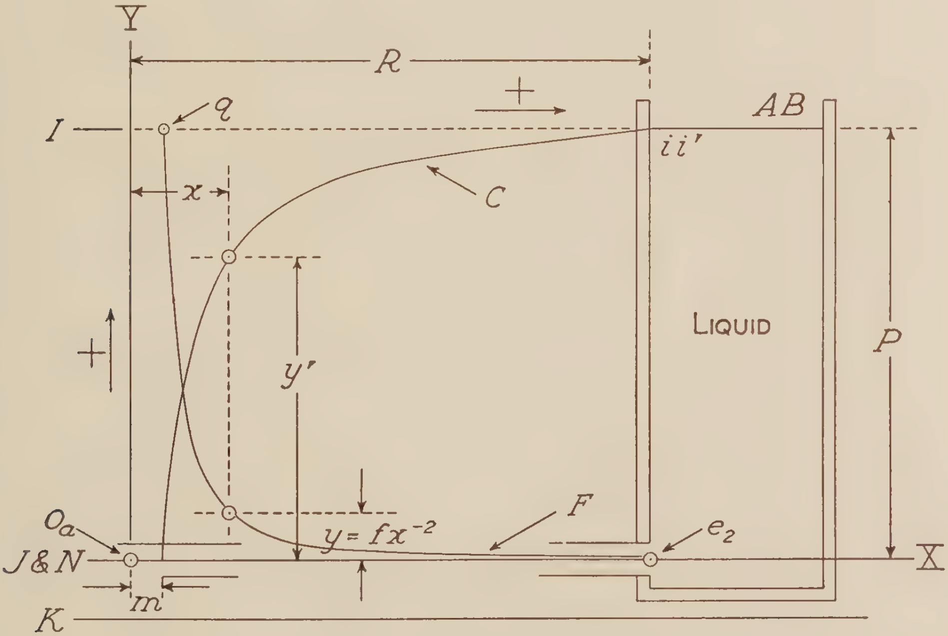


FIG. 61

present figure. It is of the same form as $B$ in Figure 60, and it lies in identically the same manner with respect to the $X$ and $Y$ axes; they differ only in the value of their constants in the equations.

As before, if we reckon from $O_a$ to $e_2$, we can say that the effect of this friction is cumulative as against $P$ for each unit of distance advanced, and being thus cumulative we can determine its curve of accumulation by integrating Equation 123. Thus we obtain

$$y' = -\frac{f}{x} + \text{a constant}$$

the latter being the usual constant of integration. When $x = m$, $y' =$ zero; therefore this constant has the value of $f/m$, and the equation becomes

$$y' = \frac{f}{m} - \frac{f}{x} \qquad (124)$$

If we wish to know the value of $f$ we can substitute the values $x = R$ and $y' = P$ into this equation. Thus we find that

$$f = \frac{PmR}{R - m}$$

The substitution of this value into Equation 124 gives

$$y' = \frac{PR}{R - m} - \frac{PmR}{(R - m)x} \qquad (125)$$

the true equation of the curve $C$ in Figure 61. This equation is not as convenient as Equation 124; let us continue with the simpler one. For Figure 60 we must transfer the origin to the proper position by substituting $P - y_2$ for $y'$, and solving for $y_2$:

$$y_2 = P - \frac{f}{m} + \frac{f}{x} \qquad (126)$$

Now we have the curve $C$ on our original figure. It indicates the position of the free surfaces of liquid in the standpipes as determined by friction alone, at rest as at $c$.

As in Figure 57, from the viewpoint of total friction head, $B$ and $C$ in Figure 60 represent opposite extremes for the system with its static pressure gradient $A$. For $B$ the internal friction is zero, and the total friction is comprised only of external friction at $O_a$. On the other hand, for $C$ the total friction is comprised only of internal friction between $e_2$ and $O_a$. In actuality the point $i$ advances toward $i'$ as the internal friction is caused to increase by altering the interior condition between the plates in any manner that will provide such an increase. For all possible positions of this point, one of which is shown at $i''$, all constants in Equations 121 and 126 are divided into $n$-fractions and $l$-fractions, where the sum of $n$ and $l$ is unity. The quantities $y_1$ and $y_2$ are to be combined in a manner which satisfies the requirement that when $x = m$, $y = P$, where $y$ is the actual pressure drop for the curve $D$, so obtained by the combination. It will be found that the conditions are satisfied when, and only when, we say that $y = ny_1 + ly_2$. The equation for $D$,[11] the kinetic pressure gradient for the system, is consequently

$$y = \frac{nk}{x^2} + l\left(P - \frac{f}{m} + \frac{f}{x}\right) \qquad (127)$$

[11] The variable $x$ is, of course, always positive, representing a distance measured outward from the orifice. Our curves, regardless of the fact that they appear in left and right positions in the figure, are entirely within the positive quadrant; that is, within the so-called first quadrant of analytical geometry.

The quantity $nk$ represents the actual velocity head at unit distance from the center; that is, it represents the portion of the pressure drop at unit distance, due to velocity alone. The quantity

$$l\left(P - \frac{f}{m} + f\right)$$

represents the remaining portion of the pressure drop at unit distance, due to internal friction alone.

The fractions $n$ and $l$ are determinable by observations on the actual gradient at convenient locations between $e_2$ and $O_a$. For tubular flow but one such observation suffices, while two are required for radial flow, as we see from the equations of $D$ in the two systems. Such observations permit us to write down the specific equation of the kinetic gradient for a particular system, and by substituting any value of $x$ into this equation the corresponding pressure drop may be easily calculated, since the values of all constants have become known.

**89.** *Tubular versus radial flow (continued)*.—Equation 127 above is clearly one for the plane curve as shown on a profile section through the $Y$ axis. The gradient is of course a surface such as would be generated by the revolution of the plane curve about the vertical axis. If in the equation we replace $x$ by

$$\sqrt{x^2 + z^2}$$

we have the equation of this surface. Then if we were to consider a line of standpipes on $M$, Figure 60, in place of those on radial lines through the center, the substitution of $L$, the distance of $M$ from the axis $Z$, into the position of $x$ in the equation of the surface would give us the equation of the gradient on $M$. Mathematically such a curve may be classed as a "witch," inasmuch as it resembles the true witch generated by the curve $B$.

The distance $R$ in the tubular and radial systems is undefined, but supposedly known. We may readily imagine a series of systems wherein $R$ varies from less than ten feet to more than ten miles. No matter how great or small this distance happens to be in a particular system, the kinetic pressure gradient, being asymptotic to the static pressure gradient, extends physically at least as far. In other words, *the drainage radius possessed by orifices in such systems as these is mathematically infinite, but physically finite, as defined by the lateral extent of the system.* If in our equations for the kinetic gradient the distance $R$ is unknown, an additional observation at some convenient point along $R$ is sufficient to determine its value; one more quantity in the equations must be treated as an unknown in the usual mathematical manner. The competency of such calculated values, like the accuracy of the equations themselves, depends upon degree of homogeneity possessed by the reservoir throughout its lateral extent.

Heterogeneous conditions within a system in no way affect the static gradient. Heterogeneous lateral conditions "warp" the curves and surfaces of the kinetic gradient, and while these continue to display the same mathematical features in general, numerical computations in accordance with the equations become unreliable. On the contrary, however, heterogeneous conditions which are confined to the vertical do not warp the curves and surfaces; the numerical computations are reliable as though all were homogeneous.

Our reservoirs of Figures 57 and 60 are hollow. We may, of course, fill the tube or the space between the plates with evenly packed porous material; for example, with fine or course sand of homogeneous texture. The internal friction would thus be modified; the static pressure gradient would remain the same, and the kinetic pressure gradient would be altered in a manner already observed; the fractions $n$ and $l$ would necessarily take other values, retaining their sum at unity.

The determination of the equations for the kinetic gradients was based partially upon the assumption that the pressure drop at the orifice is equal to the potential pressure of the reservoir. This is indeed often contrary to fact. Let us consider the simple reservoir system consisting of a circular basin of water with an orifice centrally located on the bottom. During flow the water does not assume the surface we have described as the kinetic gradient. To do so it would necessarily display a "spout" above the orifice; the free surface should ordinarily be determined by the gradient. We in fact see the water slipping radially toward the center, thus filling, partially or completely so far as we can observe, the expected spout. If we wish to show that there is radial slipping, we can start the water in a whirling motion—provided it does not start of its own accord—in order to establish a counteracting centrifugal motion. Under these circumstances the spout is formed. That the curve defined by the spout has the equation of our gradient is easily verified by mathematical analysis.[12] In our textbooks this problem is usually solved on the assumption that the internal friction is zero; consequently the equation obtained agrees exactly with ours for the gradient $B$, Figure 60.

That there should be any tendency for the fluid to whirl in our present radial system is improbable. We are safe in assuming that any such tendency is negligible, particularly if the space between the plates is filled with a porous medium. Then under the proper conditions we must expect radial slipping.

[12] The spout is that of the cyclone, or of "free vortex motion." The direction of the whirl, as seen from above, may be either clockwise or counterclockwise, depending on initial motions in the water or on irregularities in the vessel or orifice; but under ideal conditions it should be clockwise in the Southern Hemisphere and counterclockwise in the Northern Hemisphere, as a result of the earth's rotation. The whirl is easily observed in an inverted glass jug of water. The surface developed is not to be confused with that of "forced vortex motion," produced by placing a vessel of water at the center of a revolving plate. This surface is parabolic. See almost any textbook on hydraulics or hydrostatics.

What are these conditions? Simply that *the fluid can travel toward the orifice at a velocity greater than the one dictated by the residual pressure of the reservoir,* because of the small value of internal friction.

Given the reservoir system with its gradient $D$ as shown in Figure 60, what may we expect to happen when alterations in accordance with the three cases in theoretic performance are made? Let us investigate these cases briefly. For convenience we will reverse them in their usual order, and we will first assume that there is no radial slipping before or after the alteration:

CASE 3. In Figure 62 we again have a portion of the profile of our system. The gradient $D_1$ is identical with the former $D$. The constant back pressure

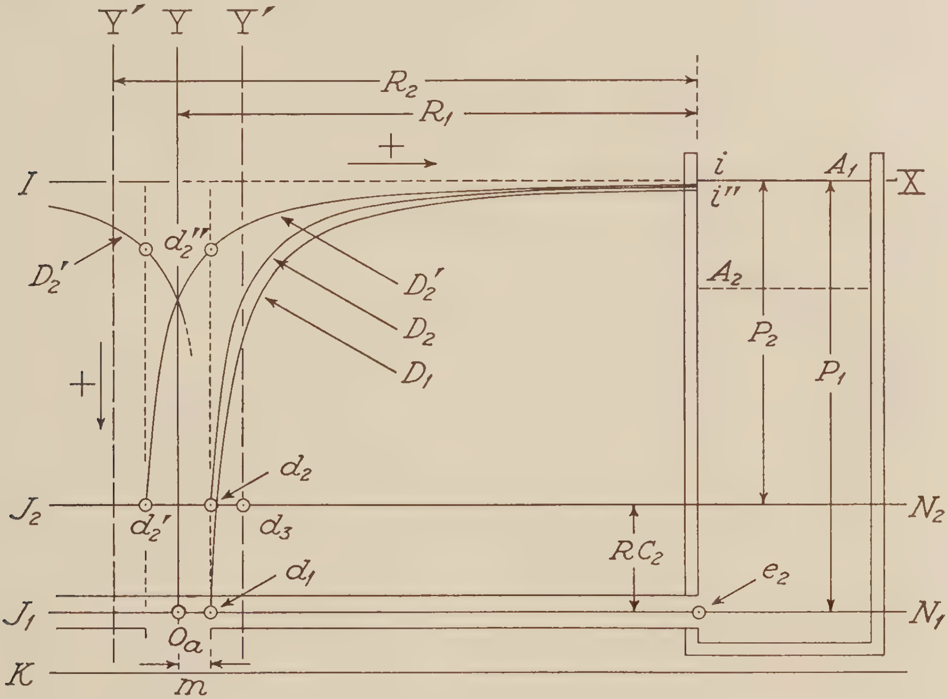


FIG. 62

at $O_a$ is increased by an amount $RC_2$; $J_1N_1$ moves to $J_2N_2$, $d_1$ moves to $d_2$, and $D_1$ is foreshortened in the vertical direction, being at the same time somewhat modified in curvature because of the change in the internal friction, becoming $D_2$. If properly stretched according to the vertical scale of Figure 60, $D_2$ would occupy a position somewhere between $D$ and $B$.

CASE 2. The static pressure of the system is reduced by lowering the height of the liquid in the tank from $A_1$ to $A_2$, an amount which we shall suppose to be equal to the distance between $J_1N_1$ and $J_2N_2$, merely as a matter of convenience. Now $I$ moves downward to coincide with $A_2$; $D$ moves downward the same distance, without deformation or modification, inasmuch as the new value of the internal friction is the same as in the preceding case.

CASE 1. Friction external to $O_a$ is increased by an amount conveniently assumed to be equivalent to a pressure represented by the distance between $J_1N_1$ and $J_2N_2$. $D_1$ becomes $D_2$ as in Case 3. We here see like effects in Cases 1 and 3. This is to be expected, for as stated in section 80, if all our reservoirs were in Hydraulic Control, the distinction between the two back pressures would be unnecessary.

There is another way of increasing the external friction; that is, it may be altered by a change in the size of the orifice at $O_a$. Let us suppose that the friction at $O_a$ is increased by diminishing the size of the orifice. $D_1$ moves laterally toward the $Y$ axis; $d_1$ takes a position in accordance with the smaller value of $m$, and the curvature of the gradient becomes modified to suit the new velocity and friction within the reservoir.

Greater, less, or contrary alterations in the three cases have their respective effects upon the gradient in the amount and direction of the shifting, and in the possible modification of its curvature.

We cannot say in advance whether a specific alteration will stop, bring into existence, or alter the radial slipping of fluid within the reservoir. Let us suppose slipping to come into existence, and give our attention to the alteration in Case 3.

The fluid slips in the direction of flow, and it carries the kinetic gradient with it. $D_2$ slips to $D_2'$, and $d_2$ takes a position at $d_2'$. Furthermore, whereas the $Y$ axis is a vertical line through the center of the orifice before the alteration, it becomes $Y'$, the surface of a right circular cylinder whose axis coincides with $Y$. The vertical generating line of this cylinder on the left belongs to the curve on the right, and so on around 360 degrees. The contour of the orifice, when projected upward, cuts the gradient surface as at $d_2''$, and below this level, on Y, the surface is closed at a point. The gradients in the remaining cases slide in the same manner.

If we should know that the alteration will permit slipping, we cannot say in advance to what extent this slipping will take place. In the water basin cited above, the slipping, in the absence of whirling, is greater than that shown in Figure 62. There the depression at the center is hardly noticeable. Given the system as in Figure 60, with slipping of fluid actually taking place, one more observation on $y$, the pressure drop, at a convenient point along $R$, is sufficient to determine the actual gradient and the amount of slipping. This observation is made necessary because of the fact that the location of the vertical axis is unknown in advance. Specifically, $R$ assumes a mathematical significance which varies slightly from its physical one. It must be treated as an unknown quantity in the equation for the gradient.

It is obvious that if we are given the system as in Figure 62, with the kinetic gradient as at $D_2'$, an alteration contrary to the one specified in Case 3 will cause this curve to slip partially or entirely back to $D_2$, depending upon the magnitude of the alteration. Under no circumstances, however, can the gradient be forced to retire from $d_2'$ to a position at $d_3$ to the right of $d_2$. If

this were possible there could be no flow between the plates from the downward projection of $d_3$, to the right of $d_1$, toward the orifice; that is, there could be no flow from the system, in spite of the fact that $P$ has a real value, and that the orifice is open. The passage of the point from the edge of the orifice toward the interior of the reservoir is an absurdity. So long as $P$ is real and the orifice remains open, the residual pressure is real, and as a consequence flow must take place at a rate precisely determined by the value of this residual pressure.

There is of course a corresponding radial slipping of fluid with tubular flow. We need not consider the facts in detail, for we shall say that the systems of Figures 57 and 59 have already served their purpose in assisting us to understand the nature of radial flow. We are now prepared to apply the correct principles to the natural reservoir capable of producing oil, gas, and water. In these we have radial flow toward the wells.

## CHAPTER XIII

# Secondary Functions of Performance
# *(Continued)*

"The ideal isolation and breaking-up of actuality, its logical separation into different divisions, is one of the favorite devices of thought; a complex, in reality inseparable, is split up into ideal elements and parts, and each of these ideal elements is then treated in isolation."—HANS VAIHINGER

**90.** *The natural reservoir.*—The more closely we abide with Nature in defining our ideals, the easier will be the task of interpreting and applying the results of analysis in practice, for fewer modifications and less of pure hypothesis will be required in attempting to arrive at actualities. For these reasons I have believed it an advantage to recognize the viscosity of fluids and the consequential internal friction which tends to oppose the movement of the fluid toward the orifice. In contrast to the present methods of analyzing the performance of reservoirs we have had for many years the well-established "Mathematical Theory of Fluid Motion."[1] It is well known that this theory is developed upon the assumptions of non-compressible liquids and perfectly elastic gases, both fluids being absolutely non-viscous in order that none but perpendicular forces can act upon each particle constituting their masses. Furthermore, the reservoirs, either as containers or as conduits, are hollow. It is perhaps clear that idealization to this extent practically prohibits the application of the results of such an analysis to those reservoirs which are known to possess a porous medium within them.

While the reservoir system of Figure 60 is an artificial one, such as we may construct with ease and maintain in Hydraulic Control by furnishing a supply of fluid equal to the rate of production, we need only imagine certain modifications to be made for the purpose of hypothetically transforming it into a natural system. Let us reason somewhat as follows:

*a*) Evenly packed sand of homogeneous texture is to be placed between the two plates of the system.

*b*) The tank proper is to be removed, and the lateral extent of the remaining section is to be increased in the order of feet to miles.

*c*) The plates are to be replaced by geological formations which are im-

[1] We need not believe that, in so far as our present requirements are concerned, the methods of treating the mathematical theory of fluid motion by means of differential equations are at all superior to the present methods borrowed from treatises on elementary theoretical mechanics and hydromechanics.

permeable to fluids. The sand grains may become homogeneously cemented together and thus constitute a porous and permeable geological formation. Stratification to any extent within the sand may be included, provided that the strata are individually homogeneous in texture throughout the lateral extent of the reservoir. *The formation may thus be heterogeneous in the vertical direction, while it is homogeneous in all horizontal directions.*

*d*) Ideal structural features are to be introduced into the system. These need only conform to the general specifications which have already been given.

In this manner of transformation we arrive at the sort of ideal natural reservoir system shown in Figure 47. Not one of these hypothetical changes affects the mathematical properties of the pressure gradients; the equations and the contours of their geometrical surfaces remain the same, for the system is equally and identically ideal before and after the transformation. In all directions these gradients extend to the limits of the reservoir, wherever these limits may happen to be located geographically. All the fluid within the porous formation, regardless of its nature, is in the reservoir, if this be taken in its physical sense. But if the reservoir be taken in its mathematical sense, as a "potential reservoir," it may happen that but a portion of all the fluid, still regardless of its nature, is in the reservoir. This distinction between the physical and the mathematical reservoir will become more evident in our study of Volumetric Control.

From this ideal natural reservoir it is but a simple step to any actual oil, gas, or water reservoir in this control, such as we know to exist in the field. The mathematical properties of the actual gradients are fundamentally the same. The static gradient continues to be a perfect geometrical surface, while the kinetic gradient is represented by a surface which is more or less warped. In the equation for this gradient the quantities $k$, $f$, $n$, and $l$ differ at least slightly at various parts of the reservoir in accordance with deviations from the ideal internal system with its porous medium.[2] The fact that these quantities possess different values here and there within the lateral extent of the actual reservoir accounts for the warping of the gradient surface. *It is obvious that changes in these quantities cannot alter the general mathematical properties of the equation;*[3] consequently the general features of the surface are not affected in any way by heterogeneous conditions within the reservoir.

Let us continue with our investigation of the ideal.

[2] Even if the productive formation were of homogeneous texture in its lateral extent, and the reservoir contained but one homogeneous fluid, slight irregularities in the structure would cause these quantities to possess different values.

[3] As "general mathematical properties" we would include the appearance of the geometrical curve, its asymptotic relation to the straight line of the static pressure gradient, and so on. The static gradient itself cannot be modified in the least by any degree of heterogeneity within the reservoir, and the kinetic gradient, regardless of warping, remains asymptotic to the static gradient.

**91.** *Interpretation of the kinetic gradient.*—We see, then, the closest analogy between the artificial reservoir system of Figure 60 and any natural system of the same control. While it is usual to regard the static pressure gradient of a system which has a small lateral extent, measured in inches or feet, as a perfect horizontal plane, it is in fact a portion of the surface of an ellipsoid that is concentric with the earth. The kinetic gradient is asymptotic to this ellipsoid. Even though our natural systems have a lateral extent measured in miles, we need not be so precise as to conform with Nature in our analysis. As a matter of fact our qualitative and quantitative results will be the same whether we regard the surface as flat or curved; we may proceed as if the static gradient were a plane. In particular our plane is determined by the line $I$ in Figure 47, corresponding to the line $A$ in Figure 60.

The standpipes in our artificial system correspond to symmetrically located wells in the field. All wells are closed except the central one. Each of these "adjoining wells" shows a certain apparent static pressure, as indicated by the height of the free surface of liquid in the standpipe, equal to the difference between $S$, the real static pressure of the reservoir, and $y$, the pressure drop at the well. The potential pressure which any individual adjoining well possesses is

$$p = P - y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (128)$$

If an individual well were opened, to the exclusion of all other adjoining wells, a particular value $p_1$ determines the fact that there shall be flow at this well, and a corresponding residual pressure $pr_1$ determines the velocity of flow. It is to be observed that while this individual well is permitted to flow the original well at the center has its velocity diminished in conformity with the kinetic gradient of the adjoining well. We may say that the second well "pulls down" the rate of production at the first one, in retaliation for the same effect which the first well has upon the second. In the ideal reservoir, where any two such wells constitute orifices that are physically identical in all respects, each has the same apparent static pressure and the same values of $p_1$ and $pr_1$. If the two wells are not physically identical, each has its particular values of these quantities; the one which has the greater velocity possesses greater values for them.

We may open a second, a third, and a fourth adjoining well, and so on, increasing their number until we consider the field "drilled up." The effects of separate gradients are cumulative for each individual well in the group. The value of $y$ for any well increases for each additional well that is opened; $p$ diminishes in value, but not by equal amounts for each successive well. If we assume all wells of a group to be physically alike, and all to be symmetrically located according to either a quadrilateral or a triangular pattern,[4] then

[4] By the expression "symmetrically located" I mean located in conformity with corners of squares or with corners of equilateral triangles. By no other pattern can an area be symmetrically covered. The regular hexagon is itself composed of six equilateral triangles; therefore the hexagonal and triangular patterns of location are identical.

we may say that the successive decrements in $p$ become smaller for each additional well; $p$ must not become zero for any number of wells, no matter how great this number may be. Let us return to this subject after a brief consideration of the multiple orifice.

If the orifice at the center of the lower plate in Figure 60 were covered with a perforated plate, such as shown in Figure 63, we might consider each

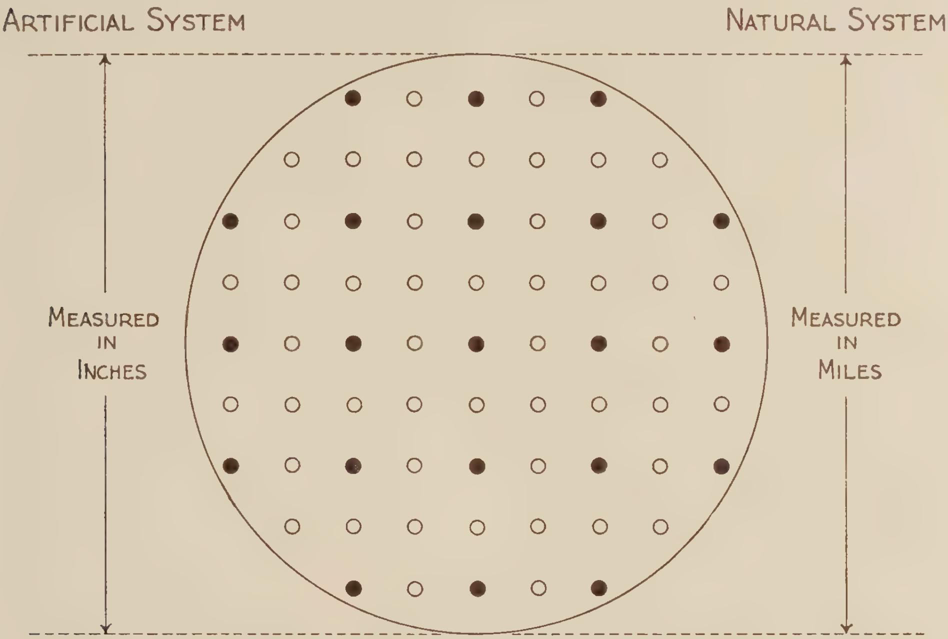


FIG. 63

perforation an individual orifice, or the entire group of them as one orifice. The individual holes must have their own kinetic gradients, each intercepting those of the others, all uniting at the border of the plate to form a composite gradient which may be said to be that of the group as one orifice. The group constitutes a *multiple orifice*.[5]

When the space between the plates is hollow, the total rate of flow from the system is largely, if not exclusively, determined by the external friction at the individual holes. On the other hand, when the space is filled with a porous medium, the total rate is determined by both the external and the internal friction, as we learned in section 84. The proportionate influence of the two depends upon the following factors: ($a$) the number of holes per unit area; ($b$) the size and physical condition of the holes; and ($c$) the permeability of the porous medium. Given a system with the factors ($b$) and ($c$) fixed, there is a minimum number of holes, $q$ per unit area, which permits

[5] See § 75.

the maximum total rate of flow from the system. In other words, *we may increase the total rate of production from a unit area by increasing the number of holes up to a certain point; thereafter, because of a regulative action of internal friction in the surrounding area, further increase in the number fails to increase the total rate.* To consider a particular example, let us suppose we have the system as in Figure 60 with a comparatively "tight" porous medium between the plates, and the central opening covered with a plate of unit area, perforated as shown by the dots in Figure 63. Now the number of holes is increased by drilling the locations which are indicated by circles, midway between the dots. Next the locations between all these are drilled; then once more, and so on, increasing the number in the ratio of 4, 9, 25, 81, and so on. At some stage in this process, as determined by the results, we meet with and exceed the number $q$. Additional holes do not increase the total rate of flow from the system. These merely reduce the rate for the earlier holes and participate in production at the same new rate.

Let us proceed with mathematics. We will say that at the time $q$ is reached each hole shows, when it alone is closed, a particular value of $p$, say $p_0$. We have in general the relation expressed by Torricelli's Theorem: namely,

$$v = kp^{1/2} \qquad (129)$$

where $v$ is a mass-velocity, and $k$ replaces the square root of $2g$ for lineal velocity. In particular we have

$$v_0 = kp_0^{1/2} \qquad (130)$$

$v_0$ being the mass-velocity for each hole at the time of $q$ holes.

The total rate for the system is clearly

$$qv_0 = qkp_0^{1/2} \qquad (131)$$

where $qv_0$ is a definite mass of fluid delivered from the system per unit of time, a quantity which will not vary when there are additional holes.

Now when there are $q + 1$ holes, the rate at each hole must be

$$\frac{q}{q+1}\, v_0 = \frac{q}{q+1}\, kp_0^{1/2} \qquad (132)$$

so that the total rate for $q + 1$ holes may be the same as that for $q$ holes. For further additional holes we have

$$\frac{q}{q+2}\, v_0 = \frac{q}{q+2}\, kp_0^{1/2} \qquad (133)$$

$$\frac{q}{q+3}\, v_0 = \frac{q}{q+3}\, kp_0^{1/2} \qquad (134)$$

and so on, for successive rates at each hole. In fact it is evident by inspection that as successive velocities, we have

$$v_1 = \frac{q}{q+1} v_0 \qquad (135)$$

$$v_2 = \frac{q}{q+2} v_0 \qquad (136)$$

$$v_3 = \frac{q}{q+3} v_0 \qquad (137)$$

and so on. Thus we know the value of the new rate for each hole in terms of the rate for one of $q$ holes. In the same manner we may know that

$$p_1^{1/2} = \frac{q}{q+1} p_0^{1/2} \qquad (138)$$

or

$$p_1 = \left(\frac{q}{q+1}\right)^2 p_0 \qquad (139)$$

Likewise

$$p_2 = \left(\frac{q}{q+2}\right)^2 p_0 \qquad (140)$$

$$p_3 = \left(\frac{q}{q+3}\right)^2 p_0 \qquad (141)$$

and so on.

Not only do the values of $p_1$, $p_2$, $p_3$, and so on, become smaller for each additional hole, but their differences grow smaller for each one. This follows from the fact that

$$\left(\frac{q}{q+1}\right)^2 - \left(\frac{q}{q+2}\right)^2 > \left(\frac{q}{q+2}\right)^2 - \left(\frac{q}{q+3}\right)^2 \qquad (142)$$

The argument here given applies equally to the artificial system of Figure 60 and to the oil or gas field in the same control. Flow from holes in the plate corresponds to flow from wells. We may now account for the statement made above to the effect that the successive decrements in $p$ become smaller for each additional well, and $p$ cannot become zero for any number of wells, no matter how great this number may be. The successive values of $v$ and $p$ at each well of course bear the relation expressed by Case 2 in theoretic performance, for the additional wells, as we know, give rise to alterations in the apparent static pressure of the reservoir.

In oil and gas fields the area for drilling is at some time restricted to the extent of the pool by the proven failure of outlying wells. The number of wells within this area is increased during the history of the field, and provided that the development proceeds sufficiently the number $q$ will be reached

and exceeded.[6] In fact the size of property holdings in the field will often determine whether or not this number is to be attained.

It is true that wells are not perfectly arranged in the manner shown in Figure 63, nor are all wells producing under identical physical conditions. *These facts do not affect the existence of* q; *they merely rule out the mathematically perfect succession of values given in the equations.* In place of 1, 2, 3, and so on, we actually encounter an erratic succession of ever-increasing numbers which are not confined to integers.[7] The successive numbers depend upon the irregularity of the pattern according to which the wells are located, that is, upon the relative proximity of adjoining wells, and also upon the physical condition of all wells concerned.

92. *Interpretation of the kinetic gradient* (*continued*).—Within the same area we may encounter one or more productive formations, as we know. Let us continue on the basis that there is but one such formation in the field, recognizing the fact that, should there be more, each one must be treated separately as though the others do not exist.

It is a feature of Hydraulic and Volumetric controls that all wells in the same field, producing from one formation, produce fluid not only from the same reservoir in its physical sense, but also from the same reservoir in a mathematical sense; that is, *all wells have one potential reservoir in common.*[8] The kinetic gradient of each well mathematically intercepts the gradients of all other wells; in other words, flow at each well affects flow from all other wells. It is to be admitted, however, that the effect which one well has upon another may be very slight on account of the distance between them. The effect is none the less real, even though it may not be measurable by test. Perhaps the most striking evidence of this small but real effect lies in the fact that any two or more wells here have the same drainage area in common.[9]

In the equation for the gradient of the individual well the quantity $m$ is obviously a very small one in comparison with $R$; $m$ is the radius of the casing, or the radius of the tubing within the casing, if the well is so equipped,

[6] The situation is not to be taken as applicable to all controls. As a matter of fact, the problem of $q$ does not enter into the consideration of production from reservoirs in Capillary Control.

[7] With a symmetrical pattern, and at the same time with wells ideally alike, the numbers are confined to integers because the effects of one well upon all others similarly located with respect to it are mathematically of equal intensity. This is certainly not true under contrary conditions.

[8] See § 19. We shall find that the situation is different in Capillary Control. In this (Capillary Control) the wells produce from the same physical reservoir, but each one produces from a separate and distinct potential reservoir, regardless of the distance between them.

[9] Water encroaches upon the pool from which the wells produce in common. The mutual effects of wells upon each other are in no case negligible with respect to volume to be produced by each one.

whereas $R$ extends to the physical limits of the reservoir. We see that $m$ holds the material portion of the gradient closely to the well, and as a consequence we may find a pressure drop of very small value at a distance of a few hundred feet from the well. When wells are few and widely separated, their mutual effects are almost, if not quite, negligible in so far as pressure is concerned.[10] The pressure drop $y$, we may say, is in this case negligible in comparison with $p$, as these quantities appear in Figure 60. Let the number of wells in a given area be great, however, then $y$ is not negligible in comparison with $p$, for each additional well in the area causes $y$ to increase and $p$ to decrease simultaneously. The mutual effects upon the rate of production from wells which are closely drilled and great in number cannot be ignored.

Given a perfectly homogeneous formation in an area, the first well to be drilled has the best opportunity to excavate for itself a chamber at its bottom which is to serve as the true orifice of the reservoir, for its potential pressure is $P$, greater than $p$ for subsequent wells. $P$ of course furnishes a greater velocity than any subsequent $p$, and the greater velocity in turn provides the well with a greater facility of excavating a chamber. If a subsequent well is the first one to strike a most favorable portion of a non-homogeneous formation, it enjoys privileges which otherwise belong to the original well in the field.

Inasmuch as we may regard a group of wells as a multiple orifice having a composite kinetic gradient formed by the union of all individual gradients, there is for such a group a radius $m$ which corresponds to that for a single well. It is a quantity to be measured from a vertical axis $Y$ at the centroid of production for the field, the location of this centroid being determined in the same way as is usual in the case of systems composed of solid particles. When all wells are located according to a symmetrical pattern, and all produce fluid at the same rate, $m$ is of the same length in all directions radially from $Y$; otherwise $m$ in general will be of different length in all directions in compliance with the irregularity in pattern and rate. *The radius* m *defines a hypothetical single orifice which is mathematically equivalent to the group of wells in the field.* The area described by $m$ has $Y$ at its centroid, and the composite gradient for the field has the usual equation wherein the variables $x$ and $y$ are measured from this axis $Y$ and the axis $X$ defined by the static gradient, respectively. Irregularities in $m$, and possibly those in $R$, prohibit us in treating the kinetic pressure gradient surface as one of revolution about $Y$. The gradients must be handled as plane curves in such directions from $Y$ as may be of interest to us. The radius $m$ is not a small quantity when $q$ wells cover an extensive area; a new well located outside the original area of a field—a so-called extension well—even though somewhat remote, may be materially affected by the pressure drop due to the gradient of the group.

[10] It follows then that their mutual effects are almost, if not quite, negligible in so far as velocity, or rate of production, is concerned.

Productive formations frequently consist of series of parallel strata that differ considerably in their texture, and therefore in their porosity and permeability. These are circumstances which are due, of course, to prevailing conditions at the time the sediments were deposited. Provided that the individual strata in a given formation are homogeneous in their lateral extent, the kinetic pressure gradient of a well which penetrates this formation is geometrically perfect; it is as though the formation were of homogeneous texture from top to bottom. The fluid within the more permeable strata may be said to be produced directly into the bottom of the well, whereas that within the less permeable strata is first produced into the adjoining more permeable strata, thence into the bottom of the well.

Lateral stratigraphic features, such as lenses of coarser or finer texture, or more extensive modifications in the petrographic nature of the formation, affect radial flow toward the orifice and, as already noted, the surface of the kinetic gradient is warped by them. Structural features have a like effect.[11] We should observe in connection with Figure 47 that the line *I* serves as the static pressure gradient of the well *W*, and while the kinetic gradient is asymptotic to this line the flexures in the formation warp its curve. The surface of this gradient cannot be one of revolution about a vertical axis at the well. We should note further that the lines *K, J, N,* and *I,* as drawn, pertain only to *W* under the given conditions of production from it; with the present distances between them these lines do not apply to such wells as *V* and *U*. In our study of the primary functions of performance we regarded the line *K* as the only one of the four which is fixed in position. *K* is fixed in Nature, or rather absolute zero pressure, which it represents, is fixed in Nature; but as a line in our drawings it is certainly not fixed until the line *I* has been definitely located. Now in connection with the secondary functions of performance we are interested in *I* as an axis *X,* and it is necessary, and convenient, to regard this line as the only one which is fixed, in order to represent properly the interrelations of such wells as *W, V,* and *U*. This change of attitude on our part contravenes Nature in no way whatever; after all, our methods of representing Nature in drawings are purely artificial, for the drawings are simply inventions of our own device. The kinetic gradients of the three wells, and those of all other possible wells in the same formation, unite in their asymptotic relation to the line *I*.

Casing-head pressure readings refer to points on the kinetic pressure gradient.[12] Figure 64 is a diagram intended to represent a producing well, equipped with casing and tubing. The axes and four horizontal lines appear as usual. We will say that under perfectly constant conditions for production oil stands in the casing at a level *N,* and that the tubing extends down-

[11] Any feature capable of affecting radial flow toward the orifice, either as to lineal velocity or as to direction of flow, necessarily affects the kinetic pressure gradient.

[12] I refer here to those readings which are taken while the well is flowing.

ward to this level, or slightly below it, if preferred. Now the kinetic gradient is $D_1$, cut by the casing at $d_2'$, so that the portion between $d_1$ and $d_2'$ breaks and becomes practically a horizontal line by the radial slipping of oil in the section between $d_1$ and $d_2$. The oil continues to stand precisely on $N$ between $d_1$ and $d_2$, while it exerts a pressure upon the gas or air above in the space be-

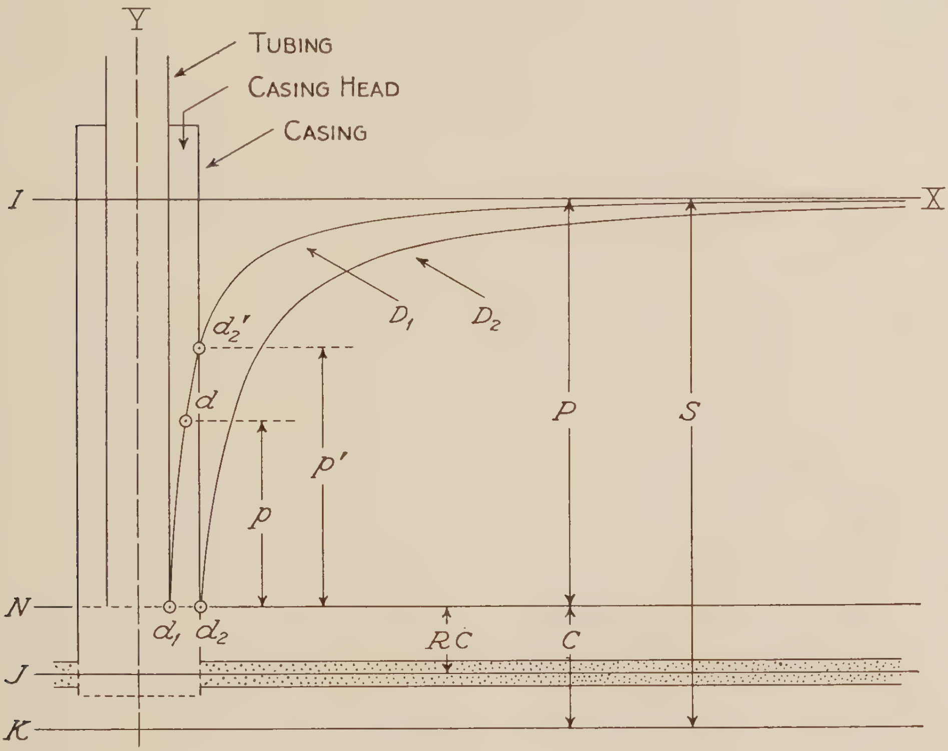


Fig. 64

tween the tubing and the casing. This pressure denotes the position of the broken gradient, and it is that pressure which is recorded by the gauge at the head of the casing. Now the pressure at $d_1$ is zero, as measured above $N$, and that at $d_2$ is $p'$.[13] The recorded pressure has some value $p$ between zero and $p'$. The broken gradient rests at a point $d$, which is defined by $p$, such that its circle about the $Y$ axis divides that portion of the gradient surface between $d_1$ and $d_2'$ into two equal parts. Thus we know the nature of the casing-head pressure.

Let us see what happens with the casing-head pressure in practice. By Case 1 in theoretic performance the gradient may be made to shift between

[13] Where the gauge is recording pressures above its zero, the corresponding distances in the figure are measured above $N$. $RC$ is not recorded by the gauge, for liquid stands in the casing from $J$ to $N$.

$D_1$ and $D_2$; as it moves outward the casing-head pressure decreases, and approaches zero as recorded by the gauge. The action is reversed when the gradient moves inward. By Case 2 the $X$ axis is shifted downward or upward by opening or closing adjoining wells; $d_2'$ moves accordingly, carrying $d$ with it; the casing-head pressure decreases or increases in compliance with this movement of $d$. Once more, by Case 3 the line $N$ is shifted downward or upward by decreasing or increasing the constant back pressure; $d_1$, $d$, and $d_2'$ move accordingly, and the gauge registers a corresponding change. Furthermore, any of these alterations may stop, bring into existence, or alter the radial slipping of oil in the formation as it moves toward the well. Under these circumstances the position of $d$ is twice affected—once by the alteration in theoretic performance and again by the alteration in slipping—and of course the gauge at the head of the casing readily responds to the situation.[14]

93. *Features within the reservoir.*—In regard to many features, though certainly not to all of them, that are classified as secondary functions of performance I believe it is more important for us to recognize their existence, and understand their behavior, than to possess their quantitative values at particular points which are close to or remote from the wells. The quantitative values of the primary functions may fortunately be made to suffice for most purposes in the operations of development and production. But these values alone cannot properly guide us, unless we know how to interpret them accurately in the light of the secondary functions. At the present time I am inclined to appreciate the kinetic pressure gradients for their qualitative significance, and to ignore any attempts to utilize their associated quantitative data.

We have given the subjects of velocity and pressure, as we find these to exist within the interior of artificial and natural reservoir systems, a thorough analysis. Let us now give our attention to volume, and investigate its associated features, at least in so far as these features are partially or entirely dictated by the gradients.

Volume of fluid is expressed either directly or indirectly in measurements of three linear dimensions. Our reservoirs, in order to contain a volume of fluid, of course possess these dimensions. While we are able to measure directly to any desirable degree of accuracy most artificially constructed reservoirs by applying a scaled ruler or tape to them, the inaccessibility of natural reservoirs partially, though not entirely, interferes with such a method. We are able to determine the volume per unit area of surface underlain by a productive formation when we have observed the thickness and the porosity of this formation from drilling records and samples. In this we assume that our observations are sufficiently accurate as data applicable to the formation at

[14] In consideration of the nature of these casing-head pressures it is evident that they possess little, if any, theoretical value in mechanics. The closed-in casing-head pressure, plus the weight of any liquid standing in the well casing, is, of course, our registered static pressure, and this, as we know, possesses value in the analysis of performance.

large. Other methods, which suit the reservoir according to its control, will be discussed later.

With respect to volume alone it will be convenient to classify all individual productive formations according to the nature of the fluids within them. As thus to be classified these will contain:

*a*) Either oil or water, one to the exclusion of the other, at least throughout an undefined extent of the reservoir, and this one to the exclusion of gas either in the dissolved or free state.

*b*) Gas alone, at least throughout an undefined extent of the reservoir.

*c*) Either oil or water as in (*a*), except that gas shall accompany the one liquid.

*d*) Both oil and water, each confined to separate sections of the reservoir or both associated in one and the same section. That is to say, the oil in a pool is completely surrounded by water, and either there is no water underlying the oil or there is such water. The liquids are present to the exclusion of gas in either the dissolved or the free state.

*e*) Both oil and water as in (*d*), except that gas shall accompany the two liquids.

With this classification we may continue our analysis of reservoirs in Hydraulic Control. We want to know of conditions and events within the reservoirs, particularly those which exist or take place during the process of production.

In the discussion concerning the kinetic pressure gradient we invariably referred to reservoirs of class (*a*) as exemplified by the system illustrated in Figure 60. The analysis pertained to natural and artificial reservoirs alike, and we found that reservoirs of class (*b*) should be included, in so far as the mechanics of liquids and gases are the same. In regard to systems which contain gas alone no question arises with respect to free surfaces within the reservoir, for gas displays no such phenomenon; it occupies under all circumstances of pressure the entire available space within the formation. But in regard to those reservoirs which contain liquid alone we may ask: Under what circumstances, if any, will the reservoir tend to provide a free surface for its liquid in compliance with the kinetic pressure gradient? And if the reservoir has this tendency, under what circumstances, if any, will it succeed?

To answer these questions let us first compare the vertical dimensions of our gradient curves with corresponding dimensions for volume; that is, let us say, with the thickness of the productive formation.[15]

[15] The true vertical dimension for volume is equal to the thickness of the formation multiplied by the factor for porosity. For example, for a sand which has a thickness of 10 feet and a porosity of 30 per cent the true vertical dimension for volume is 3 feet. But since we are here interested in the "lay of the volume of liquid" within the formation, we should consider the thickness of the formation to be the appropriate vertical dimension for the purpose.

In Figure 47 we see that the static pressure gradient passes through the points $a$ and $W'$. The latter is at a considerable height above the well at $W$, and this in turn is at a considerable height above the point $b$. In fact the thickness of the formation is very small in comparison with $bW'$. By adopting other vertical scales for pressure we can cause the static gradient to pass through any desired point located above $b$. The gradient would continue to be correct, but vertical distances would not represent feet of liquid. It is clear that the height of the gradients in our diagrams for any reservoir system whatever depends upon the adopted scale for pressure; we choose one of advantage, however, when we adopt that scale which represents feet of liquid, for with this we have harmony between vertical distances in space, including the thickness of the formation, and pressures within the reservoir. When the system of Figure 60 is provided with open standpipes in the manner shown, and when such wells as $U$ in Figure 47 remain open, the free surface of liquid which rests in them conforms with the diagram of the gradients, if this scale is adopted. We need only imagine the casings of such wells as $W$ and $V$ to be extended upward to comply with the same scheme. The scale is probably of less importance in case the fluid is a gas.

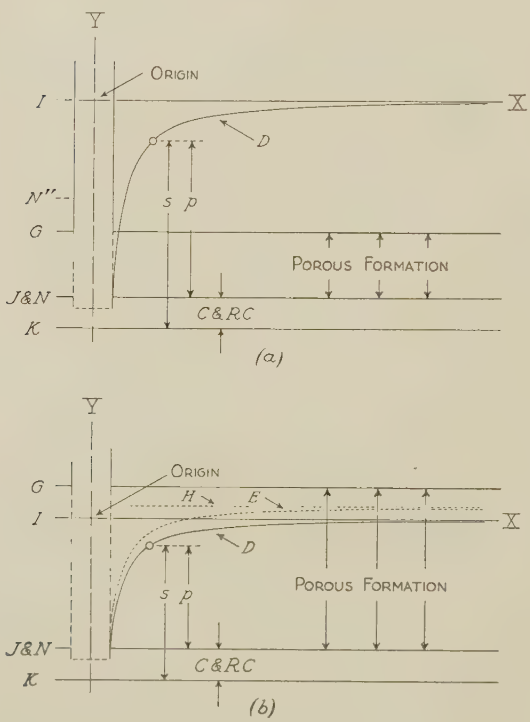


Fig. 65

We are to admit that while the ratio between the height of the gradients and the thickness of the formation is a large number in the case of the well $W$ of Figure 47, it is not always necessarily so in natural reservoirs. In Figure 65 ($a$) we have a case where the ratio is less. We may say that under the conditions of production the lines $J$ and $N$ coincide with the bottom of the porous formation, and that a line G denotes its top.[16] The static gradient stands at $I$, above $G$, and the kinetic gradient is represented by the curve $D$, the individual points of which show a pressure $p$ corresponding to a distance $x$

[16] It is not inferred that the line $G$ has any relation to the horizon $G$ of Fig. 33. It is impossible to avoid using letters of the alphabet with different meanings in separate problems. The best we can do is to avoid duplication in any instance where two or more problems are to be united in a later investigation.

from $Y$ and a pressure drop $y$ measured downward from $X$, the latter coinciding with $I$. We are still considering the case in which the reservoir produces liquid, either oil or water, alone. Is there a free surface of liquid within the reservoir? Certainly not when the liquid is permitted to rise in the well and rest at $I$. However, if the liquid is removed by mechanical means, and its level is thereby maintained at $J$ and $N$, a free surface will appear within the reservoir. Air will enter the upper perforations in the casing and allow the liquid to take a position in accordance with the kinetic gradient. The free surface thus created will lie on $D$, between the bottom and top of the formation. This surface is evidently confined to a region of an exceedingly short radius around the well, and beyond this radius all pore spaces are completely filled with liquid. If the removal of the liquid should take place at a slower rate, thereby allowing its rise in the well to some point between $G$ and $I$, the free surface disappears, for the gradient is raised vertically to a position above $G$, as in Figure 62. There $D_1$ became $D_2$ by such an increase in the constant back pressure. In the present figure $J$ and $N$ are parted, the former retaining its position while the latter shifts upward to some position $N''$.

In Figure 65 ($b$) we have the case where the ratio between the height of the gradients and the thickness of the formation is less than unity; that is, the static gradient $I$ falls below the top of the formation at $G$. Now while the level of the liquid in the well is maintained at $J$ and $N$, air enters the formation, and the free surface determined by $D$ extends throughout the reservoir. If the liquid did not rest on the line $I$ before the well was drilled, but rested at a higher position on account of the lack of air or gas within the formation, then a partial vacuum must have existed within the reservoir at the time, and this partial vacuum had to be satisfied by the inflow of air from the well before the liquid moved toward the well.

We must note the presence of phenomena due to capillary attraction in those reservoirs wherein the liquid possesses a free surface in accordance with either gradient. By this attraction the liquid actually rests on lines such as $H$ and $E$, at a definite height above $I$ and $D$, respectively. The amount of elevation is dependent upon the surface tension of the liquid and the horizontal cross-section area of the pores within the formation. These phenomena cannot exist where the liquid has no free surface. $H$ and $E$ do not denote the pressure gradients of the system; in fact, on account of the rising of the liquid the gradients take positions which lie slightly below those shown, for the attraction withdraws a portion of the liquid from the potential volume of the reservoir.

**94.** *Pressure and volume cylinders.*—Let us extend our interpretations of pressure and volume, confining our attention for the present to the simple reservoir systems that produce either liquid alone or gas alone. Figure 66 (p. 208) represents a cylinder with a circular base of radius $R$ or $r$, the latter replacing $x$ of the preceding plane figures of the gradients. The height of the cylinder

is determined, we shall say, either by $S$, the static pressure of the reservoir, or by $s$, the apparent static pressure at any point $r = x$, as shown in Figure 65. Now the cylinder, extending between $K$ and $I$ or $i$, is bounded at the top by a plane surface which represents either the static or apparent static pressure gradient, and its lateral surface represents the intensity of either of these at all points equally distant from a well $W$, if we agree that the productive formation is homogeneous in its lateral extent. Of course we can divide this cylinder into sections which correspond to the distances between the usual horizontal lines. The surfaces of the resulting sections then represent their respective pressures. With $W$ completely closed all the sections may be regarded as geometrical solids, but with $W$ open, and with production taking place in accordance with the sections, the uppermost one, representing either the potential pressure $P$ or the apparent potential pressure $p$, contains an inner surface as described by the kinetic pressure gradient. This section becomes a cylindrical solid with a centrally located "spout."

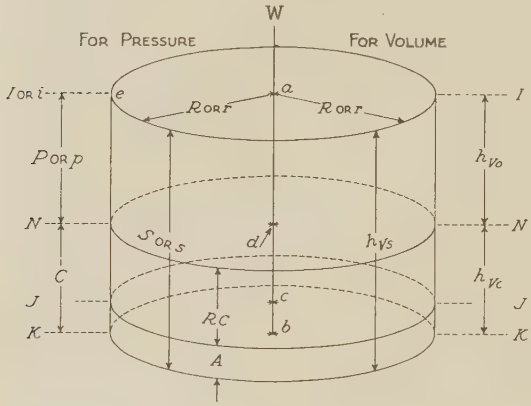


Fig. 66

The sections below $N$ are always inactive as an aid to production; they are, in fact, always active in opposition to production.[17] On the other hand, the section above $N$ is always latently active as an aid when the well is closed, and always actually active as an aid when production takes place.

When $W$ is closed we may regard the solid cylinder as being made up of a number of thin cylindrical shells, all representing, by their constant height, equal pressures in all directions, particularly in the two directions which coincide with all radii of the cylinder.[18] The fluid cannot move toward the well under these circumstances. But when production is taking place the shells diminish in height as the well is approached. On the two surfaces of each shell the perpendicular, or radially directed, pressures are unequal, for that

[17] They are evidently associated with the constant back pressure. The entire figure, however, is not to be taken too literally in the physical sense. It is properly a mathematical representation of a physical state. Perhaps this relation between a physical state and its mathematical representation can be better appreciated in connection with Capillary Control. (See § 175.)

[18] These are taken as directions of action and reaction along each radius. By *Pascal's Principle* the pressure is equal in all directions about any point on a radius, when the fluid is at rest. We are particularly interested in the directions coinciding with any radius.

one which pushes the outer surface inward is greater than the one which pushes the inner surface outward. The fluid moves in the direction determined by the greater pressure; that is, toward the well.

Corresponding to these pressure cylinders we have volume cylinders. Similarities and differences between them are to be noted. For convenience we shall refer to the same Figure 66. The radius is the same as before, but the vertical dimensions now differ, unless the scales for pressure and volume-thickness are suitably adapted to each other. This we shall assume to be the case. Contrary to our procedure in the preceding section, we should now consider the true vertical dimension for volume, as there defined. The entire cylinder then has a height $h_{Vs}$, corresponding to $S$ or $s$, which, when multiplied by $\pi$ and the square of $R$ or $r$, gives either the static volume[19] of the reservoir, or so much of it as we find to lie within the distance $r$ from the well $W$. This cylinder is divided into sections, as before.

The uppermost section has a height $h_{Vo}$, corresponding to $P$ or $p$, and thus it defines the potential volume of the reservoir within the area concerned. The sections between $K$ and $N$, having together a height $h_{Vc}$, represent the volume retained by the reservoir in virtue of the constant back pressure.[20]

If we say that the uppermost section of the volume cylinder possesses the dimensions necessary for a unit volume of fluid, then we define thereby a unit potential volume cylinder. A reservoir may contain any such number of these units as determined by the areal extent of the unit and that of the reservoir itself. The central unit alone can be represented by a solid cylinder, and all surrounding units by cylindrical rings whose thicknesses decrease as their circumferences increase. In order to determine the relation between the thicknesses and positions of these rings, let us tabulate unit volumes and areas:

| Unit Volumes | Area |
|---|---|
| 1 | $\pi r^2$ |
| 2 | $2\pi r^2$ |
| 3 | $3\pi r^2$ |
| 4 | $4\pi r^2$ |
| 5 | $5\pi r^2$ |
| .. | .... |
| $n$ | $n\pi r^2$ |

The cylinders are assumed to have the uniform height which they will have when the formation is of constant vertical thickness and of homogeneous texture in its lateral extent.

For the area in general we may write

$$n\pi r^2 = \pi r'^2 \qquad (143)$$

where $r$ is the radius of the first unit volume, and $r'$ the radius of the outer

[19] See footnote 14, § 63, page 118.

[20] For any radius $r$, less than $R$, this volume is less than $vo$ of Equation 50 (p. 91).

circumference of the ring representing the $n$th unit volume. It follows, then that

$$nr^2 = r'^2 \qquad (144)$$

or

$$r' = r\sqrt{n} \qquad (145)$$

Furthermore, since $r$ is a constant under the given conditions, we may write

$$r' \text{ varies as } \sqrt{n} \qquad (146)$$

That is to say, in order that these cylindrical rings may contain equal volumes, the radius of the outer circumference varies as the square root of the number of unit volumes circumscribed by this radius. By determining the radius for $n$ and $n - 1$ volumes, and subtracting the one from the other, we obtain the thickness of the ring which contains the $n$th volume.[21]

All space above $N$, composed of $n$ unit potential volumes of fluid, is properly represented by a solid cylinder, so long as $W$ is closed. When $W$ is open for production, the unit spaces defined by the rings continue to hold unit volumes of fluid, provided the fluid is liquid only. If gas is present, these spaces do not contain unit volumes, because of the expansion of the gas on the release of pressure. In any case, however, when $W$ is open, the same number of unit potential volumes is passing through all unit spaces during any specified interval of time, as we know from Equation 115 (p. 185).

During the process of production each unit of potential volume moves toward the well, taking up successively the positions previously occupied by the units in advance. If the fluid represented by the sections below $N$ moves, it does so only because it lies in the path of the potential volume. In general it tends to remain in place, without motion, as an obstruction to the movement of the potential volume.[22]

If the porous formation is not homogeneous in its lateral extent, we need modify the shape of the bases of the cylinders and cylindrical rings for both pressure and volume. They become irregular closed figures in accordance with variations in the texture of the formation. The principles concerning the area of the rings and the progressive movement of the fluid represented by them are not in the least modified.

[21] This geometrical proposition is not to be confused with the one represented by Equation 119 (p. 185), one which might be written as follows:

$$ve \text{ varies as } \frac{1}{r}$$

We note the following situation with respect to radial flow: When we deal with lineal velocity of flow our geometrical principle is hyperbolic, and when we deal with volume of fluid our geometrical principle is parabolic. If we have the proper analytical conceptions of velocity and volume, there is little danger of confusing the principles in any analysis of the performance of wells.

[22] In Volumetric Control the same conditions exist. When equilibrium is finally established in this finite control, fluid to an amount represented by the sections below $N$ actually remains within the reservoir.

By multiplying together the vertical dimensions of the proper sections representing corresponding portions of $S$ and $h_{V_S}$ we obtain the vertical dimensions of energy cylinders. The section representing potential energy will lie above $N$, and have a height equal to $P \times h_{V_O}$. Between $N$ and $K$ will lie the suppressed energy and the energy retained by the reservoir in virtue of the constant back pressure. Their vertical dimensions are easily computed in terms of the sections in pressure and volume. These sections of energy occupy the positions shown in Figure 33.[23]

[23] An energy cylinder modeled after Fig. 66 will possess horizons $K$, $N$, and $I$, corresponding to the present lines or planes, and $G$ will appear. ($J$ can be included, if desired. It was omitted in Fig. 33 in order to simplify that figure. In fact, as we know, $J$ can always be treated as a special case in the possible locations of $N$.)

CHAPTER XIV

# Secondary Functions of Performance *(Concluded)*

"Wherever there are qualities there are likewise quantities, but not always vice versa."—ARISTOTLE

95. *Reservoirs of oil or water, with gas.*—Our analyses of the secondary functions, based upon the nature of the volume of fluid within the reservoir, are simplified by first confining our attention to reservoirs in Hydraulic Control. These, as we already know, are regular and, in general, invariable in their performance during the lapse of time. Once having diagnosed the fundamentally important features of these reservoirs we are in a position to apply the necessary modifications in the finite controls, where time, as a function of performance, is involved in the process of production. Those reservoirs that produce a liquid alone or gas alone are the simplest in regard to conditions and events within them. The succeeding classes—three according to the list in section 93—appear to become increasingly complicated, in so far as they have special features of their own in addition to those of their predecessors.

The next class of reservoirs for us to investigate contains either oil or water, one to the exclusion of the other, at least throughout an undefined extent of the reservoir, and this liquid shall be accompanied by gas. We shall assume for the present that all this gas was dissolved in the liquid, and uniformly distributed throughout its mass, before the reservoir was provided with an orifice. No strata or pockets of free gas were present. Now that flow is taking place from one orifice—either an individual or a multiple one—both liquid and gas move uniformly toward it from all directions. According to the kinetic pressure gradient these fluids undergo a release of pressure while they so move; not only more and more gas passes from the dissolved state to the free state, but that which has already become free expands on the approach to the orifice. What happens after passing this orifice does not concern us here, for this matter was dealt with in chapter ix.

In conformity with Henry's Law, section 36, we may write

$$V_S = k'S \qquad (147)$$

where $V_S$ is the mass of gas dissolved in a unit mass of liquid at the absolute pressure $S$, and $k'$ is a constant that depends upon the solubility of the gas in the liquid. We may likewise write

$$v_s = k's \qquad (148)$$

where $v_s$ is the mass of gas still dissolved in a unit mass of liquid according to the absolute pressure $s$ at any point located at the distance $x$, or $r$, from the center of the orifice, the fluids being in motion as the result of flow, and undergoing a decrease in the value of $s$ on the approach to the orifice.[1] The constant $k'$ is the same in the two equations, for it pertains to the same combination of fluids.

Now we may subtract the second from the first equation and obtain

$$V_S - v_s = k' (S - s) \quad \ldots\ldots\ldots\ldots\ldots\ldots (149)$$

and rewrite this in the following manner:

$$v_f = k'y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (150)$$

In this expression we say that the mass of gas in the free state, per unit mass of liquid, equal to $V_S - v_s$, at any point where the absolute pressure is $s$, is equal to the pressure drop at the point, multiplied by the constant $k'$.

In section 88 the equation of the kinetic pressure gradient was found to be

$$y = \frac{nk}{x^2} + l\left(P - \frac{f}{m} + \frac{f}{x}\right) \quad \ldots\ldots\ldots\ldots\ldots (151)$$

Therefore by simply multiplying this equation by $k'$ we have

$$v_f = k'\left[\frac{nk}{x^2} + l\left(P - \frac{f}{m} + \frac{f}{x}\right)\right] \ldots\ldots\ldots\ldots (152)$$

This expression shows us that the gradient for the mass of gas in the free state, as the fluids travel toward the orifice, can be derived from the kinetic gradient by the simple expedient of multiplying all ordinates of the latter by the constant $k'$. If properly adapted to scale, the two gradients become coincident when we draw their curves. But let us rather say that $V_S = 100$ per cent, and $S = 100$ per cent; then we may take the curve for the kinetic gradient and insert the symbol "%" after all those for pressures, thereby obtaining Figure 67 (p. 214). Here we have a complete history of the gas while it moves with the liquid toward the orifice. The percentages $S$, $RS$, $P$, $C$, $RC$, and $A$ are in agreement with the identical quantities as given in section 73.

The fluids travel from beyond $e$ toward $b$. When they reach a point corresponding to $d_2$ on the curve, $y$ per cent is in the free state, $s$ per cent remains in solution, and $p$ per cent is yet to emerge from solution before the fluids leave the internal system of the reservoir.

[1] It is not essential that we make an allowance for an increase in hydrostatic pressure at depth within the mass of liquid itself. Of course there is such an increase, but our investigation at this point is in no way dependent upon it, or conditioned by it. In continuing the investigation, however, due allowance must be made, wherever more than one fluid is present. These tend to separate because of their different specific gravities.

The curve $D$ may be conveniently called the *gas distribution gradient.*[2] $Y$ may be either the center line of the individual well or a vertical axis located at the centroid of production from a multiple orifice, as discussed in section 92.

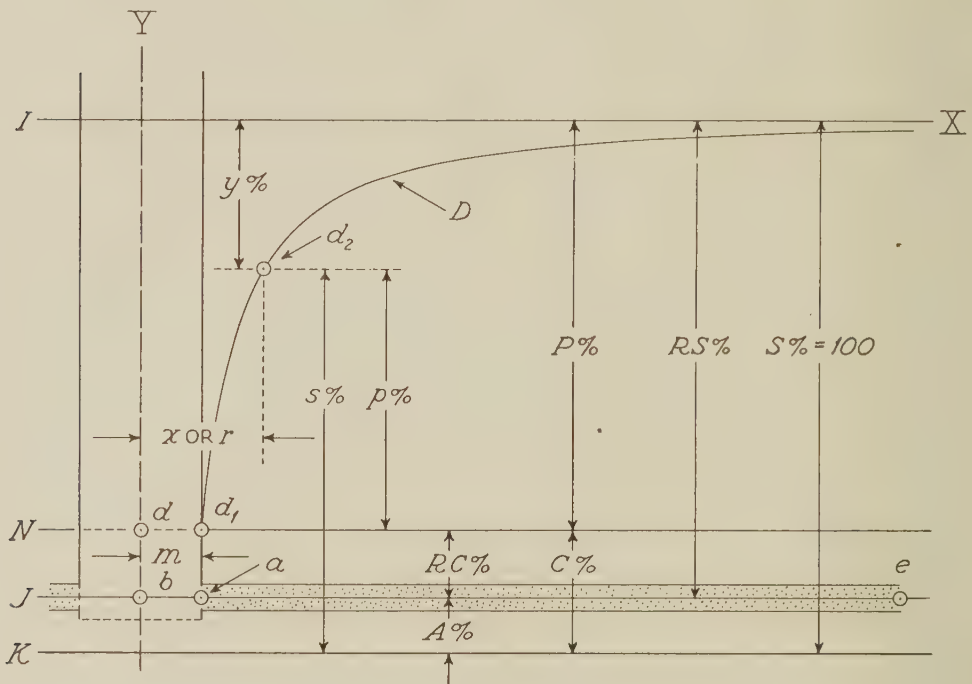


Fig. 67

96. *The gas expansion gradient.*—While the gas distribution gradient shows at a glance the proportion of the total mass of gas in the free or dissolved state, per unit volume of liquid, at every point within the reservoir, it fails to show the proportion of the space actually occupied by the free gas. Clearly the increasing pressure drop, as the orifice is approached, not only permits a greater number of unit masses of gas to become free, but it also permits each unit of mass to expand and accordingly occupy a greater space. We can determine a gradient for the space occupied by the total mass by applying the laws of Boyle and Henry to the internal situation as we now know it to be. First we must agree upon a convenient method of reckoning with space within the reservoir.

[2] The class of curves called *gradients* have abscissas representing distance and ordinates representing some function under investigation. This function is often pressure, though not necessarily so in every case. It might equally well be volume, velocity, acceleration, energy, or power. For volume we might have either mass-volume, as in the present instance, or space-volume, as in the next section.

Figure 68 ($a$) represents a unit of space. It is a column of unit horizontal cross-section area and of unit height.[3] It extends vertically from $H_0$ at the bottom of the productive formation to $H_2$ at the top. If we assume that the formation is homogeneous in its lateral extent, $H_1$, dividing the column into sections which are filled by solid matter and fluid, respectively, is fixed and invariable in its position throughout the extent of the reservoir. The section for fluid is, of course, the total of voids offered by the pores. We will take this section as 100 per cent, on the understanding that it represents the true volume thickness previously mentioned.[4] The fluid is itself divided into gas and liquid, as indicated by the line between $H_1$ and $H_2$. The position of this line is variable, but where its position is known, the proportional spaces occupied by the two fluids are defined, and vice versa. Let us conveniently assume that the gas which is dissolved in the liquid does not affect the volume of the liquid.

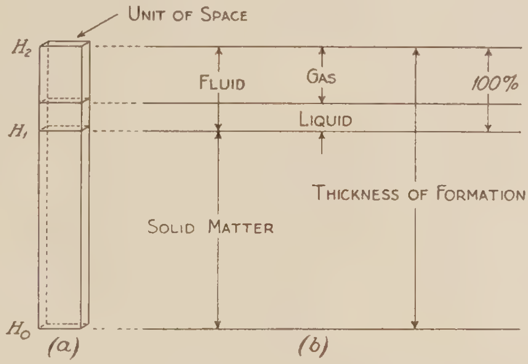


Fig. 68

From the preceding section we have the following equation:

$$v_f = k'y \qquad (153)$$

where $v_f$ is the mass-volume of gas in the free state, per unit mass of liquid, $y$ is the pressure drop according to the kinetic pressure gradient, and $k'$ is a constant which depends upon the solubility of the gas in the liquid.

Regarding the space-volume of a unit mass of gas we may say, in virtue of Boyle's Law, that

$$g_f s = k'' \qquad (154)$$

where $g_f$ is the space-volume of a unit mass of gas, $s$ is the absolute pressure exerted upon it, and $k''$ is the usual constant for this law. For $s$ we may substitute its equivalent $S - y$. Thus, on rearrangement we have

$$g_f = \frac{k''}{S - y} \qquad (155)$$

[3] Following our usual procedure, we need not define these units, nor specify a relation between them.

[4] See § 93.

Wherever there exist $v_f$ unit masses of free gas, and each mass occupies a space $g_f$, then the total space occupied by this free gas is

$$G_f = v_f g_f \qquad (156)$$

Into this equation we may substitute the values of the quantities as given in Equations 153 and 155. Thus

$$G_f = k'k'' \frac{y}{S - y} \qquad (157)$$

This is the equation between the space occupied by free gas at any point in the reservoir and $y$, the pressure drop at the point.[5] We recall that as between $y$ and $x$,

$$y = \frac{nk}{x^2} + l\left(P - \frac{f}{m} + \frac{f}{x}\right) \qquad (158)$$

a value which may, if desired, be placed in Equation 157, in order that the relation between $G_f$ and $x$ will be directly expressed. It is more convenient to handle these equations separately; for any value of $x$ a corresponding value of $y$ is known by Equation 158, and for this value of $y$ a corresponding value of $G_f$ is known by Equation 157. We shall say that Equation 157 is the equation for the *gas expansion gradient* of the reservoir.

Now we are prepared to interpret this gradient in connection with the performance of the reservoir.

It is to be noted that the laws of Henry and Boyle are necessarily fulfilled, so long as both liquid and gas are produced from the reservoir. Space occupied by gas in the free state cannot be occupied at the same time by liquid, and conversely.[6] Figure 68 is purely a mathematical representation of the situation with respect to matter as it exists between the foot and hanging walls of the productive formation. If the solid matter, as a porous mass from $H_0$ to $H_2$, were reduced to a non-porous mass, it would occupy the space $H_0$ to $H_1$. Again, if the free gas and the liquid were completely separated, they would occupy the indicated spaces. We see wherein the mathematical and physical representations of the situation must differ in regard to the solid matter, and wherein they may differ in regard to the fluids. Upon the release of pressure

[5] Equation 157 is the equation of the curve known as the "archway," or "tunnel." It has, in its application to our problems, a real and an imaginary part, and these become mutually interchanged as between investigations in space-volume and mass-volume.

[6] This is not to be interpreted to mean that gas bubbles cannot exist throughout the mass of liquid within the porous medium. We know that this situation does exist. The statement as applied to it would mean that within the space occupied by a bubble of free gas, there is no liquid (in the liquid form, of course). Consequently, within the space occupied by all bubbles of free gas, there is no liquid.

the gas is liberated throughout the mass of the liquid.[7] It comes into existence as minute bubbles, and all these which form within a single pore tend to unite into one larger bubble. If it were not for the movement of the fluids toward the orifice, these bubbles would presumably be retained in their original position by the effects of surface tension; at any rate, we are safe in saying that in virtue of the fluid motion the fluids will tend to separate on account of their different specific gravities. The gas will tend to shift toward the hanging wall and establish there its principal path of motion. A free surface will thus tend to be set up between the gas and liquid. This free surface is defined by the equation of the gas expansion gradient, revolved about the $Y$ axis. Capillary attraction may lift the curve to a position above its otherwise normal location in a manner similar to that depicted in Figure 65 (*b*).

In Figure 69 we have two drawings in one for the purpose of facilitating a comparison between curves. For convenience the horizontal scale is double

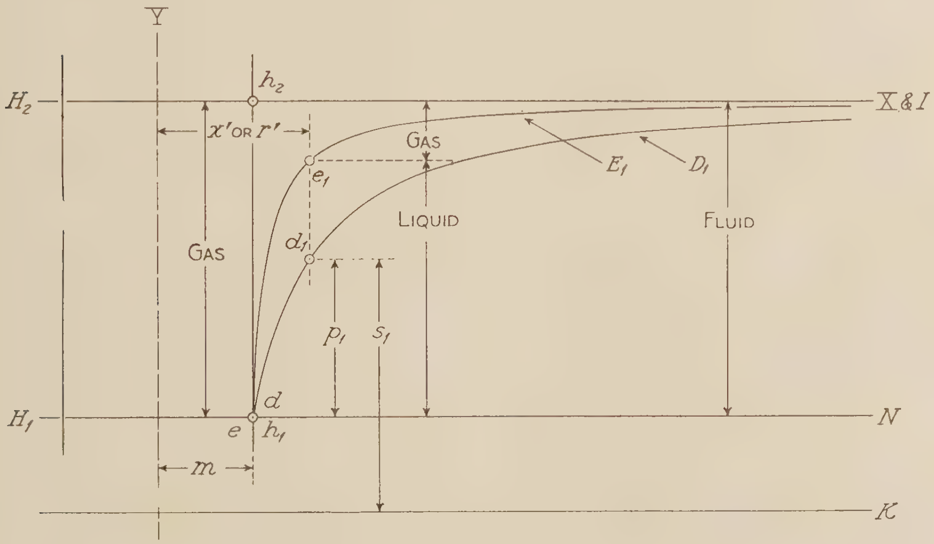


FIG. 69

that of Figure 67. $D_1$ is the same kinetic pressure gradient as $D$ in the earlier figure, both being referred to the lines $K$, $N$, and $I$, at the same distance apart. $E_1$ is the gas expansion gradient on the following conditions and assumptions:

*a*) The distance between $H_1$ and $H_2$ is so scaled that these lines conven-

[7] If, after gas is liberated from solution, the orifice is closed and $S$ is again established, it is possible that not all the freed gas will return to solution. The solution of gas under an increase of pressure takes place at the free surface between them, whereas the liberation of gas under a decrease of pressure takes place throughout the mass of the liquid. Our experiments which involve gas and liquid should be performed preferably upon a decrease of pressure, wherever measurements are to be made on their proportional amounts.

iently coincide with $N$ and $I$, respectively. The figure is simplified to the extent that the $X$ axis, as drawn, serves both gradients. The same $Y$ axis would naturally serve both in any case, for $x$, denoting distance from the center of the orifice, refers to $Y$. There would be no logical reason for adopting different scales for $x$ in constructing the two curves.

*b*) Before the opening of the orifice the liquid was saturated with gas at the given static pressure $S$. No strata or pockets of free gas existed within the reservoir.

*c*) The solubility of the gas and the distance between $H_1$ and $H_2$ are such that for $y = P$ at the edge of the orifice the gas necessarily occupies the entire space defined by these two lines.

Now the curve $E_1$ shows the proportional spaces occupied by gas and liquid within the reservoir at all points measured outward from the orifice, under the given conditions of production.

It appears under the present circumstances that only gas is produced from the reservoir. The liquid is securely locked in place by the gas on expansion.[8] Granting that the liquid itself is the source of the gas, we at once ask if the production of gas can continue indefinitely. Given the ideal combination reservoir as described in section 72 with a unique well which is centrally located with respect to a symmetrical domal structure there will come a time in the course of production when the normal conditions are upset, because the stoppage of the liquid on its attempt to enter the structure means the stoppage of the source of the gas, and the liquid on the flank of the fold prevents unlimited by-passing of the gas by fully occupying all pore space at this point. However, if the formation is a perfect horizontal plane, by-passing will continue indefinitely. The gas continually has access to the well along its principal path near the hanging wall of the formation.

We must admit that the conditions laid down for Figure 69 define a case of such a particular nature that we may seldom, if ever, find its like in practice. Let us approach the more general case. For convenience we will take the same reservoir and observe the situation for $P$ at 50 per cent of its first value. The constants in the equations are altered; we now have $D_2$ and $E_2$ as shown in Figure 70. Now a volume of liquid, with its proportionate volume of free gas, is produced. The process can continue indefinitely, without dependence upon by-passing of the gas.

But if, instead of decreasing $P$, we increase it, the situation appears as in Figure 71. The curves are now $D_3$ and $E_3$. The gas attains the full space at a point more remote from the orifice than in the first instance. The figure in fact shows $P$ equal to $S$; that is, the reservoir is producing into a perfect vacuum. $E_3$ therefore represents the limiting position for the expansion gradient under the specified physical conditions within the reservoir. Under no

[8] "Space occupied by gas in the free state cannot be occupied at the same time by liquid." (See p. 216.)

circumstances can an expansion gradient take a position to the right of the one shown.

As explained in section 39, it is not possible to produce liquid from a combination reservoir to the complete exclusion of gas. *If the application of*

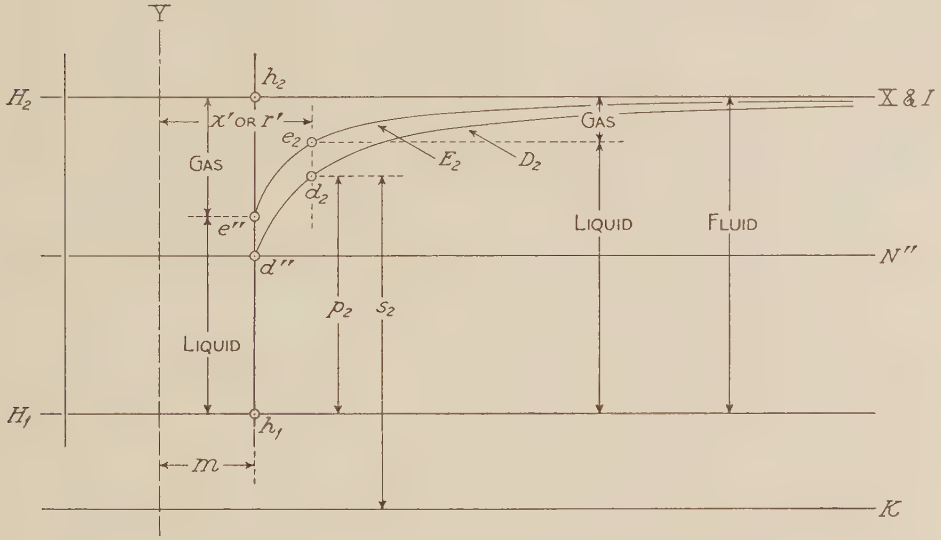


FIG. 70

*a back pressure diminishes the proportional production of gas, it does so by diminishing or preventing the by-passing of the gas.* Our three figures are based upon the assumption that no by-passing takes place. By-passing is simply a radial slipping of the gas toward the orifice. How would the curves appear in its presence?

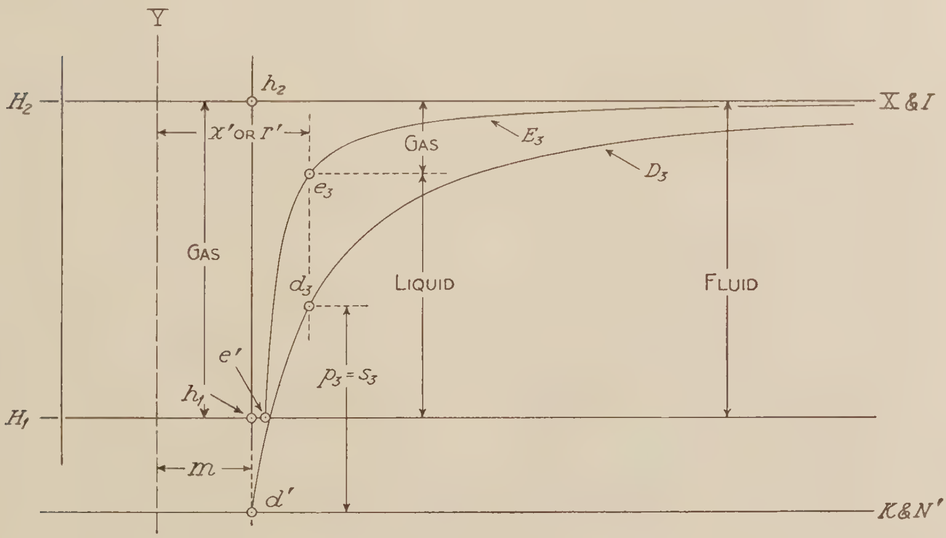


FIG. 71

In section 89 the curves $D$ were found to shift in case there is a radial slipping of the liquid. If $D$ shifts in the present reservoir it carries $E$ with it. The two shift the same distance, and this distance is independent of the by-passing of the gas with respect to the liquid. To be exact we should say that *by-passing is a phenomenon which refers to the relative motion between gas and liquid,* and not one which refers to the motion of the gas with respect to stationary matter within the reservoir.[9] If the gas slips by the liquid, the expansion curve is not affected in the least. The pressure drop $y$ determines the space occupied by gas in a given unit space within the reservoir, and $y$ is not dependent upon the total absence or the presence of any amount of gas, for, as we recall, gas is not an active agent in a combination reservoir of the open type, inasmuch as the pressure which it is capable of exerting has its sole source in the weight of the column of liquid which it supports. The space actually occupied by gas is the same whether the gas slips or not. The density of the gas in a given unit space remains the same, but the number of molecules that pass the boundaries of this space per unit of time alters as the slipping is altered.[10] In other words, volume at the instant is invariable, while velocity is caused to vary.

The curves $E$ display the fact that the greater part of the expansion takes place within a short distance from the orifice. Obviously it is advantageous to the operator to have $D$ shift, and carry $E$ with it, toward the orifice, for if these curves as shown be shifted to the left, the line $h_1h_2$ cuts them at a better place. The liquid has freer access to the orifice, being less hampered by the presence of expanded gas. Furthermore, the operator gains if he permits his well to excavate a chamber at the bottom. The removal of the solid matter between $H_0$ and $H_1$ in Figure 68 allows $H_1$ to take the position of $H_0$ at a most desirable locality.[11] The liquid has free access to the well in spite of the presence of expanded gas.

**97.** *Proportional production of gas.*—According to the preceding section a lone combination well, centrally located with respect to a symmetrical domal structure, must eventually become self-regulating in the matter of its proportional production of gas. If the gas holds back the liquid, thus interfering with its access to the structure, it literally suppresses its own supply. If a diminished quantity of liquid is to enter the structure, a correspondingly

[9] Naturally, with by-passing or with no by-passing, the gas must move with respect to stationary matter within the reservoir, if it is to flow from the orifice.

[10] The phenomenon of capillary attraction (§ 93, Fig. 65) will tend to diminish the by-passing of the gas. It maintains a more homogeneous mixture of oil and free gas between the walls of the porous medium, and as a consequence the two fluids move together in better agreement.

[11] The hyperbolic principle, as applied to lineal velocity, makes a chamber desirable in any case, and more particularly so in case gas accompanies the liquid. (See footnote 21, § 94, p. 210.)

diminished quantity of gas is to enter; therefore the suppression of the liquid will diminish. A well may begin its production with a generous proportion of gas, but there will come a time, under the specified circumstances, when the proportion is reduced to the normal amount as defined by the solubility of the gas in the liquid and the static pressure of the reservoir. In further consideration of the situation let us refer to Figure 72 (p. 222).

We have an annular reservoir system somewhat similar to those of Figures 59 and 60. There are now central plates which are portions of spherical surfaces. The space between them is evenly packed with sand of homogeneous texture. For the convenience of manipulation a reservoir of the closed type is chosen. Although the gas here has energy of its own, in virtue of compression, this fact of itself does not modify conditions and events within the section which lies between the plates, in so far as the proportional production of gas is concerned.[12] The tank proper contains liquid and gas. The system is maintained in Hydraulic Control by a supply of liquid at $l$ and compressed gas at $g$. Either the liquid is returned as fast as it is produced from standpipes on the upper plate, or a spillway for an excess is provided in the usual manner. The gas is held at a constant pressure by means of an exhaust regulator at $q$. An abundance of gas is desirable in order to agitate the liquid and saturate it at the designated pressure. Standpipes are shown to rise to a common level $N$ for the purpose of permitting us to say that the system at large is provided with one such line. The atmospheric line for this system is at $J$.[13]

$W_1$ corresponds to the centrally located well previously described. *As a lone producer—other standpipes being absent or closed in—it must deliver under normal conditions the very combination of gas and liquid which enters the space between the plates at* e. By-passing of the gas is possible only for a short time at the beginning of production.

Suppose we assume that a sufficient time has elapsed for the establishment of these normal conditions. The proportional production of gas is constant. Now a valve at $W_1$ is partially closed. Will the proportion of gas be reduced? No; not at a standpipe so ideally located on the top of a perfect "structure." Next, the valve is opened more widely than in the first instance. Will the proportion of gas be increased? Yes; temporarily so, by the by-passing of gas in the region closely surrounding the standpipe. After a few moments, however, normal conditions are again established, and the combination of fluids which enters at $e$ is once more that combination which is produced at $W_1$. When gas has been permitted to by-pass, there is liquid on the flanks of the structure that is partially depleted of its gas. Then during the process of

[12] See § 65.

[13] It is to be admitted that for a study of individual standpipes each might be provided with its own line $N$ at the proper height. In such case the atmospheric lines would appear as at $j_1$ or $j_2$, with their corresponding absolute zero lines below them, as $K$ is to $J$.

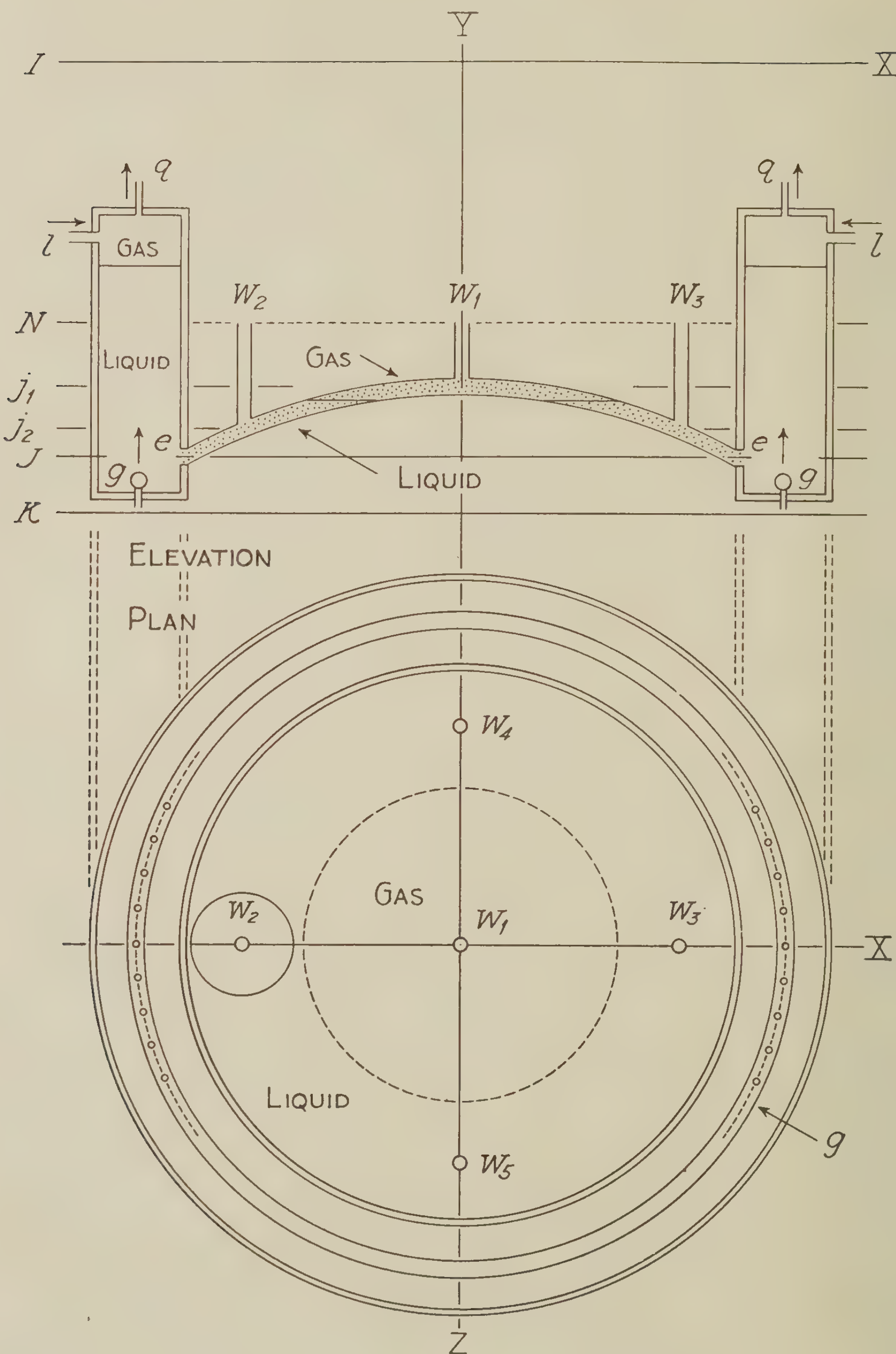
Y
I
X
q
q
l
l
GAS
LIQUID
W2
W1
W3
N
GAS
LIQUID
j1
j2
J
K
e
e
g
g
ELEVATION
PLAN
W4
GAS
W2
W1
W3
X
LIQUID
W5
g
Z

Fig. 72

establishing normal conditions *there is a gradual decline in the proportional production until such a time as this liquid is produced, and other liquid, with its normal gas content, has taken its place.* It is to be noted that the decline leads to a proportion lower than normal before normal conditions reappear.

$W_1$ was said to be ideally located at the top of the structure. Let us close it, and open the standpipe at $W_2$. Any others remain closed, as before. Obviously this standpipe has its normal proportion identical with that of any other possible one, since the combination entering at $e$ is not dependent upon the location of the pipe. When $W_2$ is opened, a pocket of gas begins to form at the crest of the structure. *Not until this pocket is fully developed in accordance with the conditions of production can the normal proportion of gas be produced at the pipe.*[14] As in the previous case, we will assume that a sufficient time has elapsed for the establishment of normal conditions. Now a valve at $W_2$ is opened more widely. Gas by-passes not only from the liquid immediately surrounding the standpipe but also directly from the pocket. A somewhat longer time is required for the establishment of normal conditions; meanwhile the pocket diminishes in size. Next, the valve is closed to its former position. The proportion of gas is reduced; liquid partially depleted of its gas arrives at $W_2$, and the pocket thereafter assumes its former size by a repeated process of accumulation on the crest.

According to the gradient most of the expansion of gas takes place within the immediate vicinity of the orifice. For $W_2$ this vicinity may be said to be included within the arbitrarily designated area as shown in the figure. The expansion outside of this area can be considered slight in comparison with that inside. All free gas within this area will not be delivered at $W_2$ while the pocket is forming on the crest. Some of it escapes from the right side and travels up the slope. It necessarily follows that, if both $W_1$ and $W_2$ are producing, the former can, and ordinarily will, produce a greater proportion of gas than the latter. Normal conditions for the two are different, although the total gas from them, in comparison with the total liquid from them, bears the usual relation in accordance with the combination entering at $e$. In general, *orifices on a given contour lose some of their gas to orifices on higher contours.* The difference in proportional production depends upon the number of orifices on the two contours, the proximity of the orifices on the separate contours, and the slope of the structure. In regard to orifices on the same contour, either they lose no gas to each other, or they lose compensating amounts to each other.

An increased number of standpipes, as shown at $W_3$, $W_4$, and $W_5$, increases the proportional production of gas at $W_1$. If these are alternately opened and closed in the course of time, we may expect the proportional production at $W_1$ to change accordingly. *As it is with two orifices, so it is with*

[14] When once fully developed, the gas pocket remains stationary in size so long as no alterations are made in accordance with theoretic performance.

*all possible orifices taken as a group; their total gas bears a constant relation to their total liquid on delivery.*[15]

It is clear that in so far as the portion of the system between plates is concerned, the situation as depicted in Figure 72 is strictly analogous to situations as we find them in natural reservoirs, provided—so far as known at the present stage of our analysis of reservoirs—that both artificial and natural systems are in Hydraulic Control. There is but one difference to be noted: time, considered as temporary for an artificial system, may be conveniently expressed in minutes, whereas for a natural system it is more conveniently expressed in weeks or months.[16]

Reservoirs that contain oil or water, with gas, are subject to the effects of Jamin action, the resistance to movement due to a series of alternating bubbles of gas and globules of liquid. This phenomenon, which has heretofore received only brief mention in section 29, is yet to be investigated in detail. We shall make no attempt here to deal with these effects, beyond noting results in connection with events already considered. In the matter of by-passing of the gas we should recognize a tendency for the gas to slip ahead of the liquid, even in the absence of bubbles of gas distributed throughout the mass of the liquid. In the presence of these bubbles, however, this tendency is far greater. How easily the liquid may become securely locked in place within the pores of the formation, thus favoring the production of gas alone from a well in a combination reservoir, can be fully appreciated when Jamin action is well understood.[17]

**98.** *Reservoirs of oil and water, with no gas.*—We come now to the fourth class of reservoirs. These contain two immiscible liquids, oil and water, and no gas. Let us investigate the movement of the contact between these liquids. We shall assume that each liquid occupies its own portion of the reservoir; the water entirely surrounds a pool of oil. In the first instance only an edge-contact exists between the oil and the *edge water,* and in the second a bottom-contact exists between the oil and *bottom water.*

As usual we proceed to idealize. The reservoir lies on a perfectly hori-

[15] This constant relation is, as we see, independent of local by-passing. It is of course necessary to include all orifices on a definite structure. In making the statement we are assuming that "normal conditions" are established; that is, specifically, that there is no blind space (a space without an orifice) wherein a gas pocket is increasing or decreasing in its size.

[16] That is to say, in an artificial system the temporary situation wherein normal conditions are being established may be of a few minutes' duration, while in a natural system the analogous situation may be of weeks' or months' duration.

[17] These effects of Jamin action, which we say are minor in Hydraulic and Volumetric controls, are easily recognized after the major effects which give rise to Capillary Control have been studied. The minor effects are discussed briefly in § 99.

zontal plane; the line of contact is geometrically perfect; the physical properties of the liquids, notably their viscosities, are identical.[18]

In Figure 73 the line of contact at $A$ is straight throughout an undefined distance. $W$ is a well at the distance $L$ from $A$. The circles with $W$ as a center define volume cylinders whose successive differences represent cylindrical rings of equal volume in accordance with their description in section 94.

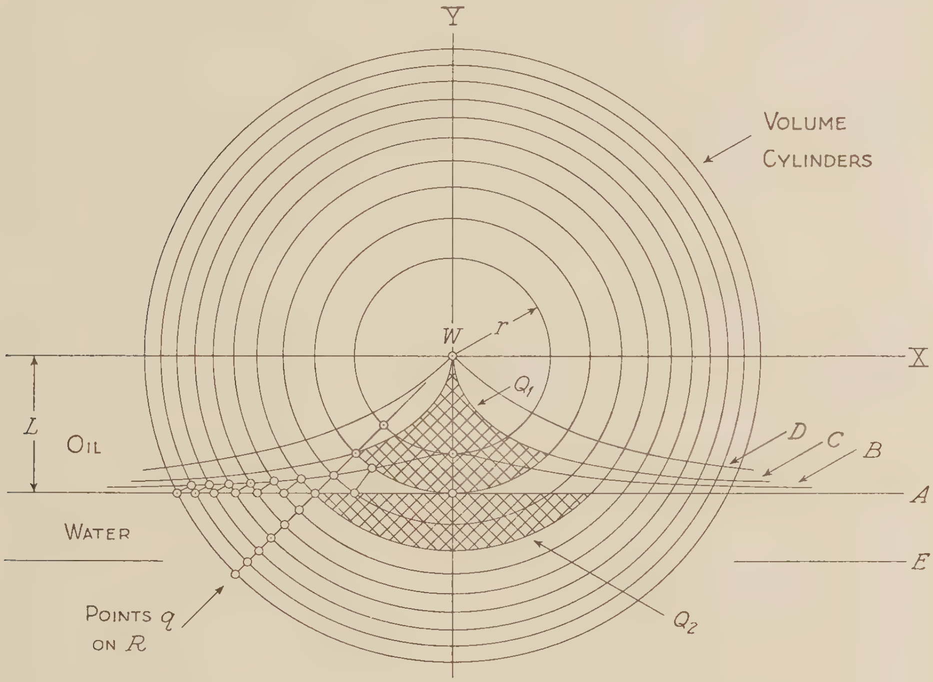


Fig. 73

The length $L$ as a radius, we shall say, defines a unit volume cylinder about $W$. This unit is inconveniently large in the present case; therefore let us deal with half-units. The first half-unit cylinder has a radius $r$, as shown. When this half-unit has been delivered at the well, all points on its circumference have reached $W$. At the same time all points $q$ on any radius $R$ have advanced one step toward $W$; that is, they have passed from one circle to the next in line. The line $A$ intersects the circles at particular points. Here we may draw particular radii and advance these points by one step. By this process we obtain a series of points belonging to the curve $B$, the contact between oil and water at this time.

The production of the first unit volume takes all the liquid from the first unit cylinder. This is entirely oil. On the particular radii we may advance

[18] These assumptions, so clearly contrary to actual conditions, are subsequently set aside.

one more step to obtain points for the curve $C$. Now a body of water which in the beginning had occupied the space $Q_2$ has taken a position $Q_1$. The present unit volume at the well contains oil and water in the areal proportion as it appears in the drawing. This combination of liquids will issue from the well in the time required to deliver the second unit volume.

Although a well may be drilled at some distance from a straight line contact, in the course of production the water "wedges in" on it. At first the wedge is narrow at its unit circular base and drawn to a long point at its vertex, but as production continues it gradually becomes wider, as for example in the curve $D$, which shows the contact line when two and one-half unit volumes have been produced. It is evident that the proportional production of water to oil must likewise increase.

In Figure 74 a circular pool of oil is entirely surrounded by water. The

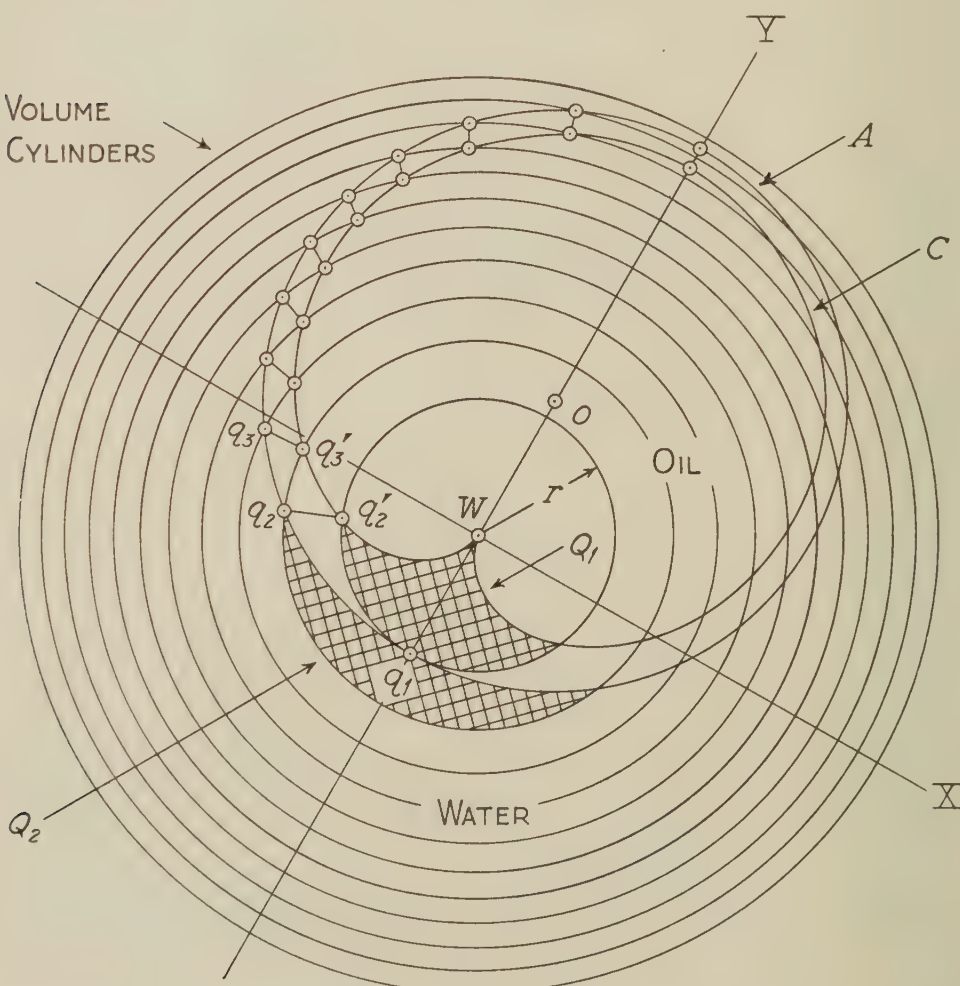


Fig. 74

center of the pool lies at *O*, the edge-contact is the circumference *A*, and the well is shown at *W*. The radius $r$ is now the radius of the unit volume cylinder. Upon the withdrawal of the first unit volume of oil the contact moves to *C*. Points $q_1$, $q_2$, $q_3$, and so on, advance a unit step to *W*, $q_2'$, $q_3'$, and so on, respectively, on radii drawn from the former to *W*.

There is but one location for a well in the present figure where the water will not wedge in. This is at *O*. The contact for a well here is continually circular, with its radius gradually diminishing in accordance with the delivery of the oil.

The reduction in the size of the pool during production is accompanied by the encroachment of water from all directions. *Wedging is merely a phenomenon due to the eccentricity in the location of the well with respect to the center of the pool.* *W*, instead of being a single well, may in fact be the centroid of production from a group of wells which constitute a multiple orifice.

Irrespective of the location of *W* in Figure 74, whether inside or outside the pool of oil as shown, all mobile oil within the pool can be produced from *W*.[19] Unless *W* is coincident with the center of the pool, an amount of water is necessarily produced before all oil is withdrawn from the pool, and this amount of water depends upon the eccentricity in the location of *W*. For example, in the present figure all liquid within the largest circle must be delivered at *W* in its given location in order to exhaust the pool. By actual measurement this requires that, of the 100 per cent liquid in the great circle, 40.6 per cent of the production is oil, and 59.4 per cent is water. Obviously the manner of diminishing the quantity of water is to move *W* toward *O*.

Of all idealizations employed in the analyses of reservoirs, I believe the present one may be truthfully considered the greatest departure from reality. The reservoir capable of producing oil ordinarily does not lie on a horizontal plane; the contact between water and oil does not lie on a perfectly straight line, nor does it describe a perfect circle; and the physical properties of the two liquids are certainly not identical. What part of our analysis remains the same, and what part must be modified, in approaching the actual case?

Where the oil pool is situated on the crest of a domal structure, the greater specific gravity of the water will retard the wedging process. The encroachment of water proceeds more evenly on all sides of the pool. In the ideal situation the progress of the wedge depends only upon the volume withdrawn from the reservoir, whereas in the case of a domal structure it depends upon volume and velocity, for in accordance with the rate of production the water

[19] By "mobile oil" I mean specifically that oil which can flow. This excludes adsorbed oil, which does not behave as a fluid. (See § 26.) Where there is structure a further restriction must be included; namely, mobile oil is that which can flow from *W* in virtue of its structural position. Oil trapped at a higher elevation on the structure is excluded.

has an opportunity to fall back, down the dip of the structure, and thus make the contact line smoother.

The graphical method of constructing the successive positions of the contact during encroachment is not dependent upon the geometrical perfection of the line. It is true that we may not be cognizant of the position of the contact on all sides of the pool. However, we shall know its position at points of greatest interest to us; that is, in the vicinity of those wells which either suffer or are about to suffer from encroachment, and here we can advantageously study the phenomenon of wedging in connection with the process of production.

A difference in viscosities gives an advantage to the liquid possessing the less value. In most cases it will be the water which is thus favored. As a matter of fact the rings shown in Figures 73 and 74 are not continuous; they must break on the line of contact, and portions of a thickness and at a distance from *W*, according to the two viscosities, will actually function together. The progress of wedging is enhanced where water is the less viscous. It is evident that differences in specific gravity and viscosity may oppose each other in the formation of the wedge, and that which is to exceed in any particular case is dependent upon obvious circumstances.

It is clear that where there is a tendency for the contact line to become smoother, such a process is more or less in conformity with the contours of the geologic structure upon which lies the pool of oil.

We considered Figure 73 to be a plan or horizontal projection of a reservoir. Now let us turn it up and consider it as a profile. *X* denotes the top of the porous formation, *E* the bottom, and *A* is a contact line between oil and bottom water. *Y* is the center line of the casing which penetrates the formation at *W* and extends, with perforations, to or below a point on *E*. The portions of circles above *X* and below *E* have no significance, while the remaining portion, between *X* and *E*, defines the position of the contact in its tendency to rise during production.

If the liquids were of the same viscosity, both would travel horizontally and enter the casing in proportion to *EA* and *AX*. Or, provided the perforations only extend from *W* to *A*, oil alone would enter the casing. However, where water is less viscous its velocity between *E* and *A* will be greater than the velocity of the oil between *A* and *X*. At the center the water will "cone up" on the perforated casing to an extent dependent upon the rate of production from the well. At a given rate it will rise to the curve *B*, at a more rapid rate to *C*, and at a still more rapid rate to *D*, and so on. Its greater specific gravity will tend to hold it down; the rise will not be progressive with volume, but only so with velocity. To some extent the perforated space may be shortened in order to meet circumstances, provided the situation is somewhat as shown by *B*. Such a shortening will not better the situation for *C*, inasmuch as the oil is securely locked in place, and the well produces only water.

We have seen that the curves *B*, *C*, and *D*, as contact lines for edge water,

are not likely to be fulfilled with a fair degree of approximation in any actual case. As contact lines for bottom water, however, they have the same possibility of fulfillment as the curves for the kinetic pressure gradient and the gas expansion gradient. Water will normally rest on a horizontal plane as denoted by the line $A$. By the usual methods of analytical geometry these curves will be found to have the following equation:

$$y^4 + y^2 (x^2 - L^2 + u^2) - L^2x^2 = 0 \quad \ldots\ldots\ldots\ldots (159)$$

wherein $L$ is the distance between $A$ and $X$, and $u$ is dependent upon the volume withdrawn from the reservoir in the case of edge water, or upon the rate of production in the case of bottom water.[20] As it stands the equation is indeterminate, but it may be easily converted into a determinate one by inserting the proper values for $L$ and $u$. As examples, for $B$ as an edge-contact, $u^2 = \frac{1}{2}L^2$, and $L = 1$; while for $C$, $u^2 = L^2 = 1$. With these values the general equation reduces to

$$\frac{2y^4 - y^2}{2(1 - y^2)} = x^2 \quad \ldots\ldots\ldots\ldots (160)$$

and

$$\frac{y^4}{1 - y^2} = x^2 \quad \ldots\ldots\ldots\ldots (161)$$

[20] Let $v$ represent the radius of $q_1$, $r$ the radius of $q_n$, and $u$ the radius of volume cylinder withdrawn. The latter pertains to the volume required to shorten $r$ to the value $v$. Then

$$\pi v^2 - \pi r^2 = \pi u^2 \quad \ldots\ldots\ldots\ldots (i)$$

or

$$v^2 - r^2 = u^2 \quad \ldots\ldots\ldots\ldots (ii)$$

By rearranging terms we have

$$r^2 = v^2 - u^2 \quad \ldots\ldots\ldots\ldots (iii)$$

The points $q_1$ and $q_n$ lie on the same radius when flow is radial; that is, the angle $\theta$ which the radius makes with the axis $X$ is the same for both points. Now

$$y = r \sin \theta \quad \ldots\ldots\ldots\ldots (iv)$$

or

$$y/r = \sin \theta \quad \ldots\ldots\ldots\ldots (v)$$

Furthermore,

$$r^2 = x^2 + y^2 \quad \ldots\ldots\ldots\ldots (vi)$$

and

$$L/v = \sin \theta \quad \ldots\ldots\ldots\ldots (vii)$$

From the latter we may write

$$v = L/\sin \theta \quad \ldots\ldots\ldots\ldots (viii)$$

By substituting values from Equations vi and viii into Equation iii we obtain

$$x^2 + y^2 = (L^2/sin^2\ \theta) - u^2 \quad \ldots\ldots\ldots\ldots (ix)$$

By Equation v we know that

$$y^2/r^2 = \sin^2 \theta \quad \ldots\ldots\ldots\ldots (x)$$

or

$$y^2/(x^2 + y^2) = \sin^2 \theta \quad \ldots\ldots\ldots\ldots (xi)$$

Now the substitution of the value from Equation xi into Equation ix gives

$$x^2 + y^2 = \frac{L^2 (x^2 + y^2)}{y^2} - u^2 \quad \ldots\ldots\ldots\ldots (xii)$$

On clearing and rearranging terms we have Equation 159.

[ § 98

The curves were made to serve a double purpose. Had we considered the bottom-contact alone, it would have been necessary to assume arbitrary values for $u$ in accordance with reasonable rates of production. These equations suffice to show the analytical properties of the curves with which we deal. If $x^2$ be replaced by $x^2 + z^2$, the resulting equation pertains to the surface generated by the revolution of the plane curve about the $Y$ axis. This is the surface described by the water as it rises on the perforated section of the casing.

**99.** *Reservoirs of oil and water, with gas.*—The fifth and last class of reservoirs contains the two immiscible liquids, oil and water, and gas. As stated in section 27, hydrocarbon gases are soluble in petroleum and in water, to a greater degree in the former than in the latter. Where these liquids are associated in the manner described in the preceding section, each has its gas in solution, and this gas is ready to pass into the free state upon the release of pressure. The conditions are similar to those which we encountered in section 95, except in so far as the situation is now rendered more complicated by the presence of two liquids instead of one.[21]

We shall assume that at the static pressure of the reservoir the gas is uniformly distributed, in solution, throughout the oil in the pool, and that it is likewise uniformly distributed throughout the water for an undefined extent of the reservoir. In each case the gas is in solution according to the solubility of the gas in the respective liquids at the specified pressure. No pocket of gas exists under the static pressure, nor can one come into existence upon the release of pressure during flow from the orifice.[22]

In conformity with the pressure drop on the kinetic pressure gradient minute bubbles of free gas originate throughout the masses of the two liquids. Each pore space within the formation tends to hold but one bubble, but as all fluids move toward the orifice, the bubbles will tend to unite within some pores, leaving other pores completely filled with liquid. Obviously the proportionate numbers of gas-filled and liquid-filled pores will depend upon the amount of pressure drop and the solubility of the gas in the liquid at a given pressure. As we know from the advance studies referred to above, the globules of liquid between the bubbles become distorted, and in this condition they offer a resistance to the movement of the fluids.[23]

The total resistance set up within a unit volume of formation, say within one cubic foot of formation, depends, aside from the texture of the formation, upon the number of bubbles present in this volume and the surface tension of the liquid. We may conveniently assume that the natural gas is ten

[21] The phenomena to be investigated here are dependent upon effects due to Jamin action. The present discussion presupposes an understanding of this action in general; consequently this section may well remain for a time when the preliminary chapters of Capillary Control have been studied.

[22] We now know under what conditions these assumptions are warranted.

[23] See in particular § 147, paragraph (5), and also the fifth paragraph of § 148.

times as soluble in the oil as it is in the water of the reservoir, and that the surface tension of the oil is one-third that of the water.[24] Now given a cubic foot of oil-bearing formation, and another cubic foot of water-bearing formation, both subject to the same pressure drop, there are ten times as many bubbles in the first volume as there are in the second one. However, each bubble in the first volume gives rise to a resistance equal to one-third of that caused by each in the second; consequently it is clear that the ratio between the total resistances set up in the two volumes is three and one-third to one, as between oil and water.

If we may consider a *pressure facility* to movement to be equal to the reciprocal of the *pressure resistance,* then we can say that the ratio between the facilities set up in the two volumes is three and one-third to one, as between water and oil. If $v$ represents lineal velocity and $pf$ represents this pressure facility, then in accordance with Torricelli's Theorem we may write

$$\frac{v_w}{v_o} = \left(\frac{pf_w}{pf_o}\right)^{1/2}$$

where $w$ and $o$ stand for water and oil, respectively. The substitution of the ratio three and one-third to one on the right of this equation gives the ratio of 1.825 to one. If the circles in horizontal planes about $W$ in Figures 73 and 74 be taken as loci of equal pressure drops, then the lineal velocity of water at all points on these circles, beyond $A$ and $W$, or beyond the subsequent positions of this contact, is 1.825 times the lineal velocity of oil at the remaining points of corresponding circles. This is in agreement with the mechanics of the kinetic pressure gradient. Where there is oil, the resistance to movement increases the pressure drop, and the water, in order to increase its pressure drop by a like amount, must travel more rapidly than the oil.

On repeating the experiments of Jamin, using the various styles of hollow and porous-filled tubes, we may conclude that *the effects due to differences in viscosity are of considerably less importance than those due to differences in surface tension and solubility.* Where the water is less viscous than the oil, the number 1.825 will be found to be less than the proper amount. Laboratory tests with the same apparatus and the same pressures, first with water and gas, and then with oil and gas, give us ratios as the result of effects due to the three factors in combination. It is this combined effect which is of value in computations rather than the separate, uncombined effects.

In regard to edge water on domal structures and to bottom water in general, we see the effects of a greater specific gravity tending to offset the effects of Jamin action and viscosity.

It is important to note in connection with natural reservoirs that a unit volume of oil and a unit volume of water possess the same amount of energy

[24] It is not inferred that these ratios are generally true. They are adopted here as ratios not unreasonable.

in virtue of the weight of the same column of liquid bearing upon them. They possess this energy in spite of the presence of gas, and not because of it.[25] Clearly, more of this energy is converted into heat where the Jamin action, in combination with the viscosity, is the greater. As a result we observe the wedging process which accompanies water encroachment and the subsequent production of water greatly facilitated.

Here the study of reservoirs in Hydraulic Control, in so far as these may be advantageously investigated apart from the others, is brought to a close. Further principles which characterize these reservoirs will come to light in the course of our study of the analytical mechanics of production from reservoirs in Volumetric Control.

[25] This statement is equally applicable to natural reservoirs in Hydraulic and Volumetric controls. It is not applicable to those in Capillary Control.

# Part III. Reservoirs in Volumetric Control

CHAPTER XV

# Ideal Performance and Its Primary Functions

"The axioms of geometry, taken by themselves out of all connection with mechanical propositions, represent no relations of real things. . . . . As soon as certain principles of mechanics are conjoined with the axioms of geometry, we obtain a system of propositions which has real import, and which can be verified or overturned by empirical observations, just as it can be inferred from experience."—HERMANN VON HELMHOLTZ

**100.** *From Hydraulic to Volumetric Control.*—We have seen that a reservoir, in the sense prescribed by us, includes all fluids which happen to be within the physical container, in so far as these play a part in the performance of the reservoir. It is essential that we understand the mechanics of this reservoir in this sense. The problems presented by a particular portion of it, say a portion defined by a pool of oil or gas, are then readily solved. The mechanics of a part cannot be properly analyzed without a consideration of the whole, if it be granted that we may know the relation between the part and the whole.

We are prepared to study reservoirs in Volumetric Control. To grasp the significant features of this control thoroughly it is proper that we confine our attention first to reservoirs of artificial construction. Under the proper conditions the two type reservoirs for Hydraulic Control, Figures 35 and 36, may be forced to alter to Volumetric Control. Thus, when the bell of the gas holder rests on bottom, any space that it may retain for gas, at a pressure not exceeding the weight of the bell, behaves as a rigidly constructed gas tank. Likewise, when the inflow of liquid into the solution tank is diminished sufficiently to allow the free surface of the liquid within the tank to lower in accordance with production, the reservoir is in Volumetric Control.

The process of changing from one control to another may be conveniently termed a *conversion of control.* Here we are concerned specifically with a conversion from Hydraulic to Volumetric Control.

In order that pressures within the gas tank may not be limited to the weight of a movable top, we shall, as stated in section 5, adopt the rigidly constructed tank as the type reservoir for this fluid. No particular shape for this tank need be specified, for the free surface of the gas, coincident with the interior surface of the tank, remains constant at all pressures. A convenient and familiar form for such a tank is a cylindrical one of circular cross-section, with its axis either horizontal or vertical, and with its ends shaped as

portions of spherical surfaces. These ends may have their concave sides either inward or outward with respect to the interior of the tank, as illustrated in Figure 75. This type tank may have either one hole which serves on filling and producing, or two which serve the respective purposes.

The same solution tank, without modification, is adopted as a type for this

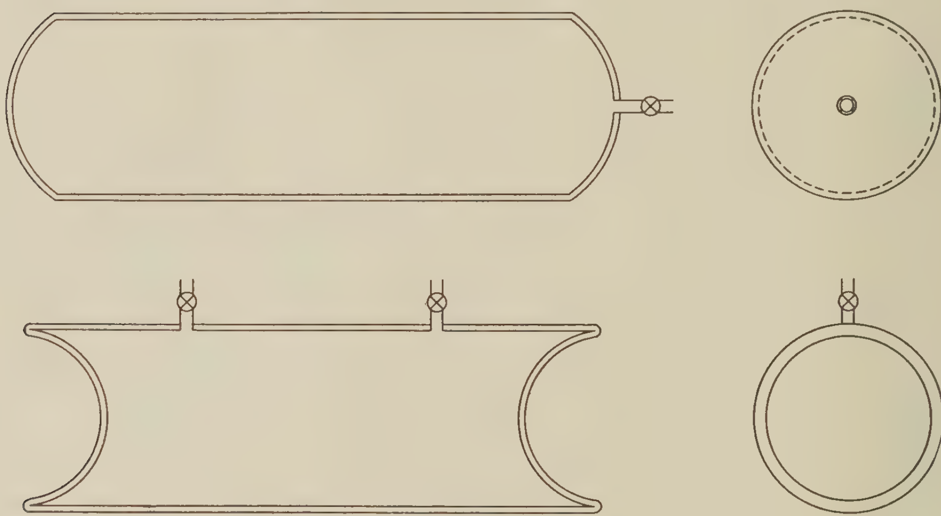

Fig. 75

control. *It is now essential that the area of the free surface of the liquid be constant while its height alters,* and this is already provided for in the fact that the sides of the tank, in its simplest form, are vertical.[1]

We shall ordinarily consider the inflow of fluid into these tanks to be reduced to zero. The case in which it is not zero will be treated as a special one. By this method of treatment, however, we are not to infer that the importance of such a circumstance is in the least ignored.

With either of our type reservoirs in this control it is obvious that if an inflow equal to the outflow be provided at any time during the process of production there is a conversion from Volumetric to Hydraulic Control.

These reservoirs have already received some attention in our studies, for we may recall the various principles established in chapter v, where we dealt with some of the elementary features in the mechanics of fluids. The reservoirs were in Volumetric Control, in so far as we were interested in changes due to a mathematical variation in the values of the various functions of performance.

In Hydraulic Control we found certain peculiar conditions to exist, in particular those which accompany ideal performance. These conditions were such as to induce us to accept this control as a mathematically deficient one.

[1] We already know that Volumetric Control is not confined to the specific forms of gas and solution tanks here adopted as types. (See §§ 40 and 44.)

With theoretic performance indeed a strange situation was encountered, for changes which might be brought about naturally in a duration of time in Volumetric Control, either in a filling process or in a producing process, were forced upon the reservoir without regard to time; that is, without setting up the fundamental conditions required for these changes and awaiting the latter to come about as a natural phenomenon. In truth we must admit, now that we think of it, that our usual experiments upon reservoirs in Volumetric Control are performed in theoretic performance of Hydraulic Control. We arbitrarily vary conditions of production, and thereafter proceed to hold them constant for a time sufficient for the purposes of observation. The results based upon the observation, we then say, are identical with those to be expected in the case of a mathematical variation with time.

**101.** *Pressure-time relations.*—Artificial reservoirs of the types described may be made to approach the ideal physical container to any desired degree. If we assume that we have them in perfection, and provide them with perfect fluids, then delivery by means of orifices that are maintained in a constant physical condition will be in accordance with ideal performance. No untimely act originating with Nature, and no act originating with us, shall disturb delivery. It is to be understood that natural changes in the value of the static pressure are not untimely.[2]

Reservoirs thus physically perfect may be assumed to be mathematically perfect. Time, as the seventh function of performance, is accompanied by changes in the values of five out of the remaining six, for, specifically, pressure, volume, velocity, energy, and power vary with time, while acceleration maintains a constant value. The mathematical relations between these six functions and time define the fundamental primary laws for delivery, and it is now our intention to derive and analyze these relations in Volumetric Control. Pressure-time relations are first in line.

In section 44 we investigated the problem of the time required to empty a vessel, and there we found, in Equation 42, the following relation:

$$t = \frac{2cc'A}{a\sqrt{2g}}\, y^{1/2} \qquad (162)$$

wherein $t$ is the time required to empty the vessel; $c$ and $c'$ are constants which care for viscosity, contraction at the orifice, and so on; $A$ is the area of the free surface of the liquid or gas; $a$ is the area of the orifice; $g$ is acceleration due to gravity; and $y$ is the pressure head at the orifice.

This equation was reduced to

$$x = k'y^{1/2} \qquad (163)$$

and this in turn was changed to

$$y = kx^2 \qquad (164)$$

Figure 23 illustrates the curve for this equation. It is a parabola.

[2] See § 23.

We now know that $y$ in any case is the potential pressure $P$, and $t$ or $x$, as the time required to empty the vessel, may be described as *time remaining.* We shall represent it by the symbol $T$. As for the constant $k$, we might conveniently replace it by $K$, say, in order that due allowance for the measurement of fluid in mass units be made. The result of all replacements is the following equation:

$$P = KT^2 \qquad (165)$$

as presented in section 45 without these details of derivation. This equation between pressure and time may be easily verified by experiment in the laboratory. It will be found to hold for liquids and gases. The presence of vapors does not disturb the relation, provided an allowance is made for any change in temperature due to them.[3]

There are two important facts to be emphasized in regard to this equation. First, we are not writing $S = KT^2$, where $S$ is the static pressure of the reservoir, nor are we writing $RS = KT^2$, where $RS$ is the registered static pressure of the reservoir—the closed-in pressure recorded by the ordinary pressure gauge. Both of these are incorrect relations.[4] Secondly, $T$ is time remaining in the life of the reservoir; it is under no circumstances time elapsed, as measured either from the beginning of production or from any subsequent instant during production. It is possible to write a correct equation between the static pressure, for example, and time elapsed. Thus

$$S - C = K\,(L - l)^2 \qquad (166)$$

wherein $S$ is static pressure, $C$ is constant back pressure, $L$ is the life of the reservoir, reckoned from the beginning or from any subsequent instant during life, and $l$ is time elapsed, reckoned from the same instant as the life.

I see no advantage to be gained in writing Equation 166, for it is equivalent to Equation 165. We have merely substituted equivalent values into one to obtain the other. The equation does show us, nevertheless, a correct relation, and it indicates the features of the pressure decline curve as shown in Figure 76. The curve *abc* shows the path of decline: $c$ denotes pressure and time either at the beginning of production or at some subsequent instant, $b$ is any point selected at random on the path, and $a$ denotes equilibrium for the reservoir under the prescribed conditions for pressure against production.

$P$ and $T$ are mathematical variables. At equilibrium $P =$ zero, and $T =$ zero. Whether or not these variables reach their limit zero, as shown at the point $a$ in the present figure, has nothing whatever to do with the question of their approaching this limit. We may deliberately abandon the reservoir

[3] Vapors simply provide for a curve with respect to a different horizontal axis. (See § 50.)

[4] For production into a perfect vacuum and into the atmosphere we would write $P = KT^2$, where $P$ takes the values of $S$ and $RS$, respectively.

before the point of equilibrium is attained, on the supposition or on the fact that our economic interest in it is no longer warranted. We shall find *a* to be of mathematical interest in spite of the fact that it may never be reached.

A variety of tanks, such as we have adopted as type reservoirs, may be imagined wherein the initial and subsequent values of *P* and *T* possess a wide range. In general, for a given fluid, these values depend upon the three dimensions of the reservoir and the size or the condition of the orifice. *P* may be less than one pound or more than five hundred pounds per square inch, and *T* may be less than one hour or more than twenty-five years. We cannot in theory define maximum and minimum values for these functions, nor can we prescribe the upper and lower extreme ratios between the two functions for individual reservoirs, granting a conventional mode of expressing the two in proper units. That is to say, $P =$ 1 pound per square inch may correspond to either $T =$ 1 hour or $T =$ 1 month, and $P =$ 500 pounds per square inch may correspond to either $T =$ 1 hour or $T =$ 25 years. Admittedly more or less extreme ratios than these are possible.[5] In practice we know approximately what to expect in given classes of reservoirs, even though we do encounter a great variety of ratios in any one class alone.[6] Figure 77 illustrates several pressure-time curves which may be said to pertain to any particular class or classes of reservoirs, provided the appropriate horizontal

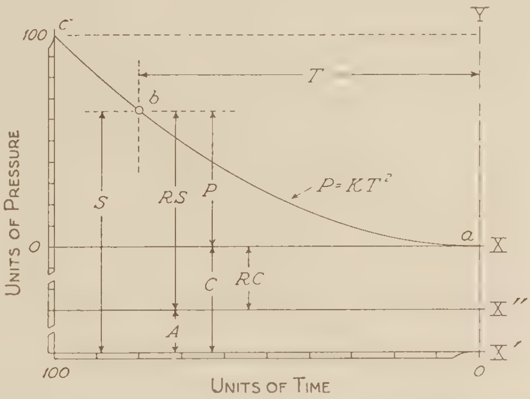

Fig. 76

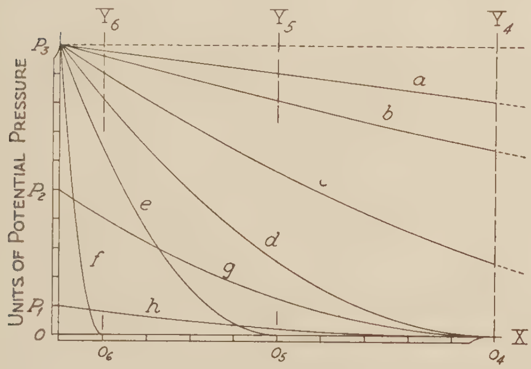

Fig. 77

[5] This is of course true among all reservoirs—artificial and natural—in this control.

[6] We well know that in natural reservoirs, as between one productive formation and another in the same field, or as between formations in different fields, different ratios are encountered.

and vertical scales are defined in quantities per unit of distance. All these curves are parabolas; they differ only in having the $K$'s in their equations at variance. No attempt is made to show very extreme ratios between the values of the functions; those shown are perfectly reasonable for any particular class of reservoirs.[7]

102. *The relative curve.*—It is necessary in reckoning with the life of the reservoir, or with time remaining for the reservoir, to know the location of the point $a$ on the $X$ axis; that is, to know in advance, for the given conditions of production, the *time-location* of equilibrium.[8] Such a requirement for the purpose of carrying out computations on reservoir performance may at first sight appear to be presumptuous. It is not so, for time remaining can be determined in advance of production, although the reservoir may be accessible only at the orifice. Nor need it be imagined that by reckoning with time remaining we introduce errors that can otherwise be avoided.

The purpose of curves and computations in reservoir performance is, of course, the forecasting of quantities as represented by the primary functions of performance. Future time, as one of these quantities, is necessarily involved in forecasting, irrespective of the particular method we may choose for procedure. It may seem to us that the heretofore popular practices in forecasting are based solely upon time elapsed, but this is fallacy. Although we may deal with time elapsed in a direct manner, time remaining must appear indirectly, to the same mathematical intent, or end, as if it were dealt with in direct terms. Indeed it will be found, in the light of principles to be established in chapter xix, that whereas we frequently believe our procedure avoids the necessity of knowing the time-location of equilibrium, we deliberately seek this point to the exclusion of all other points in time. I refer here to the method of shifting curves on logarithmic plats.[9] Why may we not profitably recognize the true import of time remaining, and deal with it knowingly in its due direct manner? We find time remaining to be the true function of performance from reservoirs in this control, and methods which do not deal directly with this function may very easily mislead us, and prevent us from thoroughly understanding production in its scientific aspects.

The simple parabolic relation between pressure and time, as derived in the preceding section, pertains only to reservoirs which meet the descriptions of our types for this control. We are not to imagine such an equation to be at all general.[10] To say that these types represent Volumetric Control, is purely

[7] The horizontal and vertical scales are presumably to be selected appropriately for each class of reservoirs.

[8] This holds true regardless of whether or not this point will eventually be reached.

[9] See, in particular, § 121.

[10] I have in mind here the class of reservoirs constituting Sub-Volumetric Control. (See § 40 and chapter xxii.) It so happens that the parabolic relation pertains as well to Capillary Control. (See § 152.)

a matter of arbitrary definition, and if we adopt this definition we thereby exclude from this control any reservoirs that do not correspond with them. We are to confine our attention for the present to those reservoirs which we know satisfy this relation between pressure and time. Let us say that we have one of these reservoirs. It is inaccessible, and perhaps intercepted from view, at all points other than at the orifice. We wish to determine its life or time remaining as reckoned from some instant during production. At one instant we shall read pressure and time, and thus be able to write

$$P_1 = KT_1^2 \qquad (167)$$

And upon repetition at a subsequent instant we shall be able to write

$$P_2 = KT_2^2 \qquad (168)$$

Now we may divide the first equation by the second, and obtain

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^2 \qquad (169)$$

In addition we have

$$T_1 - T_2 = l \qquad (170)$$

where $l$ is the time elapsed between the two readings. In 169 and 170 we have two equations containing two unknown quantities, $T_1$ and $T_2$. These are therefore determinable.

At once two questions arise concerning this method of reckoning time remaining. First, may we be certain that the $K$'s in Equations 167 and 168 are the same, so that by division they become unity, as in Equation 169? To answer we need only consider the factors upon which $K$ depends: namely, the three dimensions of the reservoir and the size or the condition of the orifice. The free surface of the fluid is constant; consequently $K$ does not alter in virtue of the dimensions of the reservoir. If the size and condition of the orifice have remained unchanged, then the $K$'s are the same, but if these have changed, of course the $K$'s are different. If it is possible—and in general it is—we should make certain the fact that the $K$'s are equal at the time of the observations.[11] Our results then require merely a modification in accordance with theoretic performance in this control, in order that they may apply to the reservoir under the normal conditions of production which prevail at times other than those of the observations. In the rare case where this procedure is impossible, I see at present no escape from the method of approximation based upon a series of observations the points of which are plotted on logarithmic paper and the consequent curve of which is shifted laterally until it appears as a straight line. Where the values of $K$ are subject to a rapid

[11] This is generally not difficult, if we understand the conditions upon which the value of $K$ depends.

fluctuation, the $P$'s may be taken from selected portions of a continuous curve described by a recording pressure gauge.

Secondly, what constitutes in any case a reasonable and safe length of time $l$ between the two observations? It should be conveniently short so as to provide either a zero change in $K$ or a facility in making its value the same at the time of the observations, and yet it should be conveniently long so as to cover a sufficient portion of $L$, the life of the reservoir which it is to determine in the guise of $T_1$ or $T_2$. If the interval is too long, its purpose defeats itself, since forecasting must anticipate future performance. I believe $l$ should be approximately equal to 10 per cent of $L$. If it is necessary to forecast at an earlier time, the results should be recast from time to time to improve their accuracy, until we are satisfied with the agreement between the predictions.

Life in Volumetric Control is finite in duration. *Whatever value life may possess at any instant during the process of flow may be represented by 100 per cent. And whatever value the potential pressure may have at the same instant may likewise be represented by 100 per cent.* It is absolutely immaterial how extreme these values of the individual functions may be, and it is likewise immaterial how extreme the ratio between the values of the two may be; we may take each at its 100 per cent value at any instant, whether this be at the beginning of production or subsequent thereto. Now if we take the equation $P = KT^2$, and substitute into it the values of 100 for $P$ and $T$, we have $100 = K\ (100)^2$, from which it is evident that under these circumstances $K = 1/100$. By substituting this into the first equation we have

$$P = \frac{1}{100}\,T^2$$

This relation I shall designate the *relative equation* between potential pressure and potential time in Volumetric Control; $K = 1/100$ is its *relative constant,* and Figure 78 is the corresponding *relative curve.* Having adopted the pressure at an instant as a standard for a basis of comparison, we see at once the proportional values possessed by the pressure at subsequent instants which define proportional values of time remaining. The curves of Figure 77, and all other possible pressure-time curves in this control, may be reduced to or derived from Figure 78. *There is but one basic curve between these functions, and this curve illustrates the law for pressure-time variation.* To suit any particular reservoir we may perform either one of two operations, as follows:

*a*) Adopt horizontal and vertical scales in pressure and time units to fit the curve in its present form, or

*b*) "Stretch" or "compress" the curve to fit any previously conceived scales in units.

The latter method gives us curves of the sort we would ordinarily construct at random in accordance with a particular experiment. Simply multiply abscissas and ordinates by the proper number greater or less than unity according to the required stretch or compression, respectively.

The relative curve is plotted from the following points obtained by giving $P$ successive values in the relative equation:

| $T$ in Per Cent | $P$ in Per Cent |
|---|---|
| 0 | 0 |
| 10 | 1 |
| 20 | 4 |
| 30 | 9 |
| 40 | 16 |
| 50 | 25 |
| 60 | 36 |
| 70 | 49 |
| 80 | 64 |
| 90 | 81 |
| 100 | 100 |

Any desired intermediate points may be supplied accordingly. If the history of pressure and time preceding the adoption of the standard is desired, values of $P$ greater than 100 per cent, when substituted into the equation, give values of $T$ greater than 100 per cent. By means of points so determined the relative curve may be extended upward to the left without limit. We shall have occasion to refer to such extensions in a later section.[12]

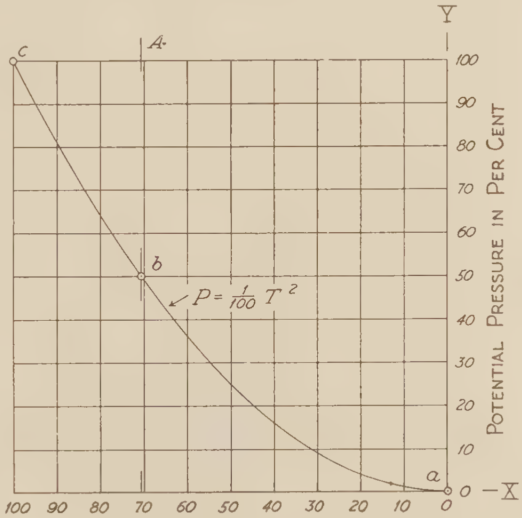


Fig. 78

To illustrate the manner of using the curve, suppose we consider a problem concerning either a rigidly constructed gas tank or a solution tank which meets with the description of one of the types.

At twelve o'clock noon the potential pressure has a value of 50 units, where these units are either in pounds per square inch or in feet of liquid according to the tank, and flow takes place from an orifice that remains undisturbed throughout the life of the process. At 4:00 P.M. the potential pressure is 32 units. What is the potential life of the reservoir?

We should proceed as follows:

$$\frac{32}{50} \times 100\% = 64\%$$

[12] See § 130.

the relative value of the pressure. By the relative curve we observe that for 64 per cent pressure, 80 per cent of life remains, or 20 per cent of life has elapsed; therefore 4 hours = 20 per cent of life, or 20 hours = 100 per cent of life, reckoned from the first observation.

As a second consideration suppose we wish to know when the tank will show 25 units of pressure.

$$\frac{25}{50} \times 100\% = 50\%$$

Now we observe by the curve that, for 50 per cent pressure remaining, 70.7 per cent of the life remains. 70.7% × 20 hours = 14.14 hours remaining after the tank shows this pressure. 20.00 — 14.14 = 5.86 hours to elapse from 12:00 M. for the attainment of this pressure; that is, at 5.86 P.M.

As a third and last consideration suppose we had neglected to take observations previous to 4:00 P.M. The potential pressure was then found to be 32 units. Subsequently, at 8:00 P.M., it was 18 units. We know that the tank was in the process of production at 12:00 M.; what was the value of the potential pressure at that time?

Regard the conditions at 4:00 P.M. as a standard for comparison, 100 per cent. Then

$$\frac{18}{32} \times 100\% = 56.25\%$$

and, according to the relative curve, to $P = 56.25$ corresponds $T = 75.00$. This percentage of life remains at 8:00 P.M.; consequently 25.00 per cent of life elapsed between 4:00 and 8:00 P.M. We see that 4 hours = 25 per cent of life, or 16 hours = 100 per cent of life. At 12:00 M. life was

$$\frac{20}{16} \times 100\% = 125\%$$

In lieu of the extension curve which we lack at the present moment, let us substitute this value into the relative equation; thus,

$$P = \frac{1}{100} (125)^2$$

from which we find that

$$P = 156.25\%$$

$$156.25\% \times 32 \text{ units} = 50 \text{ units}$$

the value of the potential pressure at 12:00 M.

*It is immaterial at what instant the functions are regarded as standard at 100 per cent for a basis of subsequent comparison, for in any and all possible cases the results are identical.*[13]

[13] It is obvious that we must necessarily have the data of observations pertaining to the instant for which "standard conditions" are adopted.

Naturally the reservoir we have just dealt with has its particular pressure-time curve in units of these two functions. The general equation was found to be $P = KT^2$, and if into this be substituted any of the corresponding values of pressure and time, as determined above, the particular equation is thereby obtained. For example, $50 = K(20)^2$; and consequently $K = \frac{1}{8}$. The particular equation is therefore $P = \frac{1}{8}T^2$. It is of interest to note that while we ordinarily do not concern ourselves with particular values for $K$ in our analysis they are none the less determinable, if we should desire them.[14]

The stretching and compressing of the relative curve itself are features of significance. To consider these let us refer to Figure 79, where the tanks

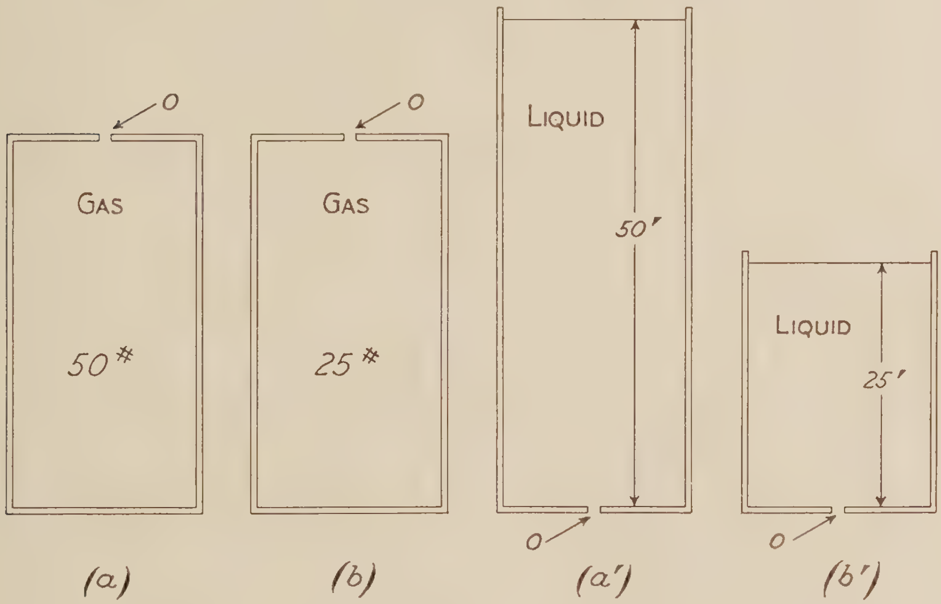


FIG. 79

($a$) and ($a'$) have 50 units of pressure, while ($b$) and ($b'$) have 25 units. Tanks ($a$) and ($b$) are presumed to be absolutely alike in their construction, and ($a'$) is absolutely the same as ($b'$), except in the matter of height. Pressures are potential, as of a constant back pressure due to the atmosphere alone; that is, production is to take place directly into the atmosphere in the manner indicated at the orifices $O$. We will first regard ($a$) and ($b$).

The orifice of ($a$) is opened, and production begins at the point $c$ in Figure 78. When the point $b$ is reached, its pressure is reduced to 50 per cent of 50 pounds; and now the two tanks are identical in all respects. If ($b$) is opened at this instant, the pressures of both travel the same path toward $a$,

[14] Instead of attempting to obtain by synthesis a value of $K$ from various known physical properties of fluids, and from the physical dimensions and conditions of the reservoir and its orifice, we have obtained its correct value by analysis.

the point of equilibrium. But had there been no tank ($a$) we certainly would have desired to take 25 pounds per square inch as 100 per cent, and in fact we might so desire in any case. Now $b$ in Figure 78 should occupy the position of $c$. Why not stretch $b$ to $c$, and carry all points of the curve from $b$ to $a$ accordingly? To do this we simply multiply abscissas between 0 and 70.71 by the number 1.414, and ordinates between 0 and 50 by 2.00. With what result? With the identical curve from $c$ to $a$ as already drawn. The curve repeats itself on stretching. *We can successively stretch as many points as we desire, from any part of the curve, including that part infinitely close to* a, *to the point* c, *and never have any but the curve with which we start.*

If we can stretch $b$ to $c$ to accommodate tank ($b$) as a standard, of course we can compress $c$ to $b$ to accommodate the resumption of tank ($a$) as a standard. In fact, $c$ can be compressed to any desired point between it and $a$, and the curve is as already drawn. Furthermore, *these operations are not confined to percentages from 0 to 100, for points may be brought in from anywhere on the extended curve, or such points as we find here may be carried out to any position on the extension, without modifying the curve in the least.*

Now it is evident that the same principles with respect to the relative curve apply to the tanks ($a'$) and ($b'$). The fact that the liquid is an incompressible fluid suggests a minor restriction: namely, that when a full tank is adopted as a 100 per cent standard, the extension curve has no significance beyond showing what the circumstances would be, if the sides of the tank were extended upward. However, if a partially filled tank is taken as 100 per cent, then a limited upward extension is possible.

These principles show us that we may approach a reservoir in the process of production and say:

"We here have a reservoir complete in itself. We may not know what this container has accomplished in the past. However, let us begin observations now, and adopt the present conditions as 100 per cent with the assurance that the reservoir will yet make a full sweep through the relative curve. This sweep is mathematically the same as the one we should have obtained, had we begun our records earlier. As soon as we have a competent record we may, in fact, compute past performance, and so obtain a complete history of the reservoir from its initial stages of production. In so far as the particular curve of the reservoir, with its scales in the customary units, is concerned, we are from now on to see the same path traversed, limited only on the left-hand side by our neglect in observing the past. Nevertheless, by means of the relative curve, this particular curve can be made complete in its extension upward to the left. And likewise, by means of the relative curve, this particular curve can be extended downward to the right in advance of performance."[15]

[15] Numerical values for past and future instants in the process of production can actually be determined either from the relative curve, or from its equation.

**103.** *Pressure-volume relations.*—After pressure-time relations we come to the relations between pressure and volume. When we have established these two for a control, all other fundamental and derived primary function relations follow in their usual course.

In sections 38 and 40 we obtained the following relation, $P = KVo$. This equation properly applies to reservoirs containing either gas or liquid. When we derived this relation we were not particularly concerned with the causes of variation between the functions; in fact we simply assumed alterations to be made independently of time as a function of performance. If we now say that these alterations are to come about naturally in the course of production, we at once may conclude that the above equation holds for ideal performance in Volumetric Control.

Pressure-volume relations are exceedingly important in production. We have seen that in Hydraulic Control these functions obey the following equation:

$$P = K \quad \text{(171)}$$

as shown in Figure 46. Now we have

$$P = KVo \quad \text{(172)}$$

in Volumetric Control, and later on in Capillary Control we shall have

$$P = KVo^{1/2} \quad \text{(173)}$$

In preparation for the derivation of the last equation I propose to reconsider Equation 172, and derive it once more. The method I intend to use may seem laborious—I grant that it would be unnecessary, if Equation 173 did not exist—but it offers the advantage of easy duplication in the most complicated control. While the methods of sections 38 and 40 fail to give any indication of Equation 173, we shall find that the present one is fully competent and sufficient to do so.

Figure 80 ($a$) (p. 248) represents a cylinder with a small orifice at the left and a piston at the right. The latter is frictionless and non-leaking. It is caused to exert a pressure $P$ upon gas or air contained within the cylinder, and this pressure is maintained at a constant value by moving the piston to the left during production.[16] If we imagine a series of spaces $S_0$, $S_1$, $S_2$, and so on, to occupy the interior of the cylinder, we see that equal pressures $P_0$, $P_1$, $P_2$, and so on, as shown in ($b$), maintain equal numbers of molecules, or equal mass-volumes, within them according to Boyle's Law, as in turn shown in ($c$). Equal pressures and equal volumes are here indicated by distances between respective pairs of parallel lines which may be presumed to be drawn to a suitable scale.

[16] These diagrams, and others to follow in Figs. 81 and 82, possess arbitrary features which need no mention here. These are in fact dictated by peculiar circumstances in Capillary Control. By introducing them in the present figures the analogy between the analytical procedures in the two finite controls will be found more complete.

In thus moving the piston we clearly maintain the individual spaces within the cylinder, for the time in which they exist, in Hydraulic Control. The conditions are those which define that control. Now with the same apparatus we may permit the spaces to be in Volumetric Control by holding the piston in its original position while flow takes place from the orifice. With this

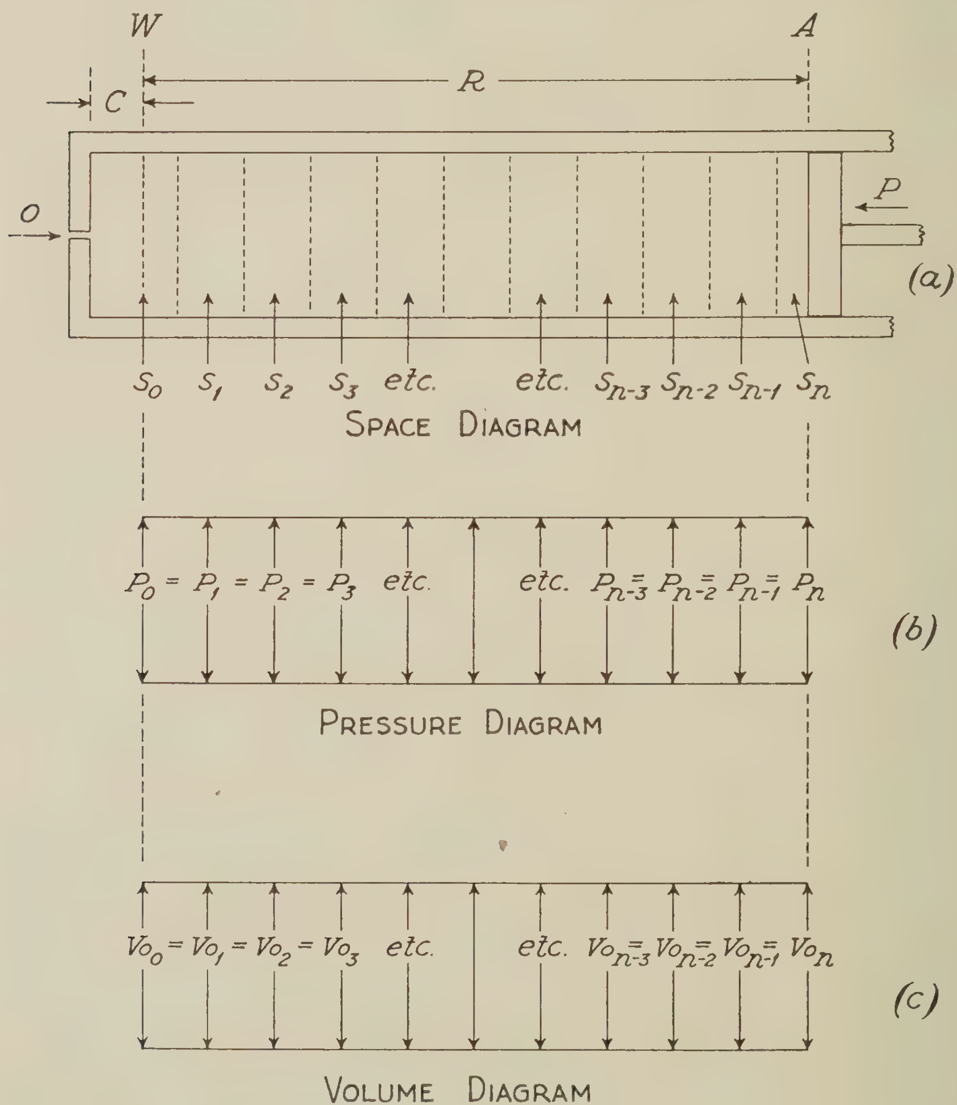


FIG. 80

provision the cylinder behaves as a rigidly constructed gas tank. Pressure and volume pertaining to the spaces decline, and the mutual relation between these functions in their variation is to be determined.

We shall hold the piston in the position mentioned during actual flow and interrupt the process from time to time for the purpose of making observa-

tions. The pressure $P$ for the spaces is of course always equal to the pressure within the cylindrical space at large, and its value may be read at any instant by means of a gauge or manometer properly attached to the apparatus. Special arrangements must be made for measuring the mass-volume within the spaces. First we should determine a sweep $R$ for the piston. This is to extend between two lines, $A$ and $W$, in the beginning, and thereafter shorten in accordance with production. The position of $A$ may be arbitrarily selected. With the piston in this position the system is allowed to assume a state of equilibrium with the orifice open; that is, the pressures inside and outside the cylinder are to become equal. Now with the orifice closed the piston is moved to the left until a selected value of $P$, say $P_0$, is indicated by the gauge or manometer. The piston in this position defines the location of the line $W$. A particular space $C$ subsequently cares for the mass-volume of gas or air which remains within the cylinder whenever the state of equilibrium is attained by production. The piston can now be returned to $A$, and the pressure $P_0$ may be set up by the introduction of a sufficient quantity of gas or air under compression. With the orifice closed the apparatus is ready for the determination of pressure-volume relations.

The volume within the spaces, and the volume within the "gas tank" at large, corresponding to any value of $P$ subsequent to $P_0$ upon production can be easily observed by interrupting production at the orifice and moving the piston to the left to re-establish the original pressure $P_0$. Successive values assumed by $R$ indicate the successive values of these volumes. The situation is depicted in Figure 81 ($a$) (p. 250), where the full sweep of the piston is conveniently divided into ten equal parts, with 0 at $A$ and 10 at $W$. We find that $R$ and the successive values of $P$ after $P_0$ vary in direct proportion; consequently we may construct the curve *abc* of Figure 81 ($b$) between the volume produced and $P$ and $R$ remaining.

The individual spaces can, as we see, be maintained either in Hydraulic or Volumetric Control with this apparatus. In the first instance the volume $Vo$ is constant,[17] and in the second it declines with production. If we were to write

$$Vo = K \qquad (174)$$

for these spaces in Hydraulic Control, and integrate the expression with respect to the pressure $P$,[18] we would obtain

$$Vo = KP \qquad (175)$$

[17] That is, it is constant within the individual spaces, while the number of such spaces declines.

[18] As to the propriety of performing this integration, see § 31. For the mathematical procedure, see footnote 15, § 64, page 120. We would ordinarily step from $Vo$ in Equation 174 to a $Vo'$ in Equation 175, and thereafter drop the prime as no longer necessary. All this is done here in one step. Obviously the two quantities are different. The second is an accumulated value of the first. In §§ 87 and 88 we took advantage of the fact that cumulative effects are obtained by integration in treating Equations 110 (p. 180) and 123 (p. 187) in this manner.

In fact this is just the operation that is performed in reasoning from (*a*) to (*b*) in Figure 81. As illustrated in Figure 82 (*a*) we integrate the straight horizontal line,

$$y = k,$$

in which $y$ is the volume within the individual spaces for any $x$ or $R$, and

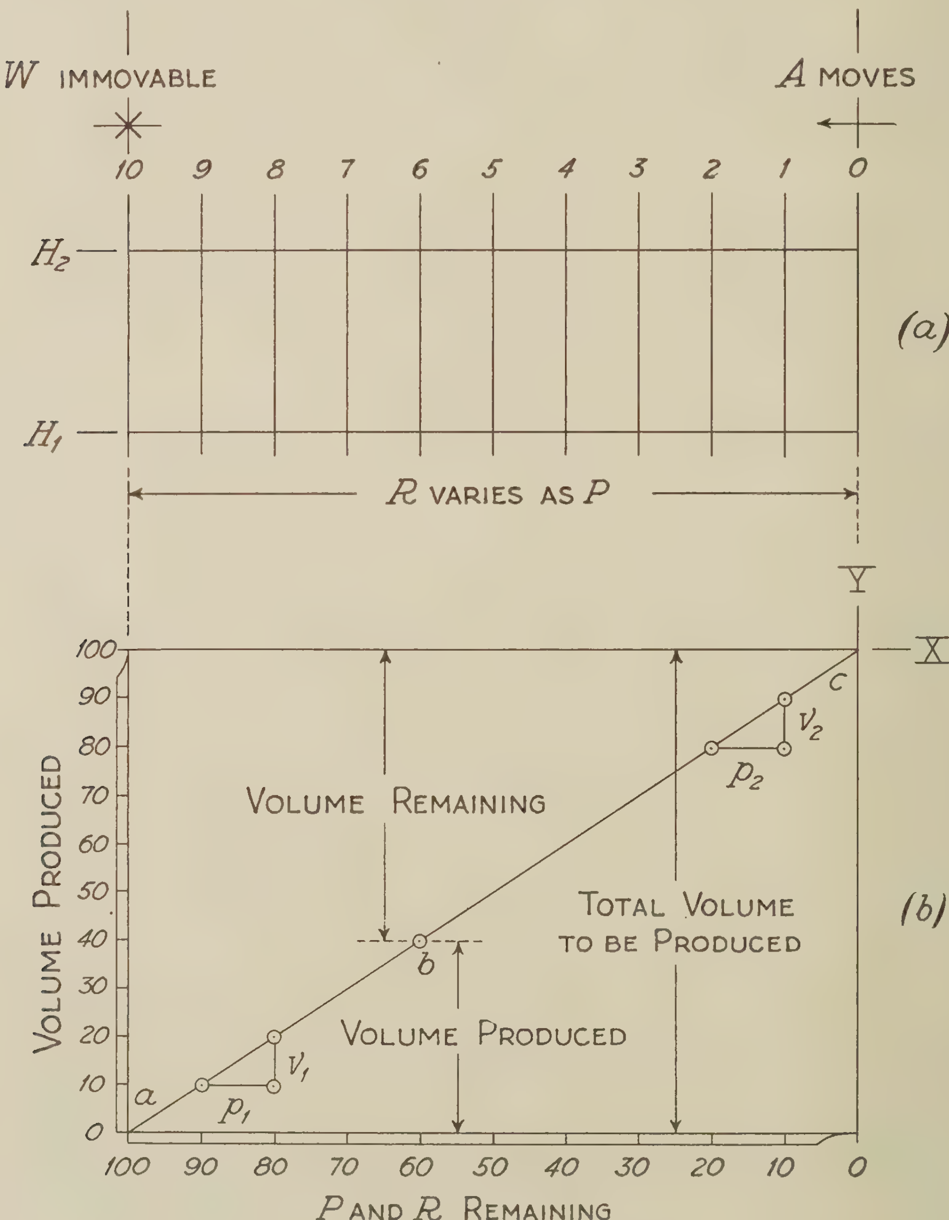


Fig. 81

consequently for any $P$ obtained by holding the piston on the $Y$ axis, with respect to $x$. The result is

$$y = kx,$$

the inclined straight line passing through the origin.

When we studied Hydraulic Control we acknowledged the fact that our

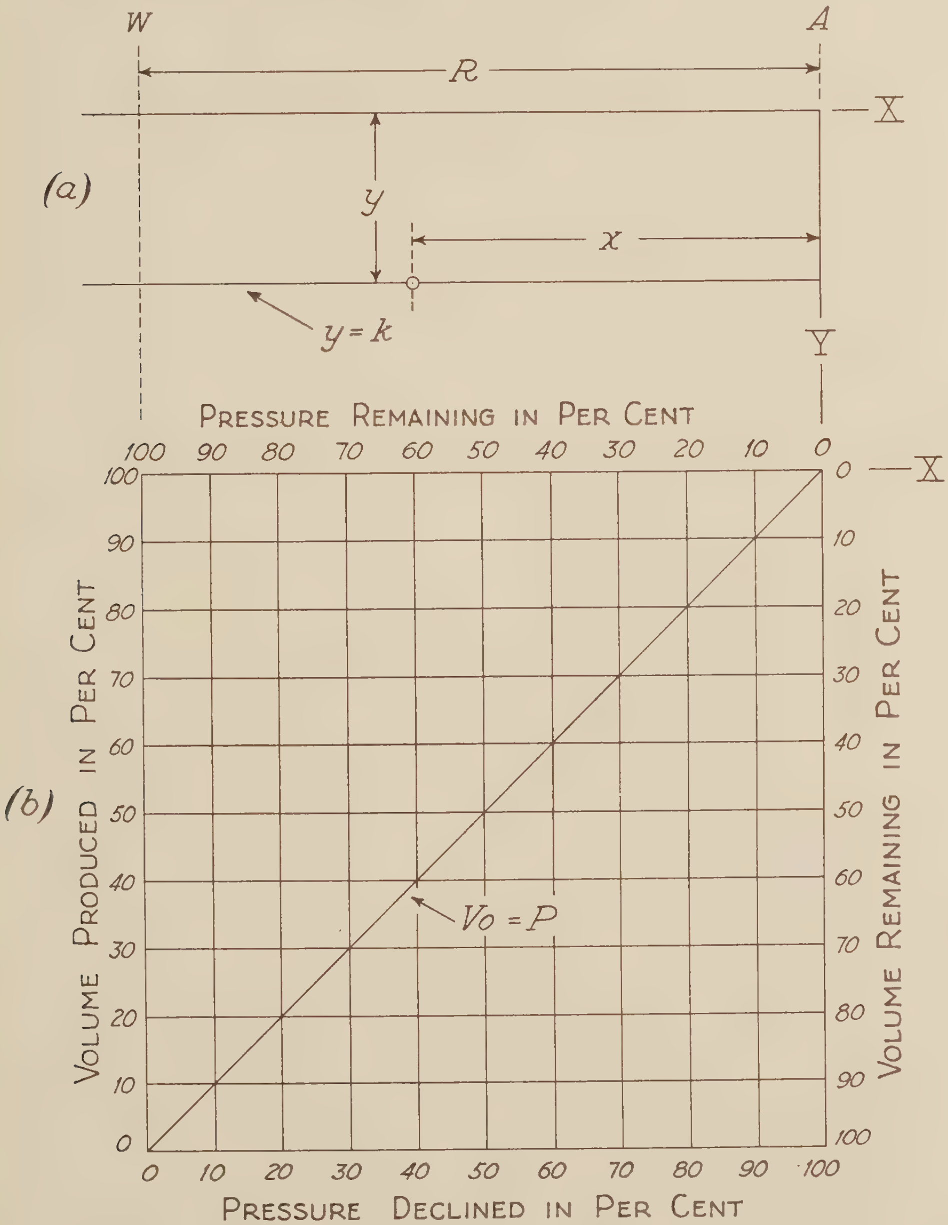


Fig. 82

method of reckoning with production from such reservoirs is artificial, in that time elapsed replaces time remaining, the origin is consequently at the left instead of being at the right, and the volume produced, in place of the volume remaining, is taken as the potential volume. These features are contrary to Nature, but they are expedient in handling a difficult mathematical situation. To step by a mathematical process from Hydraulic to Volumetric Control we must use the natural relation between pressure and volume as shown in Equation 174.

Equation 175 is of course equivalent to Equation 172. *In Volumetric Control, since we use the natural relations between the various functions of performance, a relation that pertains to individual spaces is applicable without modification to a space at large;* for example, to the entire space occupied by fluid in a particular reservoir.

In Figure 82 ($b$) we have the relative curve between pressure and volume in potential phase. Figure 81 ($b$) is simply modified so as to show the values of the functions during production in their percentage ratios. The relative constant has the value 1; therefore the relative equation is

$$Vo = P \quad \text{(176)}$$

Titles and scales in the figure are placed on all sides in order to emphasize the relation between them. Obviously there are eight possible positions for this plat; four are evident upon turning the figure in its present plane, and the other four appear correspondingly by turning, after placing the figure with its reverse side up.[19]

It is to be noted in Figure 81 ($b$) that for equal intervals $p$ in the potential pressure $P$ we have equal volumes $v$ produced or withdrawn from a potential volume $Vo$, regardless of the horizontal locations of the $p$'s with respect to $P$; that is, with respect to the value of $P$ possessed by a given reservoir at any instant during the process of production. Thus, in the figure, $v_1 = v_2$, whenever $p_1 = p_2$. This is true wherever $p_1$ and $p_2$ might be located between $a$ and $c$ on the curve.

I might say here that by a procedure analogous to the above we are to integrate Equation 175 with respect to pressure, and thereby obtain

$$Vo = KP^2 \quad \text{(177)}$$

for Capillary Control. This is clearly equivalent to Equation 173. Although, as I have admitted, the method may be somewhat laborious, and unnecessary in so far as Volumetric Control is alone concerned, I believe it is worthy of

[19] In turning or reversing we are obliged to turn or reverse the curve and its scales together. The latter, bearing their proper titles, cannot be placed on the plat at random. The $X$ and $Y$ axes obviously move with the curve. (These facts will become more evident in the same problem for Capillary Control.)

being understood, for I see no simpler and more direct way of establishing Equations 173 and 177 in Capillary Control.[20]

It is to be noted that by means of this procedure we shall have the mathematical relations between the three controls definitely established. *Two successive integrations upon pressure-volume relations in Hydraulic Control, when these relations are taken in their natural form, result in pressure-volume relations for Volumetric and Capillary controls, respectively.*

[20] See § 151.

CHAPTER XVI

# Ideal Performance and Its Primary Functions *(Continued)*

"On looking back over the history of our sciences, the first great example we find of the subjugation of a wide mass of facts to a comprehensive law occurred in the case of theoretical mechanics, the fundamental conception of which was first clearly propounded by Galileo. . . . . It was not till Leibnitz and Newton, by the discovery of the differential calculus, had dispelled the ancient darkness which enveloped the conception of the infinite, and had clearly established the conception of the *continuous* and of continuous change, that a full and productive application of the newly found mechanical conceptions made any progress."—HERMANN VON HELMHOLTZ

104. *Volume-time relations.*—We have found that in the ideal performance of reservoirs in Volumetric Control the potential pressure varies as the square of time remaining, a relation which is properly expressed by the following equation:

$$P = K_1T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (178)$$

Furthermore, we have found that at the same time the potential pressure varies directly as the potential volume, or the volume remaining within the reservoir. This relation is correctly stated by the following equation:

$$P = K_2Vo \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (179)$$

$K_1$ and $K_2$ are constants independent of the variable reservoir functions of $P$, $T$, and $Vo$, and their values for any given case in production are readily determined, if desired. The left-hand members of these equations are identical; therefore the right-hand members may be equated. Thus $K_2Vo = K_1T^2$, from which we see that we may write

$$Vo = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (180)$$

$K$ is evidently equal to $K_1/K_2$. According to this equation *the potential volume varies as the square of time remaining.*

Inasmuch as Equations 178 and 180 are both of the form $y = kx^2$, the title for the ordinates in Figure 77 may be changed to read "Units of Potential Volume," and the parabolas will then show a variety of volume-time curves for this control. When we measure pressures we automatically measure volumes remaining. Because we use different units in measuring these func-

tions, as required by their different natures, the curves that we might plot between pressure and time, and between volume and time, would ordinarily not coincide when placed upon the same plat, but they may be made to coincide by the proper stretching in the vertical direction. For a given reservoir the curves are the same except for the values of the $K$'s in their equations.[1]

To reckon with the volume which at a designated instant remains to be produced under given conditions of production may appear to be inconvenient, if not impossible, when the reservoir is inaccessible at all points other than at its orifice. Let us suppose that the orifices of our type reservoirs are accessible, and that the tanks themselves are behind a wall, beyond our sight and reach. We have in advance the two following equations:

$$Vo_1 = KT_1^2 \quad \text{.......................} (181)$$

and

$$Vo_2 = KT_2^2 \quad \text{.......................} (182)$$

in which the volumes $Vo_1$ and $Vo_2$ correspond to two instants $T_1$ and $T_2$. These volumes are as yet unknown, since we cannot directly measure the tanks. On dividing the first equation by the second we have

$$\frac{Vo_1}{Vo_2} = \left(\frac{T_1}{T_2}\right)^2 \text{.......................} (183)$$

If we measure the volume produced between the instants $T_1$ and $T_2$ we have

$$Vo_1 - Vo_2 = v \quad \text{....................} (184)$$

where $v$ is this volume. Now in Equations 183 and 184 there are four unknown quantities, and unless we can reduce the number to two no solution is possible. But we have already seen that $T_1$ and $T_2$ become known by reading the corresponding $P_1$ and $P_2$. Thus we need observe these values of the pressure in order to make our problem a determinate one. With $Vo_1$ and $Vo_2$ as the unknown quantities the two equations may be solved simultaneously in the usual manner.

As a matter of fact we need not consider $T_1$ and $T_2$, as the quantities of time required to produce $Vo_1$ and $Vo_2$, at all. It is sufficient to read the pressures and measure the volume produced between the readings. Our two equations are then

$$\frac{Vo_1}{Vo_2} = \frac{P_1}{P_2} \text{........................} (185)$$

the pressure-volume relations for this control, and Equation 184 as given.

The relative curve between volume and time is shown in Figure 83 (p. 256). It is the same as that for pressure and time. The relative constant has

[1] This is a true statement only for reservoirs in Volumetric Control.

the value 1/100. All volume-time curves may be reduced to, or derived from, this one by stretching or compressing, as required. The relative curve continually repeats itself, when it is subjected to stretching and compressing, in the same manner as we observed with the pressure-time curve. The curve $ab_2$ becomes the curve $ab_2c$ by multiplying all abscissas by 1.414, and all ordinates by 2.00. The significance of this feature, with respect to the potential volume of fluid contained in the reservoir, is clear.

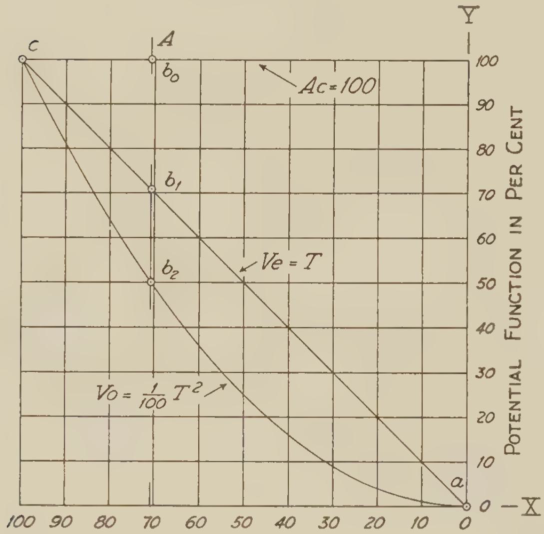


Fig. 83

The volume-time curve in practice is represented by the cumulative production curve. It is easily seen that the latter is the inversion of the former. The reservoir contains at the initial instant of production, or at some subsequent instant we are to adopt as zero time elapsed, a definite potential volume of fluid that is to be produced during life, under the given conditions of production, and if we are to measure the fluid which has left the reservoir instead of that which yet remains within it, then we are actually measuring a quantity $v$, Figure 84, and plotting it as $v'$. $Vo + v$ is the definite potential volume of fluid that is to be produced during life. If the lower half of this figure is turned about the line $A$, it assumes the position as shown in the upper half. The equation of the curve is not altered, provided the origin of co-ordinates retains the same position with regard to the curve.

Let us refer once more to the illustrative problems in section 102.

We shall say that the tank ($a$) of Figure 79 produced 3,600 cubic feet of gas between 12:00 M. and 4:00 P.M. What potential volume remained within the tank at the latter instant?

Now we found that at 4:00 P.M. 64 per cent of the pressure remained, and corresponding to this amount in Figure 82 ($b$) it appears that 36 per cent of the original potential volume had then been produced. That is, 3,600 cubic feet = 36 per cent, or 10,000 cubic feet = 100 per cent. This amount the tank contained at 12:00 M. Consequently at 4:00 P.M. it still contained

$$10{,}000 - 3{,}600 = 6{,}400 \text{ cubic feet.}$$

As a second consideration, how many cubic feet of gas remained within the tank at 5.86 P.M.? And how many cubic feet had been produced at that

time? Having previously learned that the life of the reservoir at 12:00 M. was 20 hours, at 4:00 P.M. there are 14.14 hours remaining. Therefore

$$\frac{14.14}{20.00} \times 100\% = 70.71\%$$

of life remains at this time. In Figure 83 the volume is seen to be 50 per cent when life is 70.71 per cent, and consequently

$$50\% \times 10{,}000 = 5{,}000$$

cubic feet of gas remained to be produced at 5.86 P.M. The same amount had been produced at that time.[2]

Had we said that the tank ($a'$) of Figure 79 produced 360 gallons between 12:00 M. and 4:00 P.M., our results would have been numerically similar to the ones obtained.

*Given the pressure and volume data at two instants during production, we can construct the complete cumulative production curve, for the past and for the future, by means of the relative curve between volume and time.*

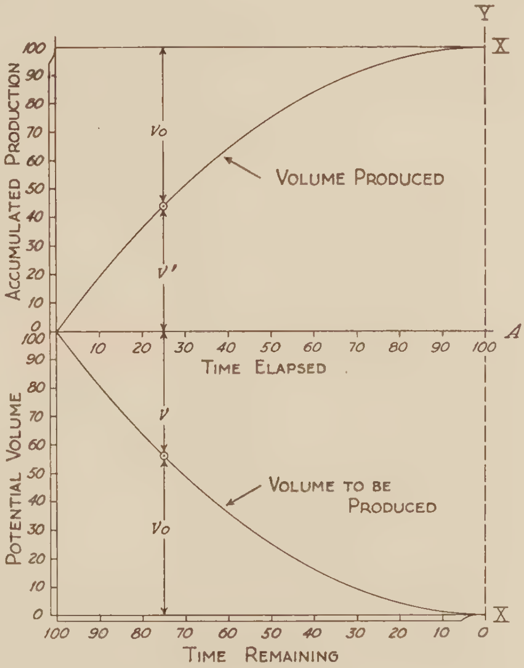


FIG. 84

**105.** *Velocity-time relations.*—According to section 13, if the volume of fluid within a reservoir at any instant may be defined as a function of time, or, in other words, if the relation between volume and time may be expressed by an algebraic equation, then the first derivative of the expression, with respect to time, is an equation showing the relation between the velocity and time. Between volume and time we have

$$Vo = K_1T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (186)$$

and this by differentiation becomes

$$Ve = KT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (187)$$

wherein $K$ is equal to twice the value of $K_1$. The equation shows us that *the velocity of production is directly proportional to time remaining.*

[2] It is to be noted that in these problems we are calculating the amount of gas to be produced under the given conditions of production, and not the amount of gas within the tank. (See footnote 1, § 38, p. 61.)

Corresponding to the variety of pressure-time curves in Figure 77 we have the straight lines for velocity as shown in Figure 85. The vertical scale is assumed to be compatible with the units in which this function is measured. The use of any other scale results in curves that may be obtained from these by stretching or compressing.[3]

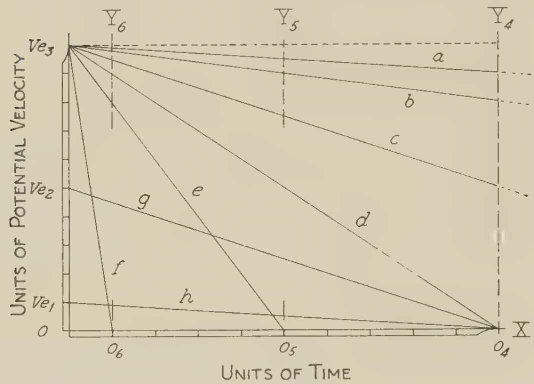


Fig. 85

If Figure 85 is superimposed upon Figure 77, each curve for velocity overlies its correspondent for pressure, as in Figure 34 the curve *C* overlies the curve *B*. When the velocity and pressure curves are drawn with their vertical scales so as to cause both to start from the same point in the upper left-hand corner of the plat, it is perfectly proper to say that one of them over- or underlies the other.[4] *Regardless of the fact whether this starting-point represents the initial instant or some subsequent instant in the life of the reservoir, the relative positions of the two curves on the plat remain invariable.*[5]

We cannot say that the curve for one of the two functions of a given reservoir declines more or less rapidly than that for the other. The slope of a curve at any point depends upon the units of measurement, the scales for plotting, and the distance between the point and the zero of time remaining. In Figure 34 we see that *B* declines more rapidly than *C* in the upper section of the curve, while the situation is reversed in the lower section; and yet we know that the initial point in common at the upper left-hand corner of the plat might represent any one of an infinite number of instants in the life of the given reservoir.

Provided two or more reservoirs belong to the same finite control, it is proper to compare their curves for the same functions, and say that one declines more rapidly, or less rapidly, than another.

The area subtended by any mass-velocity-time curve whatever represents mass-volume. The area subtended by a potential velocity-time curve represents potential volume. Given a potential velocity-time curve, squares of unit

[3] Since the positive direction for time remaining extends from right to left, the positive direction along the rate of production curve extends from the lower right to the upper left corner of the plat, opposite to the direction dictated by production. This is of course true of all curves in the finite controls.

[4] See footnote 2, § 58, page 107.

[5] This situation is not dependent upon ideal performance with its geometrically perfect curves.

width and unit height may be defined, such that the area covered by each represents a unit volume of fluid that is already delivered, or is to be delivered, from the reservoir. By measuring the number of these squares, and fractions thereof, subtended by the curve we learn of the corresponding volume. The entire curve, or portions of it as defined by vertical lines in accordance with particular instants in the life of the reservoir, may be handled in this manner.[6]

The geometrical relation between velocity and volume is illustrated in Figure 86. Here both curves start from a point $q$ as a matter of convenience, and not of necessity. The following equation may be written:

$$\frac{\text{Area } Oab}{\text{Area } Ode} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

The areas subtended by portions of the velocity-time curve are directly proportional to corresponding ordinates of the volume-time curve.[7] Again, the areas subtended by portions of the velocity-time curve are directly proportional to corresponding ordinates of the pressure-time curve. The latter follows from the fact that there is a direct variation between pressure and volume in this control.

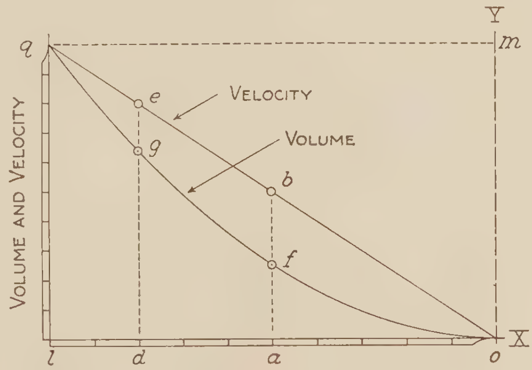

Fig. 86

In the present figure it is to be noted that the area of the triangle $Olq$ is one-half the area of the rectangle $Olqm$. The same would be true for any other triangles and their inclosing rectangles as might be defined by points between $O$ and $q$ on the velocity-time curve. We may therefore say that *the potential volume of a reservoir in this control, at any instant during its life, is equal to one-half the potential velocity at the instant, multiplied by the time in life remaining.*[8]

[6] Where the curves are not ideal, these areas may be conveniently measured by a planimeter. This instrument is in fact a mechanical integrator. It is mechanical in the sense that its act of integrating is not dependent upon a perfection of curves in accordance with equations.

[7] These portions of the velocity-time curve must be continuous, and they must extend to the right as far as the zero point of equilibrium.

[8] A given reservoir in Volumetric Control will deliver its potential volume in twice the time required of the same reservoir to deliver the same volume of fluid in Hydraulic Control. This is one of the earliest propositions known about flow from vessels. To us it is evident from the geometry of the two velocity-time curves with their subtended areas.

Obviously the units for velocity, volume, and time must correspond. For example, if velocity is expressed in gallons or cubic feet per hour, time remaining must be in hours, and volume will result in gallons or cubic feet, respectively.

The relative curve for velocity-time relations is shown in Figure 83. The relative constant has the value of 1; therefore the relative equation[9] is $Ve = T$. Like its predecessors, this curve repeats itself when stretched or compressed. It always remains a straight inclined line in accordance with its equation. For any point on the line the abscissa and the ordinate have equal values, and these are multiplied by one and the same quantity in the operation of stretching or compressing.

**106.** *Acceleration-time relations.*—As stated in section 14, if the relation between velocity and time may be expressed by an algebraic equation, then the first derivative of the expression, with respect to time, is an equation showing the relation between acceleration and time. Now by differentiating

$$Ve = K_1 T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (188)$$

with respect to $T$ we obtain the following expression in its most complete form:

$$Ac = K_1 T^0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (189)$$

But any quantity raised to the zero power is equal to unity and $K_1$ is more generally expressed as $K$; therefore this equation reduces to

$$Ac = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (190)$$

*Acceleration, or the change in the velocity or rate of production, is constant in this control.* Its value is equal to whatever constant necessarily appears in the equation between velocity and time, and obviously this constant cannot be zero.

We have performed the second differentiation upon the equation between volume and time in obtaining Equation 190.

To illustrate our mathematical operations let us consider that the tank ($a'$) in Figure 79 at the beginning holds 10,000 gallons, and that, in accord-

[9] Thus, from

$$Vo = \frac{1}{100}T^2$$

we have $Ve = T$. In performing the operations of calculus upon a relative equation (either differentiation or integration), we may either "carry through" the arithmetical relations to obtain the new relative constant, or, after the operation, deliberately place the proper constant in the equation without further consideration. By either method the constants are identical. (See footnote 5, § 77, p. 151.) The latter method is, of course, the simpler one.

ance with the observations previously made, its life as a reservoir is 20 hours. Into the equation $Vo = KT^2$ these values may be substituted. Thus

$$10{,}000 = K(20)^2,$$

from which it appears that $K = 25$. The volume-time equation for the tank is therefore $Vo = 25T^2$; that is, the volume remaining within the tank at any instant during production, in gallons, is equal to the square of the number of hours remaining in life, multiplied by 25. This equation by differentiation once becomes

$$Ve = 50T$$

The rate of production, in gallons per hour, is equal to the number of hours remaining in life, multiplied by 50. Thus we see that when $T = 20$, $Ve = 1{,}000$ gallons per hour, and that when $T = 19$, $Ve = 950$ gallons per hour.

By differentiation a second time the equation becomes

$$Ac = 50$$

The change in the rate of production, that is, the decline in the number of gallons per hour for each hour during life, is 50. It is clear that acceleration is properly expressed in gallons per hour per hour in the units here used. The amount 50 is already noted in the paragraph preceding.

The relative curve for acceleration is the simple straight horizontal line as indicated in Figure 83. Its equation is $Ac = 100$. In stretching or compressing this curve the abscissas are multiplied by the same number used for those of the other functions, and the ordinates are multiplied by 1. Obviously the curve repeats itself.

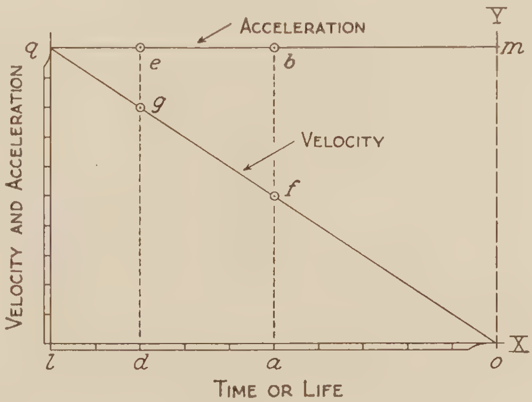


Fig. 87

Areas subtended by portions of the acceleration-time curve are directly proportional to corresponding ordinates of the velocity-time curve.[10] For Figure 87 we may write the following equation:

$$\frac{\text{Area } Oabm}{\text{Area } Odem} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

[10] We are to apply the same restrictions here as we did in the preceding section. (See footnote 7, § 105, p. 259.)

In section 105 we found that

$$Vo = \tfrac{1}{2}\, VeT$$

and now we see that

$$Vc = AcT$$

It naturally follows then that

$$Vo = \tfrac{1}{2}\, AcT^2$$

Evidently the value of $K$ in the volume-time equation is equal to one-half the value of acceleration. This fact might already have been noted above, where we observed the mathematical relation between the $K$'s in the volume-, velocity-, and acceleration-time equations.

This successive differentiation with respect to time, resulting in acceleration, reminds us of Galileo's laws of motion for freely falling bodies near the earth's surface. These are $s = \frac{1}{2}\, gt^2$, $v = gt$, and acceleration $= g$. In these equations $s$ is the distance through which the body falls from rest, say in feet; $t$ is the time elapsed since the body left the position of rest, say in seconds; $v$ is the velocity at the end of the time, then in feet per second; and $g$ is the value of acceleration due to the attraction between the earth and the body, in feet per second per second. A standard value of 32.17 feet per second per second is often used in computations. This is equivalent to its true value at sea level near latitude 45.

Acceleration in this control is perfectly analogous to $g$.[11] A solution tank produces its fluid by the action of gravity, and a rigidly constructed gas tank in turn produces its fluid in virtue of a pressure which is in all respects, from the point of view of mechanics, equivalent to a head in feet of liquid. For a given body, at a given locality, $g$ can be diminished by causing the body to move along a smooth inclined plane, and correspondingly, for a given container with its given fluid, $Ac$ can be diminished by partially closing the orifice. The two differ, however, in one detail: namely, that $g$ cannot be increased, unless it has been previously decreased, while $Ac$ may be increased or decreased without a corresponding restriction.[12]

To say that production from our type reservoirs is analogous to the falling of a body from rest is not strictly correct; we should say that *production is analogous to the rising of a body that is projected upward, either vertically or on an inclined plane.* For both the reservoir and the projected body a state of equilibrium is being approached, so long as there is motion, and if there is no interference this state will be attained. Here the analogy actually ceases,

[11] This is evident in consideration of Torricelli's Theorem. (See footnote 4, § 42, p. 67.)

[12] Our system of mechanics is in agreement, then, with Galileo's system. It is also in agreement with Newton's system. The latter, as we know, is based upon his three laws of motion.

for the body at once begins to fall from its instantaneous position of rest, while the reservoir remains inactive.[13]

*Where we reckon with time elapsed for a body falling from rest, we necessarily reckon with time remaining for the same body projected upward; and in the same manner we necessarily reckon with time remaining in production from the type reservoirs in this control.*

**107.** *Energy-time relations.*—Potential energy, as we already know, is equal to the product of potential pressure and potential volume. This is expressed by the following equation:

$$E = KPVo \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (191)$$

where $K$ is a constant whose value, on the assumption that the units for the three functions mutually involved are consistent, depends solely upon the control.

If we multiply pressure-time relations,

$$P = K_1T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (192)$$

by volume-time relations,

$$Vo = K_2T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (193)$$

we obtain the following energy-time relations:

$$E = K_1K_2T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (194)$$

a relation which may be expressed simply as

$$E = KT^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (195)$$

*The potential energy possessed by a reservoir in this control varies as the fourth power of time remaining.* The curve defined by this relation is a parabola of the fourth power. While the value of the constant $K$ appears to be equal to the product of the $K$'s in the original equations, as a matter of fact we shall find in the next section that to compute the amount of potential energy in a reservoir at any instant during life the fourth power of time remaining must be multiplied by one-half the product of the $K$'s in the pressure and volume equations.

Inasmuch as Figure 77 serves for volume and time as well as for pressure and time, all ordinates of the variety of curves there shown may be squared and stretched vertically to a suitable scale, thus resulting in Figure 88 (p. 264). The points where the curves strike the $X$ axis are not affected in any way.

[13] For the rising body the equilibrium, when attained, is, we might say, unstable, whereas for the reservoir the equilibrium is stable. The analogy, as here presented, cannot be carried into Capillary Control.

The potential energy possessed by a unit volume of liquid immediately at the orifice of a solution tank is equal to that possessed by another unit volume at the free surface of the liquid. The two energies differ in their separate values of the elevation head $z$ and the pressure head $p/w$, as these appear in Bernoulli's Theorem. Nevertheless the sums of these heads for the two unit volumes are equal. The same principle holds in regard to gas within a rigidly constructed tank. If the orifice is at the bottom of such a tank, a unit volume of gas in its immediate vicinity possesses potential energy equal to that of a unit volume at the top of the tank.[14] If we say that the elevation head for gas is negligible, because of its low density, it follows that the pressure head possessed by these two unit volumes is practically the same, and we may place the orifice in any position with respect to the tank, without experiencing an appreciable change in the energy possessed by two units in such extreme positions. Where unit volumes of liquid, or unit volumes of gas, have equal amounts of potential energy irrespective of their positions in the tank, all unit volumes occupying intermediate positions must have like amounts.

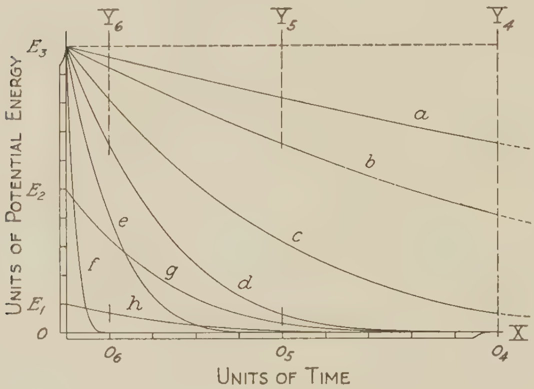

Fig. 88

*In Volumetric Control the potential energy possessed by a unit of the potential volume of fluid varies as the square of time remaining, and the number of unit volumes within the reservoir simultaneously varies as the square of time remaining; consequently, as we see in Equation 195, the amount of potential energy within the reservoir varies as the fourth power of time remaining.* We measure potential pressure, and therefore potential energy, with the orifice closed. Under this circumstance we are concerned with Bernoulli's Theorem in the following form:

$$E = W\left(z + \frac{p}{w}\right) \quad \ldots\ldots\ldots\ldots (196)$$

[14] The unit volume, being a mass-volume, occupies less space at the bottom than at the top of the tank, because of a compression due to the weight of the superincumbent gas. The pressure energy of the gas, anywhere within the tank, is, by Bernoulli's Theorem,

$$E_p = W\frac{p}{w}$$

where $p$ and $w$ now alter in the same proportion, as we pass from the bottom to the top. Consequently the fraction $p/w$ remains constant. (See § 43.)

where the symbols have the meanings attached to them in section 43. Now so long as we confine our attention to $W$ = one unit volume of fluid, then

$$E = \left(z + \frac{p}{w}\right) = K'_1 T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (197)$$

but where we admit that

$$W = Vo = K'_2 T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (198)$$

then from Equation 196

$$E = K'_1 K'_2 T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (199)$$

(The primes are introduced merely for the purpose of avoiding a conflict with the discussion immediately following Equations 194 and 195.)

In practice the curve between energy and time enters in problems concerned with accumulated gas production from combination reservoirs. This subject we take up in a later section.

The relative curve for energy-time relations appears in Figure 89. The relative constant, in order that

$E = 100$ when $T = 100$,

is by substitution found to be

$$K = \frac{1}{1{,}000{,}000}$$

This curve is subject to stretching and compressing in the same manner as the preceding ones, and it likewise repeats itself continually. For example, the point $b_4$ may be stretched to $c$ by multiplying all abscissas by 1.414, and all ordinates by 4.000. The resulting curve is that which is already shown.

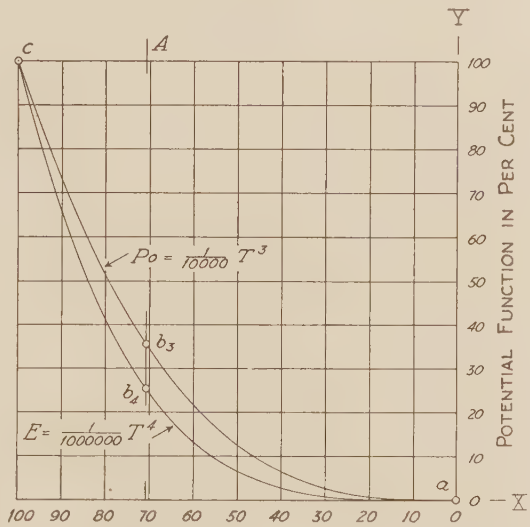


Fig. 89

108. *Power-time relations.*—If the relation between energy and time may be expressed by an algebraic equation, then the first derivative of the expression, with respect to time, is an equation showing the relation between power and time. Or, power is the product of pressure and velocity. Where we perform these mathematical operations upon expressions in potential phase, the result is an expression involving potential power.

To repeat our derivation of energy-time relations we accept the two following equations:

$$P = K_1T^2 \qquad (200)$$

and

$$Vo = K_2T^2 \qquad (201)$$

as proper, and multiply them to obtain

$$E = K_1K_2T^4 \qquad (202)$$

as in Equation 194 (p. 263). This by differentiation becomes

$$Po = 4K_1K_2T^3 \qquad (203)$$

But if power is the product of pressure and velocity, we might differentiate Equation 201 for

$$Vc = 2K_2T \qquad (204)$$

in agreement with Equation 187 (p. 257), and multiply this by Equation 200. Thus

$$Po = 2K_1K_2T^3 \qquad (205)$$

Obviously Equations 203 and 205 disagree. How shall we decide between them? Before doing this let us analyze power-time relations in general.

The two equations reduce to

$$Po = KT^3 \qquad (206)$$

where the value of $K$ is as yet undecided. *Potential power, or the rate of displacement of potential energy, varies as the cube of time remaining.* The curve defined by this relation is a cubic parabola. Corresponding ordinates in Figures 77 and 85 may be multiplied together, and stretched to a suitable scale, to obtain the variety of curves in Figure 90. Once more, the points where the curves strike the $X$ axis are not affected in any way.

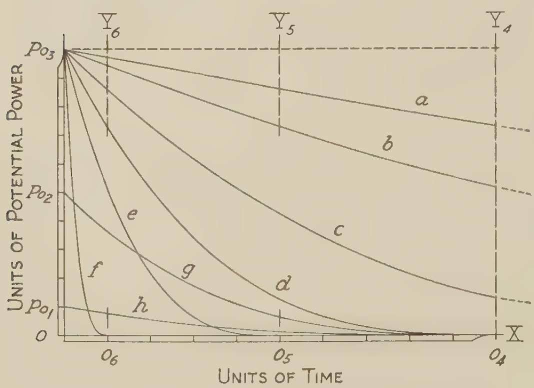


Fig. 90

Corresponding to Figure 86 for volume and velocity we may construct Figure 91 for energy and power. The following relation holds in this figure:

$$\frac{\text{Area } Oab}{\text{Area } Ode} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

The areas subtended by portions of the power-time curve are directly proportional to corresponding ordinates on the energy-time curve.[15] The area subtended by the power-time curve is equal to one-fourth that of the inclosing rectangle; that is, in the present figure the area *Olq* is one-fourth that of *Olqm*. And the same is true for any intermediate areas, such as *Ode* and *Oden*. We may therefore say that *the potential energy of a reservoir in this control, at any instant during its life, is equal to one-fourth the potential power at the instant, multiplied by the time in life remaining.*[16] As in the earlier case with velocity and volume, so now the units for power, energy, and time must obviously correspond.

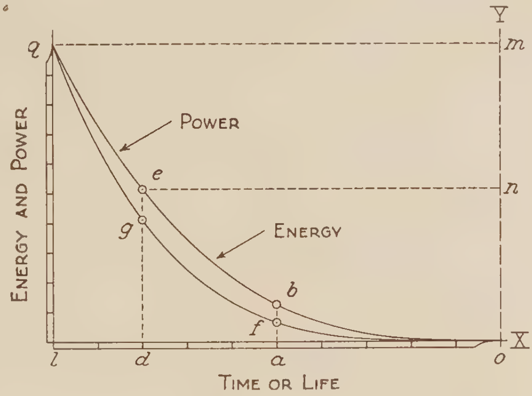


Fig. 91

Now let us refer to the tank $(a')$ of Figure 79 as we found it in its initial state at 12:00 M. We have the following data:

$$P = 50 \text{ feet of liquid,}$$
$$T = 20 \text{ hours,}$$
$$Vo = 1{,}000 \text{ gallons, and}$$
$$Ve = 100 \text{ gallons per hour.}$$

From these we can set up the following specific equations for production from the tank: $P = \frac{1}{8} T^2$, $Vo = 2\frac{1}{2} T^2$, and $Ve = 5T$. According to the values of $P$ and $Ve$ at the instant, it is clear that $Po = 5{,}000$ units per hour.[17] The total potential energy possessed by the reservoir at the instant is, in accord with the above rule, $E = \frac{1}{4} \times 5{,}000 \times 20 = 25{,}000$ units. On the other hand, if we multiply the values of $P$ and $Vo$ at the instant, we obtain $E = 50{,}000$ units, just twice the preceding amount.

The area subtended by the curve representing potential pressure-volume relations in turn represents units of potential energy. In Figure 82 $(b)$ this subtended area is clearly one-half the rectangle formed by the product of

[15] We are to apply the same restrictions here as we did in the preceding sections. (See footnote 7, § 105, p. 259.)

[16] The area subtended by any parabolic curve, with the equation $y = kx^n$, is $\frac{1}{n+1}$ times the area of the inclosing rectangle.

[17] For the dimensions of these units see footnote 9, § 15, page 23. Now we would express gallons in terms of cubic feet of liquid.

$P$ and $Vo$,[18] and we may therefore say with assurance that for Volumetric Control, $E = \frac{1}{2} PVo$. Accordingly we may know that the first of the two amounts above is the correct one.[19] Likewise we may know that Equation 205 is correct, and that Equation 203 is incorrect.

The relative curve between power and time is shown in Figure 89. The relative constant has the value of

$$K = \frac{1}{10,000}$$

The curve also repeats itself on stretching and compressing. To stretch the point $b_3$ to $c$ all abscissas should be multiplied by 1.414, and all ordinates by 2.828.

The points for the two relative curves of this figure are computed by substituting successive values of $T$ into the relative equations. These are as follows:

| $T$ in Per Cent | $E$ in Per Cent | $T$ in Per Cent | $Po$ in Per Cent |
|---|---|---|---|
| 0 | 0.00 | 0 | 0.00 |
| 10 | 0.01 | 10 | 0.10 |
| 20 | 0.16 | 20 | 0.80 |
| 30 | 0.81 | 30 | 2.70 |
| 40 | 2.56 | 40 | 6.40 |
| 50 | 6.25 | 50 | 12.50 |
| 60 | 12.96 | 60 | 21.60 |
| 70 | 24.01 | 70 | 34.30 |
| 80 | 40.96 | 80 | 51.20 |
| 90 | 65.61 | 90 | 72.90 |
| 100 | 100.00 | 100 | 100.00 |

Intermediate points, and any on extension, may be obtained in like manner.

**109.** *Summary of the fundamental relations.*—We have analyzed the six fundamental primary function curves for Volumetric Control. At whatever instant we begin our reckoning in time, whether it be the initial instant of production or any subsequent convenient instant, life, or time remaining, is

[18] The fact that Fig. 82 ($b$) is a relative curve is immaterial, for we may stretch or compress it to accommodate actual units, and the same proposition holds. We recall that in Hydraulic Control $E = PVo$, the constant $K$ for the general equation being unity. The area there subtended by the curve is a complete rectangle. (See footnote 10, § 15, p. 24.) It is essential that we appreciate the difference between qualitative and quantitative definitions in this work. The former, we may say, pertains only to the individual unit squares subtended by a curve, while the latter pertains to these and to the number of squares subtended. This is not only the situation with respect to Pressure $\times$ Volume = Energy, but equally so with respect to Velocity $\times$ Time = Volume, and Power $\times$ Time = Energy.

[19] As a consequence, liquid in the type solution tank may be treated as if, during flow, it were continually concentrated on a horizontal plane passing through the center of mass of the liquid.

mathematically finite. And if we begin our reckoning at a subsequent convenient instant, the paths traveled by the functions of performance in their decline are right-hand portions of their complete paths, as these are customarily traced on co-ordinate plats. Furthermore, reckoned from any instant whatever in the life of the reservoir, all functions make a complete sweep through their respective relative curves.

Our curves belong to the parabolic family, of the general equation, $y = kx^n$, wherein the exponent $n$ is either zero or a positive integral number not exceeding four, and the constant $k$ is a positive number, either integral or fractional. We place the origin of the curves at the right, and reckon time in terms of time remaining.

The following table gives the six relations as we have found them in the preceding sections:

VOLUMETRIC CONTROL

FUNDAMENTAL PRIMARY FUNCTION RELATIONS

| | |
|---|---|
| Pressure-Time | $P = KT^2$ |
| Volume-Time | $Vo = KT^2$ |
| Velocity-Time | $Ve = KT$ |
| Acceleration-Time | $Ac = K$ |
| Energy-Time | $E = KT^4$ |
| Power-Time | $Po = KT^3$ |

Inasmuch as these symbols in the right-hand column are employed for representing only the potential functions, as stated in section 18, the equations as written refer only to the potential axis $X$.[20] Although in our discussion no $X$ axes other than the potential ones have been mentioned, except in connection with Figure 76, we must admit that in every case we also have the absolute and atmospheric axes.[21] As we learned in the beginning, the potential axes do not coincide with the absolute axes unless the reservoir produces into a perfect vacuum, nor with the atmospheric axes unless the reservoir produces into the atmosphere.

Now we may on a composite plat superimpose the potential axes, the absolute axes, or the atmospheric axes, for all functions pertaining to a given reservoir upon production. *So long as our interests are centered in the performance of the reservoir in its delivery of fluid, it is not proper to superimpose any but the potential axes. Nor is it proper to compare the decline in the various functions except with respect to their respective potential axes.* It is inconceivable that one or more of the potential functions may have a zero value while the others possess a positive real value. When one of the potential

[20] To obtain values of the functions for plotting curves we may use either a table of squares and cubes, a table of common logarithms, or a slide-rule.

[21] In studying Hydraulic Control we considered the three axes in connection with the curves for each function. I have assumed a repetition of this procedure to be unnecessary in this, as well as in the next, control.

functions reaches the value of zero, all others must do the same. Obviously time remaining must then be the same for all potential functions of a given reservoir, for all reach equilibrium at the same instant. The forecasting of performance must be based upon the potential axes only, for these are being approached on decline.

When the six primary functions of performance are arranged according to the exponents of time with which they are associated, they appear as follows:

| Function | Exponent |
|---|---|
| Acceleration | 0 |
| Velocity | 1 |
| Volume and Pressure | 2 |
| Power | 3 |
| Energy | 4 |

Already we have observed that the areas subtended by the velocity and power curves represent units of volume and energy, respectively. Now we may say that, in virtue of the fact that the exponents differ successively by 1 in this list, the areas subtended by the curve for any of the functions represent units of the function succeeding in the list. The conversion of areas into units is possible if the proper conversion factor be determined from the data. Ordinarily it is convenient and desirable to compare portions of the areas subtended, in order to learn relative values of the functions which these areas represent. By thus carrying ratios the need of conversion factors is avoided.

In section 84 the functions of performance were classified as intensive and extensive in their nature. Now we have observed that pressure, velocity, and power, as intensive functions, are determinable from a single reading; acceleration, as a change in an intensive function, requires two readings, with an interval of time between them; and volume, energy, and time, as extensive functions, require two readings likewise, provided the reservoir is inaccessible except at its orifice. Inaccessibility offers no hindrance to the measurement of these extensive functions.

The constants $K$ which appear in all our primary function equations are determinable, but they need not be determined if we resort to the use of comparative data, as exemplified by the relative curves. In regard to the values of the relative constants $K$ we are now in a position to observe by inspection that, if $n$ is the exponent of $T$ in the fundamental primary function equation, then

$$K = \frac{1}{10^{2n-2}}$$

that is, the relative constant $K$ is equal to one divided by ten raised to the "two $n$ minus two" power. Of course this is the same as

$$K = 10^{2-2n}$$

I believe the latter to be the more convenient form of expression.

§ 109 ]

It will be advantageous to have all relative curves for Volumetric Control on one plat; therefore these are grouped in Figure 92. This chart is accurate, in so far as graphic representations may be made so. Where the accuracy of the data warrants, the use of relative equations is to be preferred. In general the chart is sufficient for our needs; and it has the desired property of presenting at a glance a clear picture of the control.

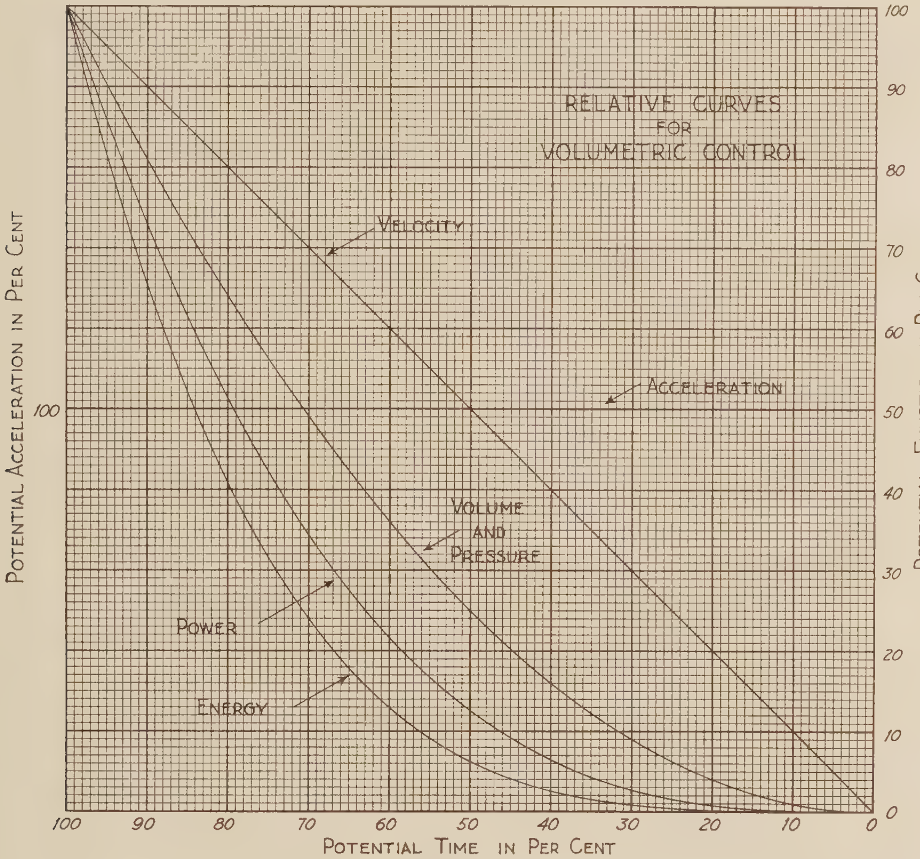


FIG. 92

That a reservoir of this control in its performance may fulfill these relative curves, each one of which depends upon $T$ as a function of performance, requires performance itself to be ideal, and this in turn requires the reservoir to be ideal. Now ideal reservoirs are unknown to us; consequently we cannot expect these curves to be exactly fulfilled in practice. *These curves do nevertheless define the laws of delivery from the type reservoirs, and the laws of delivery from all other reservoirs that approximate them in their physical condition.* Not only do these laws serve as a basis for reckoning

performance with respect to time, but they serve as well as a basis of determining the derived primary function relations, all of which are independent of $T$ as a function of performance, and some of which are independent of time as a matter of definition, as explained in section 41. Furthermore, these laws serve as a basis of theoretic performance in both Hydraulic and Volumetric Controls.[22]

[22] We have already taken advantage of these laws in our study of the first control.

CHAPTER XVII

# Ideal Performance and Its Primary Functions *(Continued)*

"The most direct, and in a sense the most important, problem which our conscious knowledge of Nature should enable us to solve is the anticipation of future events, so that we may arrange our present affairs in accordance with such anticipation. As a basis for the solution of this problem we always make use of our knowledge of events which have already occurred, obtained by chance observation or by prearranged experiment. In endeavoring thus to draw inferences as to the future from the past, we always adopt the following process: We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in Nature of the things pictured."—Heinrich Hertz

**110.** *Derived primary function relations.*—If we are to forecast the future performance of a reservoir in Volumetric Control by methods which involve the phenomena associated with decline, we should understand, I believe, the general mathematical properties of decline. Now we possess a set of six equations between pressure, volume, velocity, acceleration, energy, and power, on the one hand, and time on the other. These quantities appear to be the important functions of performance, and the equations which express definite relations between them define, so we have found, the laws of the delivery of fluid.

While a reservoir is never ideal in the sense we define it, no great stretch of the imagination is required to picture one that meets with all the specifications of the ideal. In fact artificially constructed reservoirs can be made to approach the ideal as closely as we may desire. Let it be granted that we have a reservoir so nearly perfect; it is at once evident that ideal performance is critically dependent upon one function: namely, the function of time. We have seen that time plays a double rôle in connection with delivery: first, as the function $T$, and, secondly, as a matter of definition, in so far as time must be mentioned in accurately specifying the mathematical nature of some of our functions. It is clear that $T$ may be eliminated between any two of the six fundamental equations, with the result that relations which are not dependent upon $T$ in any manner are obtained. There are, as stated in section 9, fifteen of these derived primary function equations. The list for this control follows.[1]

[1] The table of equations is given again in Appendix B, where comparison with corresponding tables of §§ 69 and 160, for Hydraulic and Capillary controls, respectively, may be made easily.

DERIVED PRIMARY FUNCTION RELATIONS

| | |
|---|---|
| Pressure-Volume | $P = KVo$ |
| Velocity-Pressure | $Ve = KP^{1/2}$ |
| Acceleration-Pressure | $Ac = K$ |
| Pressure-Energy | $P = KE^{1/2}$ |
| Pressure-Power | $P = KPo^{2/3}$ |
| Velocity-Volume | $Ve = KVo^{1/2}$ |
| Acceleration-Volume | $Ac = K$ |
| Volume-Energy | $Vo = KE^{1/2}$ |
| Power-Volume | $Po = KVo^{3/2}$ |
| Acceleration-Velocity | $Ac = K$ |
| Velocity-Energy | $Ve = KE^{1/4}$ |
| Velocity-Power | $Ve = KPo^{1/3}$ |
| Acceleration-Energy | $Ac = K$ |
| Acceleration-Power | $Ac = K$ |
| Power-Energy | $Po = KE^{3/4}$ |

We did indeed use velocity-pressure relations as expressed by Torricelli's Theorem to obtain pressure-time relations, and pressure-volume relations as given above to obtain volume-time relations. Our listing these original relations now among the derived relations may seem incongruous; it is for convenience of classification that they are included in the present table. It is true that these relations can be derived from the proper equations in the table of section 109 by eliminating $T$ in the usual way, but in this operation we are reversing earlier procedure and returning to the relations with which we started.[2]

Any of the derived relations may be expressed in the order reversed to that given above. For example, we may write either

$$\text{Pressure-Velocity } P = KVe^2, \text{ or Velocity-Pressure } Ve = KP^{1/2}$$

and

$$\text{Pressure-Volume } P = KVo, \text{ or Volume-Pressure } Vo = KP.$$

Thus the order is optional in any couplet we may form with two functions. We have already exercised our privilege of using first one and then the other of the examples here cited.

To be consistent and systematic we should adopt the first-mentioned function as the one to be represented by ordinates of the plotted curve, and the second as the one to be represented by abscissas. The equation for any couplet can be written down by rule; all we need to remember are the exponents between the various functions and time. Let $F_1$ be the first-mentioned function and $F_2$ the second; then $F_1 = KF_2^{n/m}$, wherein $n$ is the exponent of $T$ in the fundamental equation for $F_1$, and $m$ is the exponent of $T$ in the fundamental equation for $F_2$. The present exponent $n/m$ may be a fraction that is subject

[2] In other words, we are not to consider a derivation of velocity-pressure and pressure-volume relations from the fundamental equations of this control as a proof of their correctness. These relations are, as we know, founded only on experience. Their derivation now merely serves to show a consistency in our system of mechanics.

to simplification by removing a common divisor, as in the case of pressure-energy relations, and $m$ may have the value of one, leaving $n$ as an integer. For acceleration $n$, and consequently $n/m$, is zero when this function is the first mentioned in the couplet.[3] Obviously the reversed order for two functions forming a couplet results in an exponent which is the reciprocal of that for the original order. The plotted curve is given a double turn by so reversing the order: first, a clockwise turn of 90 degrees in the plane of the plat; and, secondly, an overturn as with the page of a book in front of us. The $X$ and $Y$ axes are thus interchanged.[4]

Relative curves can be constructed for all couplets showing derived relations. The value of the relative constant is found in the same manner as in the case of the fundamental relations: namely, by the equation $K = 10^{2-2n}$, where $n$ now has the value just described as $n/m$. As examples let us consider the following:

| Couplet | $n$ | $K$ |
|---|---|---|
| Pressure-Velocity | 2 | $10^{-2} = \frac{1}{100}$ |
| Velocity-Pressure | ½ | $10^{1} = 10$ |
| Pressure-Volume / Volume-Pressure | 1 | $10^{0} = 1$ |
| Pressure-Power | ⅔ | $10^{2/3} = 4.642$ |
| Power-Volume | 3/2 | $10^{-1} = \frac{1}{10}$ |
| Velocity-Energy | ¼ | $10^{1.5} = 31.62$ |
| Acceleration-Energy | 0 | $10^{2} = 100$ |

It is not necessary to construct these relative curves, unless we desire them for the picture which they present to us.[5] Corresponding values to be

[3] It is clear, then, that we need only have memorized the equations for the fundamental primary function relations to be able to write down the derived relations in accordance with the rule as expressed by the equation between $F_1$ and $F_2$. The rule applies to the three controls alike. In Hydraulic Control, wherever we obtain for $n/m$ the value 0/0, the result is indeterminate. The mathematical relation between the functions cannot be interpreted, except in so far as to say that the relation has no meaning. For this reason there are blank spaces in the table of derived primary functions of § 69. In the present control $n/m$ is infinite when acceleration is taken as the last function mentioned in the couplet. The curve is then the straight vertical line paralleling the $Y$ axis.

[4] They are interchanged only because we desire to retain $X$ horizontal for abscissas and $Y$ vertical for ordinates. The axis and the vertex of the curve itself of course remain unchanged with respect to the curve, and the scales must therefore be left unaltered in their positions with respect to the curve. (Compare with footnote 19, § 103, p. 252. There we were not reversing the order of the functions.)

[5] See footnote 20, § 109, page 269. We shall later have occasion to consider three of these "pictures," as follows:

Fig. 138, § 145: Pressure-Volume,
Fig. 159, § 160: Velocity-Pressure
Fig. 160, § 160: Velocity-Volume.

In these we compare the two finite controls.

shown by these may be obtained with ease by simply locating the given value of one function on its respective time-curve in Figure 92, and there cutting the entire plat by a vertical line. Where this vertical line intersects any other time-curve, there we find the value sought. While the cut gives us values that are independent of *T,* we can make note of the value possessed by this function, or at least a value indicated for it, if we desire. The significance of an indicated value, and the use that may be made of it, will be explained in a following chapter on theoretic performance.[6]

111. *Forecasting by pressure and volume.*—Although we may eliminate the function *T,* and so confine our attention to the derived primary function relations, we cannot escape the idea of time in all couplets. Velocity, acceleration, and power involve time in their definitions; consequently all couplets which include one or two of these functions are dependent upon time. In section 41 we observed, however, that pressure, volume, and energy are not dependent upon time in this way. The relations between pressure and volume, pressure and energy, and volume and energy are therefore absolutely independent of time. Regardless of alterations in the size or condition of the orifice during the course of production these relations are accurately fulfilled. Their relative curves define paths that are traveled in a perfect mathematical manner—each of the functions, in order to pass from one value to another, must pass through all intermediate values, and this it must do purely as a result of production from the orifice. Regardless of alterations in the value of the static or constant back pressure during the course of production these relations are also fulfilled with accuracy. With these alterations, however, the paths defined by the relative curves are not traveled over in one continuous "declining" sweep. Points on the curves are reached by "jumps" that are specified by each individual alteration, and evidently these jumps may be either upward to the left or downward to the right. They are made independently of production at the orifice.

*These relations should be given preference in forecasting the future performance of a reservoir in virtue of their independent nature.* Where a reservoir produces either liquid alone or gas alone, predictions should be based upon pressure-volume relations. Where a reservoir produces both liquid and gas, predictions upon the liquid should be based upon pressure-volume relations, and predictions upon the gas should be based upon either pressure-energy or volume-energy relations. By means of the latter we compare the volume of liquid with the volume of gas.

The so-called "graphic solution by Boyle's Law," mentioned in section 37, is of course based upon pressure-volume relations in potential phase. This method has been used with success where reservoirs produce gas, for with

[6] See chapter xxi, §§ 130, and so on.

these the data on pressure and accumulated production were available without specific solicitation on the part of those who were responsible for the forecast.[7]

The relation between pressure and volume in Volumetric Control permits us to say that *equal amounts of fluid are produced for equal amounts of decline in the pressure.* Let us consider three successive readings on pressure and their relation to the corresponding volumes of fluid remaining within the reservoir:

$$P_1 = KVo_1 \qquad (207)$$

$$P_2 = KVo_2 \qquad (208)$$

and

$$P_3 = KVo_3 \qquad (209)$$

Now by subtraction we obtain

$$P_1 - P_2 = K(Vo_1 - Vo_2) \qquad (210)$$

and

$$P_2 - P_3 = K(Vo_2 - Vo_3) \qquad (211)$$

The first of these may be divided by the second:

$$\frac{P_1 - P_2}{P_2 - P_3} = \frac{Vo_1 - Vo_2}{Vo_2 - Vo_3} \qquad (212)$$

Here the numerator and denominator on the left represent differences of pressure,[8] while those on the right represent differences in the volume remaining within the reservoir; that is, they represent the volumes produced in the interval of time between the readings on pressure. Now if the numerator and denominator on the left are equal, those on the right must likewise be equal. This is the law frequently spoken of as that of "equal production per pound decline." As a law it possesses the following restrictions:

*a*) It is correct only for reservoirs in Volumetric Control.

*b*) It is correct as applied to one—any one—reservoir in this control, and it is not correct as applied to more than one reservoir unless both or all happen to have the same physical dimensions.

*c*) It is correct as applied to liquid or gas, when the reservoir produces either one of these fluids alone. If the reservoir produces both liquid and gas, it is correct as applied to the liquid, but not as applied to the gas.

Obviously the possibility of such a law for reservoirs in Hydraulic Control is absurd, for there is no decline in pressure.[9] In Capillary Control we

[7] See § 9 for the list of five relationships with which we are familiar in practice.

[8] We might also have written on the left

$$\frac{S_1 - S_2}{S_2 - S_3} \text{ or } \frac{RS_1 - RS_2}{RS_2 - RS_3}$$

inasmuch as $C$ or $RC$ vanish by the subtraction.

[9] If there were a decline, the reservoir would not be in Hydraulic Control.

shall develop a corresponding law by starting with three equations based upon the relation $P^2 = KVo$. The result will be

$$\frac{P_1^2 - P_2^2}{P_2^2 - P_3^2} = \frac{Vo_1 - Vo_2}{Vo_2 - Vo_3} \qquad (213)$$

an equation which shows us that there is equal production for equal differences of squares of pressure.[10]

If we withdraw a given volume of fluid from each of several solution tanks, or from each of several gas tanks, it is clear that the decline in pressure occasioned by such a withdrawal depends upon the lateral dimensions of the solution tanks, and upon the space dimensions of the gas tanks. Then where any of these tanks differ in their dimensions, equal decline in pressure does not mean equal volumes of fluid withdrawn. We cannot compare different tanks on the basis of equality, but we can, however, compare them on the basis of proportionality. The numerator and the denominator of both members in the equation between differences simply do not bear the relation of 1 to 1 when the tanks differ. We are permitted to say that in general, *with one or more reservoirs in Volumetric Control, proportional amounts of fluid are produced for the same proportional amounts of decline in pressure.* The law in equality is clearly a special case of the law in proportionality; the latter is reduced to the former in case we deal with one reservoir only, or with two or more reservoirs that are alike in their dimensions.

When a law may be expressed clearly and concisely in a few words, and is therefore simple in its practical application to natural phenomena, it is a worthy expression. That it may be so requires an extreme simplicity in the mathematical relation upon which it is based. In the present example we have seen that the basic equation is the simple relation between pressure and volume in this control: namely, $P = KVo$. We should therefore doubt the utility of the corresponding law for reservoirs in Capillary Control, and we should likewise doubt the utility of a law which provides for proportional changes in the volume of gas delivered from a combination reservoir and the decline of pressure within such reservoir. These situations are better handled by means of their basic equations, without any attempt to interpret and preserve them verbally. By algebraic methods we treat the simple and the complicated relations with equal facility.

112. *Law of expectation.*—Whereas a forecast may with equal propriety refer to a future value to be possessed by any one of the various functions of performance in the course of production from a given reservoir, custom appears to have favored a particular forecast which refers directly to the volume of fluid that is to be delivered from the reservoir during its life. This we call

[10] See § 161.

"expectation." The volume, of course, is that which we specifically designate as the potential volume.

A law of expectation is at best founded on the relation between pressure and volume. Thus in comparing the performance of two reservoirs we may write

$$P_1 = K_1 Vo_1 \qquad (214)$$

and

$$P_2 = K_2 Vo_2 \qquad (215)$$

where the equations pertain to their respective reservoirs. It is clear that if

$$K_1 = K_2 \qquad (216)$$

the reservoirs must have like dimensions, inasmuch as the value of $K$ in pressure-volume relations depends only upon the dimensions of the reservoir. Granted that the $K$'s are equal for the two reservoirs, we see at once that when at any time $P_2$ becomes equal to $P_1$, then $Vo_2$ must be equal to $Vo_1$. For these reservoirs we have a law of equal expectation: *Whenever reservoirs of the same dimensions possess a previously specified value for the potential pressure, they thereafter produce equal volumes of fluid.*

Ordinarily we need not expect Equation 216 to hold, except in one important instance. Let us suppose that one of our type reservoirs in this control possesses two or more orifices that are physically alike in all respects; particularly in the case of the solution tank they are located at the same level, and in any case they are of the same size and condition. Now each orifice has its own potential reservoir, in spite of the fact that they serve a common container. The latter circumstance merely provides the possibility that the $K$'s are equal, and guarantees the fact when the orifices are physically alike. The law of equal expectation now holds for these orifices.

But if we may suppose that the orifices are not alike, or if we may suppose that they do not serve a common container, we may say that there is at least a law of relative expectation which does hold. The ratio between the expectations is exactly that as expressed by the ratio between the $K$'s. In fact we should in general assume the $K$'s to be unequal; then if the ratio proves to be 1 to 1, we may be assured of an equal expectation in place of merely a relative expectation.

To say that the dimensions of two or more reservoirs are the same, and that the orifices are physically alike in all respects, provides equal values for all $K$'s in the fundamental and derived primary function relations, respectively. In particular the $K$'s in the separate velocity-time equations are identical in value; consequently a law of equal expectation may then with propriety be based upon the velocity, or rate of production from the orifices: *Whenever reservoirs of the same dimensions, having orifices that are physically alike in all respects, possess a previously specified value for the rate of*

*production, they thereafter produce equal volumes of fluid.*[11] This is a special case of a more general law of relative expectation, likewise based upon velocity.

113. *Pressure as a basis for computations.*—We can appreciate the freedom from restrictions when we base forecasts upon the relations between volume and pressure. The results of computations are accordingly the most accurate that can be obtained. It would likewise be most satisfactory to forecast the values of other functions by using pressure as a basis for computations. Pressure, as we know, is easily and accurately observed at the orifice. All functions can be expressed in terms of pressure in the manner described in section 110. Let us review them:

$$Vo = KP$$

$$Vc = KP^{1/2}$$

$$Ac = K$$

$$E = KP^2$$

$$Po = KP^{3/2}$$

and in fact we may add

$$T = KP^{1/2}$$

It will be convenient to have a chart which shows the relative curves for these equations. This is given in Figure 93. As usual the curves may be extended upward and to the left as far as desired.

In general it may be said that velocity, and likewise power, should be treated as dependent functions. Their values as observed today serve at best as a basis for computing future values, not of other functions, but of their own kind. And when the future time becomes present time, actual observations may be compared with the computed ones. Thus we may know of the physical condition of the orifice at the time, at least on a comparative basis.[12] If the observed velocity or power is less than the computed value, the orifice has become partially obstructed, and if it is greater than the computed value, the orifice was not in the best condition at the time of the first observation;

[11] In stating the laws of equal expectation I do not pretend to quote the original authors, Carl H. Beal and James O. Lewis. If the present versions have the least advantage, it is only because they clearly indicate the restrictions that must be included in their application.

[12] Velocity, as previously stated, refers only to the rate of production of liquid or gas, when these are produced alone. Power refers to the rate of production of gas when both fluids are produced.

it has in the meantime cleared itself, and further computations should be based upon the newly observed values.[13]

The curves of Figure 93 bear the usual consistent relations between themselves. The area subtended by the volume-pressure curve represents, as we have seen in section 108, units of potential energy, and this area is one-half

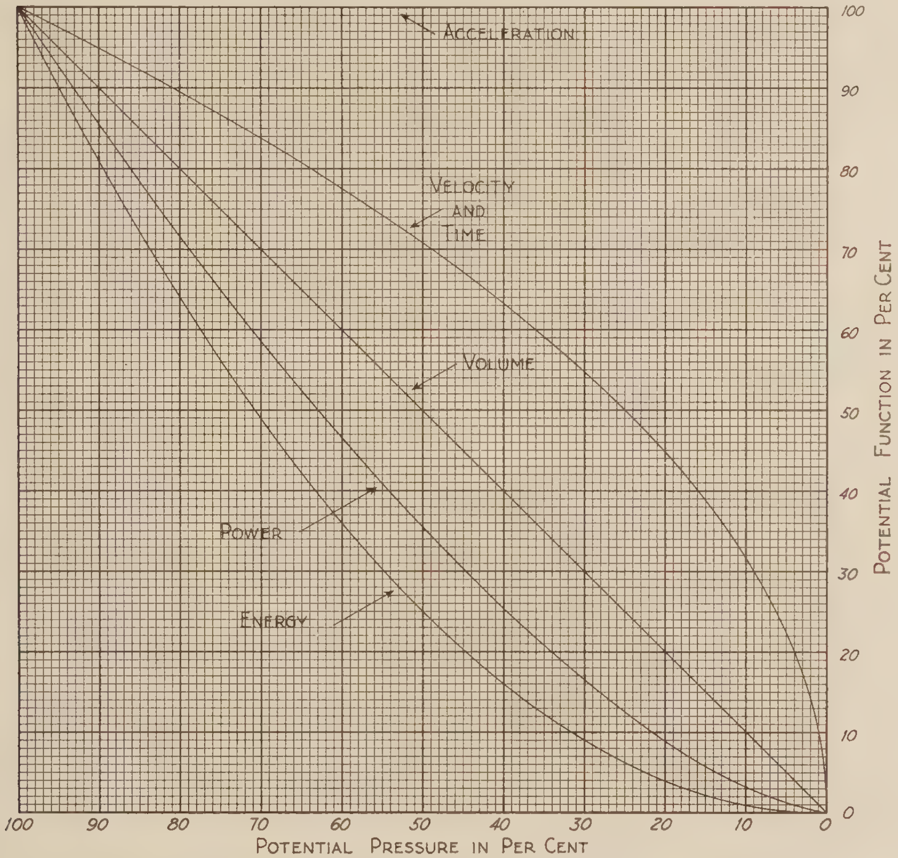


FIG. 93

that of the rectangle inclosing it. The integration of volume-pressure relations, with respect to pressure, gives energy-pressure relations; thus

$$Vo = KP$$

becomes

$$E = KP^2$$

[13] This statement is based on the assumption that production has proceeded continuously, without partial or complete interference at the orifice. In the case of partial interference due allowance must be made in accordance with theoretic performance, and in the case of complete interference its time must be taken out of the record of production.

The areas subtended by portions of the volume-pressure curve are directly proportional to the corresponding ordinates of the energy-pressure curve.

The curve between velocity and pressure should impress upon our minds the fact that when the equation is expressed as $Ve = KP^{1/2}$, we deal with a path traveled by velocity with respect to pressure. We are not merely concerned with single values as ordinarily determined by the equation $v = \sqrt{2gh}$ for lineal velocities.[14] $Ve$ and $P$ are true mathematical variables, such as are usually represented by $y$ and $x$, respectively, in algebraic solutions of problems, and in curve tracing in connection with studies in co-ordinate geometry. $Ve$ and $y$ are dependent variables, we say, while $P$ and $x$ are independent variables. The former travel in specific directions along definite paths; in our present example $Ve$ travels from a position at the upper left to one at the lower right corner of the plat in accordance with $P$, as this declines in the process of production from the reservoir.

The area subtended by the velocity-pressure curve represents units of potential power. The integration of velocity-pressure relations, with respect to pressure, gives power-pressure relations; thus

$$Ve = KP^{1/2}$$

becomes

$$Po = KP^{3/2}$$

The areas subtended by portions of the velocity-pressure curve are directly proportional to the corresponding ordinates of the power-pressure curve.

In introducing the relative curve for pressure the statement was made to the effect that in this control all possible pressure-time curves, in actual units of the functions, may be reduced to or derived from the one relative curve between pressure and time. It is evident that the same statement may be made concerning all possible velocity-time curves, volume-time curves, and so on, with respect to their corresponding relative curves. Now we might in fact make the statement in regard to curves between the various functions and pressure.

As a general proposition, we can make the following comprehensive statement: *Any verbal law that may be expressed with regard to production from a reservoir in Volumetric Control may, provided it is correct, be reduced to, or derived from, the curves as shown in the charts of Figures 92 and 93.*

We shall have corresponding curves that differ from these for reservoirs in Capillary Control.

**114.** *The combination reservoir.*—Our type reservoirs for this control are provided for the production of liquid or gas, one to the exclusion of the other, except in so far as the liquid in the solution tank may hold gas or air in the dissolved state by virtue of the pressure of the atmosphere. Naturally this

[14] See § 70.

gas would make itself known by passing into the free state only when production takes place into a vacuum. Now we shall consider a type of tank which of necessity produces liquid and free gas. In its simplest form such a tank appears as in Figure 94. It is of the closed type, with the orifice at the bottom. Upon this orifice the liquid bears its weight $h$ per unit area, and the gas under compression exerts a pressure of intensity $p$ upon the free surface of the liquid. The potential pressure at $O$ during the course of production is always $P = h + p$, since the constant back pressure is composed only of the pressure of the atmosphere. As in our earlier analyses of reservoirs we have the usual four horizontal lines: $K$, the line representing absolute zero pressure; $J$, the line representing the atmospheric pressure; $N$, now coinciding with $J$, representing potential zero pressure; and $I$, the line representing, by means of its distance from $K$, the static pressure of the reservoir. Together these lines define the following pressures:

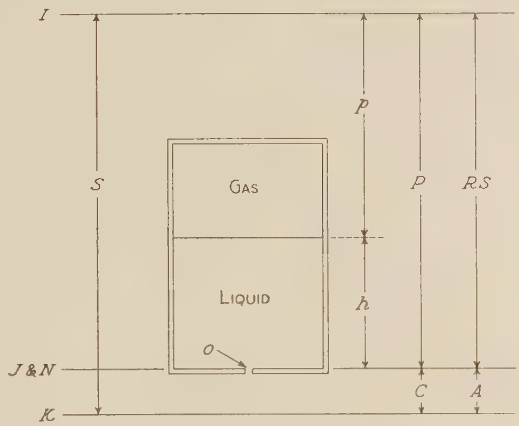

FIG. 94

$S$, the static pressure of the reservoir,
$A$, the pressure of the atmosphere,
$RS$, the registered static pressure,
$C$, the constant back pressure,
$P$, the potential pressure, and
$RC$, the registered constant back pressure,

the last one being zero in the present figure. (These pressures were discussed in sections 62 and 71.)

During the process of production these pressures behave precisely as in Figure 76. This tank is a reservoir in Volumetric Control, and the mechanics are the same as in the cases of the type reservoirs. The primary functions decline in the manner shown in Figure 92; $Vo$ now refers to the liquid, and $E$ to the volume of gas remaining within the reservoir and yet to be produced in accordance with $P$. Furthermore, $Ve$ refers to the rate at which the liquid is produced, and $Po$ the rate at which the gas is produced, from the orifice.

There are three possible cases to be noted in connection with a reservoir of this sort:

*a*) *There is insufficient gas to expel all the liquid.* When equilibrium is established the free surface of the liquid stands above the level of the orifice. This position actually locates the line $N$ above the position shown in the figure. All primary functions make a full sweep through their respective curves, continually maintaining the same values for the $K$'s.

*b*) *The quantity of gas present is just sufficient to expel all the liquid; there is neither a deficiency nor an excess.* This is the condition assumed in the present figure. All primary functions make a full sweep through their respective curves, continually maintaining the same values for the $K$'s.[15]

*c*) *The quantity of gas present is more than sufficient to expel the liquid.* The line $N$ is now located below the position shown in the figure. The curves for all the primary functions approach a definite point on the horizontal, or time, axis, but before this point is reached they "break." This happens at the instant all liquid has left the reservoir. Immediately the tank ceases to be a combination reservoir and assumes the status of the ordinary gas tank, producing gas alone. New values for the $K$'s are assumed at the instant of the change.

The tank shown in Figure 95 is somewhat more complicated. The orifice is here at the bottom of a tube which extends downward through the top. In

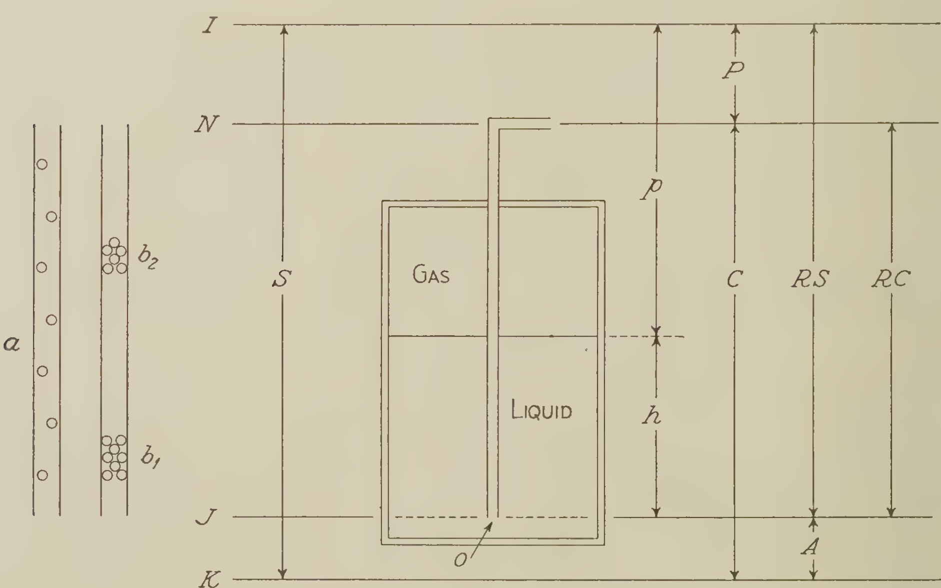


FIG. 95

[15] We are here considering the tank as a reservoir in the physical sense. If we limit our consideration to the potential reservoir, as defined in § 19, only item (*b*) holds true. The amount of gas determines precisely the size of the potential reservoir within any given tank of this sort. As we know, we learn of the size of this potential reservoir by measuring the area subtended by the velocity-time curve for the liquid.

the same manner as before we have $h$ and $p$. The constant back pressure is composed of the weight of the fluid in the tube, plus the atmospheric pressure. The registered constant back pressure is not zero; therefore $J$ and $N$ do not coincide.

While the line $N$ is given a definite position in this figure, its location may not be easily determined. Here it has its maximum height above the line $K$ on the supposition that the bubbles of free gas are sufficiently separated to permit the liquid to exert a full pressure for its height against the orifice at the bottom of the tube. The column actually weighs less on account of the displacement due to the presence of the bubbles, but the pressure is the same regardless of this fact. The situation is shown in the figure at $a$.[16] But if the bubbles are able to form a "bridge" within the column, either at points as $b_1$ and $b_2$ or throughout the entire height, the pressure at the orifice is partially and perhaps almost entirely, diminished in its intensity.[17] $N$ takes a position accordingly, somewhere between its present location and that of the line $J$.

The tank in Figure 95 acts on the principle of the siphon-bottle provided with carbonated water. *It is a reservoir in Volumetric Control, as we must know, and the behavior of a natural reservoir cannot be compared with its behavior unless the natural reservoir is in the same control.*[18]

During the course of production from either of the combination tanks, we have now considered gas as liberated from solution in virtue of the decline in the static pressure.[19] This free gas immediately passes upward through the remaining liquid, and collects in what we may term a "pocket." The fact that there is liquid with its gas below the level of the orifice in the second tank is immaterial in so far as the mathematical properties of the curves of decline are concerned. The reservoir simply behaves as if this liquid were not present, and at the same time, as if the gas were more soluble in the liquid than it actually is. In other words, the laws of delivery are the same, and they are fulfilled in the same precise manner, whether such liquid with dissolved gas is or is not present. Merely certain constants $K$ in the gas equations for the reservoir possess values which depend upon its presence or its absence, in

[16] The situation is a simple one in accordance with the principles of hydrostatics: *The intensity of pressure exerted by liquid on the bottom of its container is dependent upon the height of the liquid in the container, and independent of the shape of the container.* Here the separate bubbles have the effect of altering the shape of the container. In their presence the column of liquid is not cylindrical.

[17] The phenomenon is clearly that of the gas lift.

[18] Specifically, the behavior of natural reservoirs in Capillary Control cannot be compared with the behavior of the siphon-bottle. Events and conditions in the two reservoirs are extremely different.

[19] The two tanks are obviously reservoirs of the closed type. Reservoirs of the open type may likewise be constructed. As we know, the mechanics of the two types of combination reservoirs are the same, regardless of the fact that the rôle of the gas, with respect to the energy of the systems, is different. (See § 65.)

addition to their general dependence upon the solubility of the gas in the liquid.[20] If the reservoir were inaccessible, except at the orifice in so far as we may reach it through the tube, we would never know from its performance whether such liquid exists or not.

The two tanks possess energy due to a twofold source. Both the liquid and the gas exert their pressures against the orifice. However, if these tanks were very shallow and very extensive laterally, and if they were maintained perfectly horizontal and equipped with orifices consisting of perforated tubes which extend from top to bottom, the weight of the liquid is not to be reckoned with. The energy may then be said to be due solely to the pressure of the gas.[21]

**115.** *Proportional production of gas.*—The amount of gas dissolved in the liquid, and therefore the amount of gas produced with the liquid from an ideal reservoir, is dependent upon the static pressure of the reservoir in accordance with Henry's Law. Is it not clear, then, that if the static pressure continuously diminishes during the process of production from a reservoir—a circumstance the truth of which is unquestioned in regard to this control—the amount of gas per unit volume of liquid issuing from the orifice must also diminish? And that the amount of gas per unit volume of liquid and the static pressure continually diminish in the same proportion? Unless we have reason to deny Henry's Law, we must accept this argument as the truth.

In Figure 76 we found the path of the static pressure to be represented by the curve *cba* with respect to the absolute, or $X'$, axis. This curve must likewise be the path traveled by the proportional amount of gas to liquid during production from a reservoir in this control. At any time during the process of production we can measure the amount of gas issuing per unit volume of liquid, taking the sole precaution against the possible by-passing of the gas, where the mechanical structure of the reservoir permits it.[22] We are to know the static pressure of the reservoir at the time of the observation, and the conditions under which the gas and liquid are produced and measured. Consequently we may apportion the gas in percentages according to all pressures indicated in Figure 76,[23] conveniently taking $S$ per cent as 100 per cent. We thus have the following:

$S$ per cent, the basis of comparison, representing the proportional amount

[20] See footnote 15, § 73, page 136.

[21] In other words, any so-called "hydrostatic head" is quite negligible.

[22] Thus "normal conditions" (§ 97) are assumed. Where the reservoir is flat, and shallow in comparison with its broad lateral extent, the continual submergence of the orifice is not easily maintained, as it is, without effort, in the reservoirs of Figs. 94 and 95. At a slow rate of production by-passing is at a minimum. Measurements made directly upon production include the factor of degree of saturation and the "as if" behavior of gas and liquid described in the preceding section.

[23] See §§ 73 and 74.

of gas that would be delivered in the free state if production were to take place into a perfect vacuum.

*RS* per cent, the proportional amount of gas delivered in the free state when production takes place into the atmosphere.[24]

*A* per cent, the proportional amount of gas still remaining in solution when the liquid is delivered at atmospheric pressure.

*P* per cent, the proportional amount of gas free from solution immediately at the orifice, in case of production against any constant back pressure *C*.

*C* per cent, the proportional amount of gas still in solution immediately at the orifice, in virtue of the constant back pressure.

*RC* per cent, the proportional amount of gas freed from solution in the flow-line between the orifice and final delivery at atmospheric pressure at some downstream point.

The quantity *P* per cent, corresponding to the potential pressure of the reservoir, is important. It varies in its amount directly with the value *P* itself. We may therefore write

$$P\% = (T\%)^2,$$

where *T* per cent represents life remaining. The relative curve for potential pressure and time is also the relative curve for the proportional amount of gas free from solution immediately at the orifice. It is obvious that if we know the value of the pressure *P* and *S*, we can determine the value of *P* per cent gas, since *S* per cent is taken as 100 per cent. In section 74 this variable quantity of gas was called "potential gas." In the tanks of Figures 94 and 95 it is precisely that gas the energy of which is utilized in the process of production. After the fluids reach the orifice of the tank in the latter figure, none of the gas, either already in the free state or about to enter it, can be said to aid in the process of production, except in so far as it may lower the position of the line *N*; that is, except in so far as it may increase the value of the potential pressure by the action of a "gas-lift" within the tube. Now presumably we have properly located *N* for the existing conditions of production; consequently the gas-lift is already taken into account. By declaring the orifice to be located at the bottom of the tube we conveniently, and advantageously, separate the reservoir system into its internal and external portions, and thereby make it possible to distinguish clearly between the action of the reservoir in bringing the fluid to the orifice and the action of contrivances, such as pumps and gas-lifts, which operate upon the fluids in the flow-line exterior to the orifice.[25]

24 For this amount it is immaterial whether production takes place directly from the orifice into the atmosphere or at some downstream point in a flow-line.

25 It is improper to analyze the action of gas without making a clear distinction between events in the internal and external systems of the reservoir.

In discussing the two tanks just cited we made note of the formation of a "gas pocket" above the free surface of the liquid. Let us now refer to Figure 72. We shall suppose the liquid within the annular tank to be saturated at a high pressure and $g$, $q$, and $l$ completely closed. The system is now capable of producing fluid according to the laws of Volumetric Control. Let there be one standpipe open for production at $W_2$. If we say that the atmospheric line is $J$, passing through $e$, the proportional production of gas from this pipe is perfectly normal; that is, gas percentages abide by the dimensions of Figure 76. A gas pocket must form at the crest of the structure defined by the plates exactly as in the tanks of Figures 94 and 95. But if this reservoir system were inaccessible except at $W_2$, from our observations we should place the atmospheric line at $j_2$, a horizon determined by the true orifice of the standpipe. In this case the proportional production of gas would appear to be perfectly normal, except in the fact that the proportional amount of gas, if actually measured at stated intervals during the course of production, would continually appear to be ahead of the schedule previously determined by the test upon the solubility of the particular gas in the particular liquid; that is, the proportional amount of gas would continually be greater than that prescribed by the static pressure and the solubility of the gas in the liquid. However, we may yet claim that the proportional production of gas is normal, for the percentages of Figure 76 are exactly fulfilled. The gas only appears to be more soluble than it actually is.[26]

Now let there be five standpipes, all producing, as shown in the figure. $W_1$ is unique in its location; it will continually show a greater proportional production of gas—provided it does not produce gas alone—than its neighbors on the lower contour, for it draws from the pocket which they form, or attempt to form. The proportional production of gas from the field at large, however, is perfectly normal, as of the atmospheric line $J$ passing through $e$. As the fluids enter in combination at $e$, so must they be produced from the five standpipes. In fact these five pipes together constitute a multiple orifice.[27]

[26] This is in accord with the "as if" behavior of gas and liquid described in the preceding section. Percentages are presumably based upon observation at $W_2$. Without doubt the inferred analogy between the reservoir of Fig. 72 and the natural reservoir—be it either in Hydraulic or in Volumetric Control—is quite obvious.

[27] For the action of a multiple orifice see § 91. The principles, as there illustrated, apply as well to reservoirs in the present control.

CHAPTER XVIII

# Ideal Performance and Its Primary Functions (Concluded)

"A relative truth, a proposition which is only true on the basis of an arbitrary presupposition, and which deviates from entire reality in a carefully defined sense—just such a proposition is incomparably more capable of permanently advancing our comprehension than a proposition which endeavors at one stroke to come as close as possible to the nature of things, and in doing so carries with it an inevitable and, in their full range, unknown mass of errors."—F. A. LANGE

**116.** *Gas-time relations.*—According to section 74 the gas velocity, or the rate at which gas is produced from a combination reservoir, is equal to the proportional production of gas multiplied by the velocity for the liquid.[1] Where, as only in Hydraulic Control, both of the latter quantities are constant in their values throughout the course of production, it is a simple matter to determine, and thereafter reckon with, the relations between gas velocity and time. Now in Volumetric Control these quantities are varying with time, the first with respect to an absolute axis $X'$, and the second with respect to a potential axis $X$. To deal with gas velocity and time relations properly we must not fail to take this variation in a twofold manner into account. No difficulty arises in complying with this necessity, for if we will refer once more to Figure 76 we see that the static pressure $S$ may be conveniently divided into two parts by the potential zero, wherever this may be located on the absolute scale of pressures. Our potential axis $X$ is given its location in accordance with this potential zero, and from this axis we can measure both $P$ and $C$, upward as positive for the former and downward as positive for the latter. At the same time we are dividing the proportional amount of gas, 100 per cent corresponding to $S$, into two significant parts, the upper one denoting $P$ per cent variable in the course of production, and the lower one denoting $C$ per cent constant in the course of production.[2]

[1] Qualitative and quantitative definitions are identical in Hydraulic Control, where the $K$'s are unity and the subtended areas are rectangles. In the finite controls the definitions are different, for the $K$'s are not unity and the subtended areas are not rectangles. The present definition is purely a qualitative one. (See footnote 18, § 108, p. 268.)

[2] $C$ and $C$ per cent have their usual components $RC$ plus $A$ and $RC$ per cent plus $A$ per cent. To abbreviate our investigation we shall not give the latter quantities their full consideration. While we may regard $A$ as a special value of $C$, for production directly

The variable and constant quantities with which we must reckon are shown graphically in Figure 96. They are, as we see, pressures and velocities, which, by multiplication, become powers, or rates of displacement or accumulation of energy in the process of production from the reservoir system. Thus we have

$Ve$, the potential velocity for the liquid,

$P$, the potential pressure of the reservoir,

$C$, the constant back pressure exerted against production, and

$ve$, a velocity for the liquid corresponding to $C$.

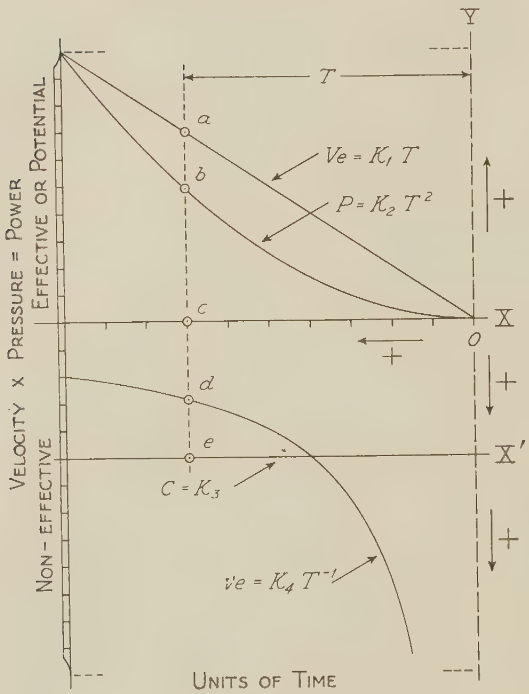


Fig. 96

The last of these requires special consideration.

In section 74 the conceptual idea of velocity $ve$ was introduced, a velocity which in time would give an accumulated volume $vo$ within the reservoir. Now in Volumetric Control we must understand clearly the significance of both of these quantities. If we will refer to Figure 66, and view it in reference to this control, we can say that $R$ or $r$ represents the initial or subsequent radius of the reservoir, that the fluid above $N$ is $Vo$, and that the fluid below $N$ is $vo$. Obviously $vo$ is a definite quantity of fluid to be retained by the reservoir at the time equilibrium is established in accordance with $C$. Its amount can be computed from the simple relation[3]

$$\frac{P}{Vo} = \frac{C}{vo} \quad \text{.......................} \quad (217)$$

that is,

$$vo = \frac{CVo}{P} \quad \text{.....................} \quad (218)$$

from the orifice into the atmosphere, it most frequently happens that we have both $C$ and $A$ with which to reckon, for production against a load greater than $A$, yet a subsequent, final delivery at $A$.

[3] The equation following is in agreement with pressure-volume relations in Volumetric Control.

According to our conception we are to approach a reservoir at any instant during its life, and imagine a volume $vo$ accumulated within this reservoir during time remaining in life. Then $ve$, the rate of accumulation, will have a value in accordance with

$$ve \text{ varies as } \frac{1}{T} \quad \ldots\ldots\ldots\ldots (219)$$

where $T$ is the time in which $vo$ is to be accumulated. By introducing a constant $K$ into this variation we may write it as an equation. Thus

$$ve = \frac{K}{T} \quad \ldots\ldots\ldots\ldots (220)$$

where the value of $K$ is indeed equal to the value of $vo$, a fact which may perhaps be more evident if we write the equation as follows:

$$veT = K \quad \ldots\ldots\ldots\ldots (221)$$

At whatever instant we begin our reckoning with the performance of the reservoir, $ve$ is such that, if multiplied by $T$, a volume $vo$ will be accumulated within the reservoir, to be retained at equilibrium.[4] The curve for these equations is a rectangular hyperbola.[5]

There are four curves, then, in Figure 96. All are drawn with respect to the axes $X$ and $Y$, with the positive directions for measurement indicated by arrows. The values of the four functions, at any instant during production, are to be multiplied together for the purpose of obtaining the four following terms:

$$PVe, \quad CVe, \quad Pve, \quad \text{and} \quad Cve.$$

[4] Of course Equation 221 is the same as $ve = KT^{-1}$.

[5] It is to be observed that the value of $ve$ is determined by the hyperbola, and that when once determined it is thereafter to be treated as a constant. The hyperbola represents the loci of all possible values for this constant $ve$. It is not a path traveled by $ve$ in the sense that the straight line $Ve = KT$ is the path traveled by $Ve$. In Fig. 116 (p. 340) and in Fig. 171 (p. 483) we have hyperbolic loci in the curves $A$ determining the values of the variable function $Ve$ in Case 1, theoretic performance, for the two finite controls. Perhaps in these figures the nature of hyperbolic loci is more readily observed.

Equation 221 may be written as follows: $veT = vo$. It is of interest to note the meaning of this expression in Hydraulic Control. We there find that $vo$ is finite, as determined by $C$, and that $T$ is infinite in accordance with the control; therefore $ve$ is zero to satisfy the equation. For any interval of time short of infinite time Equations 81, 82, 83, and 84 (p. 141) have their two last terms equal to zero. It follows, then, that Equations 89, 90, 91, and 92 (pp. 141–42) have their last term equal to zero, while Equations 93 and 94 vanish completely. (See footnote 23, § 74, p. 142.) As a result of these zero quantities for finite time Hydraulic Control possesses the simplicity previously referred to, but ignored, in § 74. A reflection of this simplicity is observed in connection with the apportionment of the gas, inasmuch as this apportionment can be made strictly in accordance with $S$, $P$, and $C$. This cannot be done in the finite controls. See the text to follow in the present section.

For example, at the instant denoting $T$ in time remaining, we have the following ordinates multiplied in couplets:

$$cb \times ca, \quad ce \times ca, \quad cb \times cd, \quad \text{and} \quad ce \times cd.$$

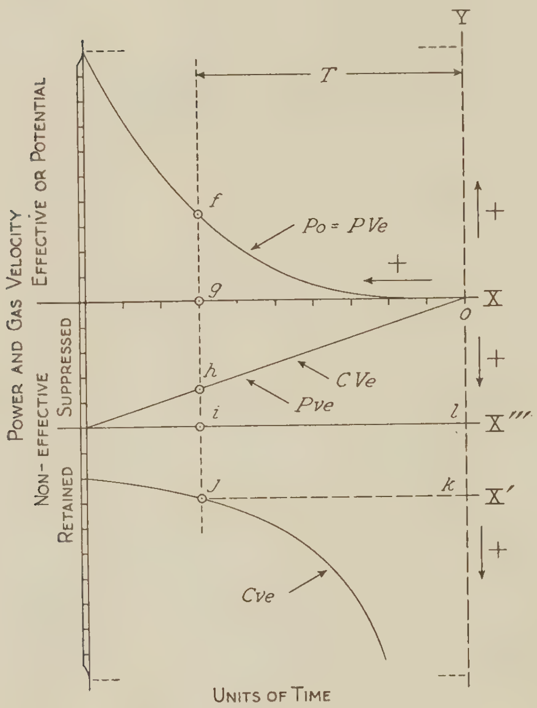


Fig. 97

From these we have the data for Figure 97, wherein

$$cb \times ca = gf$$

$$ce \times ca = gh$$

and

$$cb \times cd = gh$$

as measured from the axis $X$. For the amount

$$ce \times cd = ij$$

it is advantageous to introduce a horizontal axis $X'''$, corresponding to the horizon $G$ in Figure 33.[6] As we already know, the $X$ axis corresponds to the horizon $N$, and the $X'$ axis corresponds to the horizon $K$.[7]

The subtended areas in Figure 97 have the following significances:

*Ogf* represents *PVo*

*Ogh* represents *CVo*

*Ogh*[8] represents *Pvo*

and

*lijk* represents *Cvo*

[6] If desired, the same axis can be placed in Fig. 96.

[7] The position of $X'$ shifts in accordance with the position of $j$. We are to remember that our scheme for representing pressures by diagram is a purely artificial one. The shifting of $X'$ here has no significance with respect to the performance of the reservoir other than a conceptual one in agreement with *ve*.

[8] The line *Oh* represents two curves; consequently $h$ represents two separate points, say $h_1$ and $h_2$. In analytical work, where no numerical values are given, we can see as many curves as we like in one line. There is no reason for separating them. If we did, we would only make the figure more complicated in appearance.

The energies on the right are those of Figure 33. The area *lijk* is constant, as it must be, regardless of the duration of time $T$.[9]

In accordance with the four terms in power we have gas velocities, and in accordance with the four terms in energy we have gas volumes.[10] The curves and subtended areas in Figure 97 show the relation between gas velocities and gas volumes on the one hand, and time on the other.

By a definite procedure in section 74 we obtained a set of equations for the relations between gas and time in Hydraulic Control. Let us now do the same for Volumetric Control. We begin with the following four equations:[11]

$$Ve = K_1T \qquad (222)$$

$$P = K_2T^2 \qquad (223)$$

$$C = K_3 \qquad (224)$$

and

$$ve = K_4T^{-1} \qquad (225)$$

We again take for convenience $Ve + ve = 100\%$, and $P + C = 100\%$, for the apportionment of the corresponding percentage values of the gas involved in the process of production. As before, we shall omit the sign for per cent in our equations.

In the finite controls we cannot apportion the gas directly in accordance with the percentage relations between $S$, $P$, and $C$. As a matter of fact, if we take $S$ per cent of the gas as 100 per cent, the observed proportional production of gas will be greater than 100 per cent, but this is not to be regarded as an absurd situation. We already know that quantities greater than 100 per cent merely imply a selection of a standard base for comparison at values smaller than the greatest possible values.[12] How shall we then apportion the gas? I believe the method can best be understood by considering an illustrative example that is to follow the development of the gas-time equations for this control. Meanwhile we will assume that we can write down the four equations with respect to the proportional production of gas to liquid, as follows:

$$G_{Pp} = K_2T^2 \qquad (226)$$

$$G_{Cp} = K_3 \qquad (227)$$

$$G_{Sp} = K_2T^2 + K_3 \qquad (228)$$

and

$$G_{RSp} = K_2T^2 + K_5 \qquad (229)$$

[9] Rather, the amount of energy represented by the area is constant. Different horizontal and vertical scales, and different units of measurement in pressure, volume, and time, will cause the area to alter while the amount of energy remains constant.

[10] See § 73.

[11] The equations appear here in the same order as in §§ 74 and 163, where we have the gas-time equations for Hydraulic and Capillary controls, respectively.

[12] This is evident in the fact that we may extend the relative curves of Figs. 92 and 93 upward to the left as far as we desire.

in which all quantities are such as explained in section 74. These equations now show that the potential gas and the total gas vary with time in the process of production, while the gas in the dissolved state immediately at the orifice is constant with time.

The total power—the total rate of displacement and accumulation of energy—involved in the process is

$$Po_a = PVe + CVe + Pve + Cve \quad \ldots\ldots\ldots\ldots\ldots (230)$$

The terms on the right are those given on page 291. By means of Equations 222 to 225, inclusive, we may write

$$Po_a = K_1K_2T^3 + K_1K_3T + K_2K_4T + K_3K_4T^{-1} \quad \ldots\ldots (231)$$

The integration of Equation 230 with respect to time gives

$$E_a = PVo + CVo + Pvo + Cvo \quad \ldots\ldots\ldots\ldots\ldots (232)$$

as we know. Again, the integration[13] of Equation 231 gives

$$E_a = \tfrac{1}{4}\, K_1K_2T^4 + \tfrac{1}{2}\, K_1K_3T^2 + \tfrac{1}{2}\, K_2K_4T^2 + K_3K_4 \quad \ldots\ldots (233)$$

From these four equations all gas velocity and gas volume equations are derived.

We have

$$Po = PVe \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (234)$$

and

$$G_{Ve} = K_1K_2T^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (235)$$

The latter shows the relation between the rate of production of potential gas and time. We also have

$$E = \tfrac{1}{2}\, PVo \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (236)$$

taking into account the previously determined constant that is required, as between energy, pressure, and volume, in this control. To this corresponds

$$G_{Vo} = \tfrac{1}{4}\, K_1K_2T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (237)$$

This is the relation between the cumulative production of potential gas and time.[14]

[13] In reality we cannot say that the last term is obtained by true integration. The last term of Equation 231 is simply multiplied by $T$. Thus

$$\frac{K_3K_4}{T} \times T = K_3K_4$$

For the sake of brevity we shall continue to speak of integrating the terms as they appear in Equation 231, to obtain those as they appear in Equation 233.

[14] Equation 237 may be obtained either by integrating Equation 235 with respect to time, or by substituting for $P$, the value $K_2T^2$, and for $Vo$, the value $\tfrac{1}{2}\, K_1T^2$, the latter being derived from $Vo = \tfrac{1}{2}\, VeT$, as explained in § 105. The cumulative production curve is the inverted volume-time curve. (See § 104.)

The second and third terms on the right in Equations 230 and 231 represent, when taken together, *suppressed power,* because of their relation to the same terms in Equations 232 and 233, the latter representing *suppressed energy*.[15] Accordingly we have

$$Po' = CVe + Pve \quad \text{.......................} (238)$$

$$G'_{Ve} = K_1K_3T + K_2K_4T \quad \text{..................} (239)$$

$$E' = CVo + Pvo \quad \text{.......................} (240)$$

and

$$G'_{Vo} = \tfrac{1}{2} K_1K_3T^2 + \tfrac{1}{2} K_2K_4T^2 \quad \text{.............} (241)$$

Equations 239 and 241 refer to *suppressed gas.* It is produced with the potential gas, but the power and energy to which they pertain are themselves suppressed, since they are prevented from performing useful work in the process of production.[16]

In the sum of Equations 235 and 239, and likewise in the sum of Equations 237 and 241, we have the total quantities of gas issuing from a combination reservoir, as we would measure them without apportionment. In each case the three quantities appearing on the right would be measured as one quantity.

As a certain amount of energy remains within the reservoir in virtue of the constant back pressure, so also a certain amount of gas remains there. In accordance with the last terms of Equations 232 and 233 these are

$$E'' = Cvo \quad \text{........................} (242)$$

and

$$G''_{Vo} = K_3K_4 \quad \text{......................} (243)$$

respectively. The corresponding rate equations may be written from the last terms in Equations 230 and 231. This energy and gas is not available under the given conditions of production. They can only be produced from the reservoir by reducing $C$ to zero value.[17]

As an illustrative problem in the use of our gas-time equations, let us consider a combination tank, such as the one in Figure 95, with the following data at hand:[18]

At 12:00 M.:

$$\begin{aligned} P &= 50 \text{ units of pressure} \\ RC &= \ \ 9 \text{ units of pressure} \\ A &= \ \ 1 \text{ unit of pressure} \\ \hline S &= 60 \text{ units of pressure} \end{aligned}$$

while

$$Ve = 100 \text{ gallons per hour}$$

[15] See §§ 51 and 57.

[16] As stated previously, by "useful" I refer to the viewpoint of getting the fluid out of the reservoir. (See footnote 4, § 51, p. 92.)

[17] See footnote 23, § 74, page 142.

[18] With respect to the liquid we shall take the data of § 102. The present problem may then be regarded as a continuation of the one of that section.

At 4:00 P.M.:

$$\begin{aligned} P &= 32 \text{ units of pressure} \\ RC &= 9 \text{ units of pressure} \\ A &= 1 \text{ unit of pressure} \\ \hline S &= 42 \text{ units of pressure} \end{aligned}$$

while

$$Ve = 80 \text{ gallons per hour}$$

Furthermore, by observation at 12:00 M. we learn that for final delivery into the atmosphere there are 32.0 cubic inches of gas per gallon of liquid, separation of the fluids having taken place at the pressure of the atmosphere upon their issuance from the tank.

By means of the relative curves we know that at 12:00 M. the life of the reservoir is $L = 20$ hours, and by means of the relation between velocity and volume in this control we also know that $Vo = \frac{1}{2} \times 100 \times 20 = 1{,}000$ gallons, the amount of liquid to be produced at the given constant back pressure.

The four specific equations for production from the tank are as follows:

$$Ve = 5T \qquad (K_1 = 5)$$
$$P = \tfrac{1}{8} T^2 \qquad (K_2 = \tfrac{1}{8})$$
$$C = 10 \qquad (K_3 = 10)$$

and

$$ve = 200T^{-1} \qquad (K_4 = 200)^{19}$$

By addition Equations 235 and 239 become

$$G_{Ve} + G'_{Ve} = K_1K_2T^3 + K_1K_3T + K_2K_4T$$

Into this we substitute the values of the $K$'s and the value of $L = 20$ for $T$. Thus

$$G_{Ve} + G'_{Ve} = (\tfrac{5}{8} \times 8{,}000) + (50 \times 20) + \left(\frac{200}{8} \times 20\right)$$
$$= 5{,}000 + 1{,}000 + 500 = 6{,}500$$

[19] We have

$$\frac{P}{Vo} = \frac{C}{vo}$$

Therefore at 12:00 M.,

$$\frac{50}{1000} = \frac{10}{vo}$$

or $vo = 200 = K_4$. Since $veT = vo$, it follows that at 12:00 M. $20ve = 200$ gallons, or $ve = 10$ gallons per hour. Again, at 4:00 P.M.,

$$\frac{32}{640} = \frac{10}{vo}$$

or $vo = 200 = K_4$. Consequently $ve = 12.5$ gallons per hour. But we have no need for these values of $ve$. Our conceptual $ve$ vanishes in practical applications. We use only the value of $K_4$.

These are units of power and gas velocity pertaining to $Ve$ for the liquid at 100 gallons per hour. Obviously there are 65 units pertaining to each gallon of liquid.[20]

The amount of gas issuing from the tank, 32.0 cubic inches per gallon of liquid, does not refer to the constant back pressure $C$, but in fact to the pressure of the atmosphere $A$, less than $C$ by the value of $RC$. To meet these conditions we have an auxiliary specific equation for production, as follows:

$$RC = 9 \qquad\qquad (K'_3 = 9)$$

By using the value of $K'_3$ in place of $K_3$ in the gas velocity equation the second term becomes 900 instead of 1,000, and the total of the three terms becomes 6,400 instead of 6,500. We may therefore make the following computation:

$$\frac{6{,}500}{6{,}400} \times 32.0 = 32.5$$

Had the fluids, on final delivery into the atmosphere, been separated in a perfect vacuum, and the gas then measured at atmospheric pressure, the proportional amount of gas thus measured would have been 32.5 cubic inches per gallon of liquid.[21] This is the amount which is associated with the gas velocity equation above. With it the various terms are measured on the same basis, and they may therefore be placed in comparison. The most important comparison is the following:

32.5 cubic inches = 65 units of gas velocity

From this we know that

0.5 cubic inch = 1 unit of gas velocity[22]

It is now a simple matter to apportion the gas, as of 12:00 M. From the equation we have

2,500 cubic inches = potential gas velocity
750 cubic inches = suppressed gas velocity
3,250 cubic inches = total gas velocity

[20] This value holds for the one instant at 12:00 M. We do not need to state the dimensions of these units. We know, however, that the unit of power can be reduced to foot-pounds per hour. (See footnote 9, § 15, p. 23.)

[21] Of course we cannot separate the fluids at abolute zero pressure, but we can observe the separation on successive steps toward this pressure, and extrapolate for the separation at this pressure. Procedure of this kind is frequently followed by the physicist.

[22] With one stroke we here wipe out all reference to corresponding percentage values between the various functions with which we deal. Applications often permit abbreviated forms of calculation that are prohibited in analysis. We do not lessen the importance of percentage values by striking them out in a problem of this sort.

Thus 2,500 cubic inches of gas are actually doing useful work in the production of fluid, while 750 cubic inches are escaping from the tank without the performance of this type of work.[23]

We furthermore have the data with which to write the following:

$$G_{Pp} = \frac{1}{16} T^2$$

$$G_{Cp} = 5$$

$$G_{Sp} = \frac{1}{16} T^2 + 5$$

$$G_{RSp} = \frac{1}{16} T^2 + 4.5$$

The first two equations are written in accordance with the first and second terms in the gas velocity equation. The third and fourth follow from the preceding ones, including the circumstance that $RC$ is nine-tenths of $C$. These are equations in actual units of gas.[24] They are not mere percentage equations.

By addition Equations 237 and 241 become

$$G_{Vo} + G'_{Vo} = \tfrac{1}{4} K_1K_2T^4 + \tfrac{1}{2} K_1K_3T^2 + \tfrac{1}{2} K_2K_4T^2$$

Into this we substitute the values of the $K$'s and the value of $T = 20$. Thus

$$\begin{aligned} G_{Vo} + G'_{Vo} &= (\tfrac{1}{4} \times \tfrac{5}{8} \times 160{,}000) + (\tfrac{1}{2} \times 50 \times 400) + (\tfrac{1}{2} \times \frac{200}{8} \times 400) \\ &= 25{,}000 + 10{,}000 + 5{,}000 \\ &= 40{,}000 \end{aligned}$$

[23] If the combination tank were one of the open type instead, we should say that the energy possessed by the liquid, to an amount equivalent to 750 cubic inches of gas, is escaping from the tank without the performance of this type of work. In the reservoir of the open type the gas has no energy intrinsically its own. It is not the gas, but the column of liquid, that performs the work of production.

[24] At 12:00 M.,

$$G_{Sp} = \frac{1}{16}(20)^2 + 5 = 30 \text{ cubic inches per gallon.}$$

The observed amount was 32 cubic inches per gallon. Then if $G_{Sp}$ is taken at 100 per cent, the actual amount observed is

$$\frac{32}{30} \times 100\% = 106\tfrac{2}{3}\%$$

a quantity greater than 100 per cent, as previously mentioned in this section. As a matter of fact, $G_{Sp}$ per cent is not identical with $S$ per cent in the finite control, unless production takes place into a perfect vacuum. They are always identical in Hydraulic Control, inasmuch as the conceptual velocity $ve$ is there zero for any specified finite time.

These are units of energy and gas volume pertaining to $Vo$ for the liquid at 12:00 M.

The gas can now be apportioned in the following manner:

| | |
|---|---|
| 12,500 cubic inches | = potential gas volume |
| 7,500 cubic inches | = suppressed gas volume |
| 20,000 cubic inches | = total gas volume |

We already know that the liquid retained by the tank at the time of equilibrium is $vo = 200$ gallons. The energy retained at this time is

$$Cvo = K_3K_4 = 2{,}000 \text{ units};$$

therefore the amount of gas retained is 1,000 cubic inches.

Now we know the state of the tank as a combination reservoir at 12:00 M. If we wish to know the state at any subsequent instant we need only substitute the corresponding value of $T$ into the gas velocity and gas volume equations. For example, to know the state at 4:00 P.M. we should substitute the value $T = 16$ into the equations. The difference between the gas volume at 12:00 M. and at 4:00 P.M. represents the volume of gas that has been produced with the liquid in the interim. All gas quantities, either in velocity or in volume, for any subsequent instant in the life of the reservoir, can be changed from the basis of $C$ to that of $RC$.

While our analysis has proceeded on the assumption that the tank is one as shown in Figure 95, we can now say that it applies as well to such a reservoir system as shown in Figure 72. In considering production from the separate orifices, or from the group taken as a multiple orifice, no modifications are necessary, provided gas production is "normal," a condition defined in section 97.

If for Volumetric Control we confine our attention to energy and gas displaced or produced from the reservoir, the equations between power and gas velocity, on the one hand, and time, on the other, possess the following form:

$$\left.\begin{array}{r}\text{power}\\ \text{gas velocity}\end{array}\right\} = k_1t^3 + k_2t$$

Similarly, the equations between energy and gas volume, on the one hand, and time, on the other, possess the following form:

$$\left.\begin{array}{r}\text{energy}\\ \text{gas volume}\end{array}\right\} = k_1t^4 + k_2t^2$$

For the retained quantities we would add the following terms to the respective forms: $k_3t^{-1}$ and $k_3$. It is evident that the terms do not reduce to the simple form which we found in Hydraulic Control. Now we have the function $t$ with different exponents, and these do not permit a reduction to one term.

In Hydraulic Control we need not discriminate between potential and suppressed functions, but in the finite controls it is essential that we do so.[25]

117. *Paths on producing from reservoirs.*—Let us return now to the performance of our simple type reservoirs in Volumetric Control for the purpose of investigating production from a somewhat different point of view. We have found the paths of the functions to be perfectly mathematical in ideal performance. The reservoir is itself ideal; it is equipped with an orifice the size and physical condition of which remain perfectly constant during the process of production; we open this orifice and "allow Nature to take its course." As a result we see definite paths traveled by the various functions in their successive values, while a state of equilibrium is being approached. I propose to designate this process as *percentage rate production,* in order to distinguish between it and a *constant rate production* that is made possible by a continual or intermittent regulation of the size of the orifice, presumably by manipulating a valve attached thereto.

First, why may the expression "percentage rate production" be applicable to the natural process of production? Let us suppose that we have a reservoir the orifice of which is fitted with a valve. Now with this valve fully opened we shall say that the velocity of production would describe the line $aO_2$ in Figure 98. But instead of permitting production to take place in this manner we deliberately close the valve partially, with the result that at the beginning the rate is expressed by the vertical distance $ld$, a definite fractional portion of $la$. The velocity of production describes the line $dO_1$. At any instant during production we might open the valve fully for the purpose of determining the rate at "open flow." These values will lie on the line $aO_1$, as at $a$, $b$, $c$, and so on, and on returning to the partially closed position of the valve we shall find the actual rate always bearing a definite fractional value of the open rate; that is,

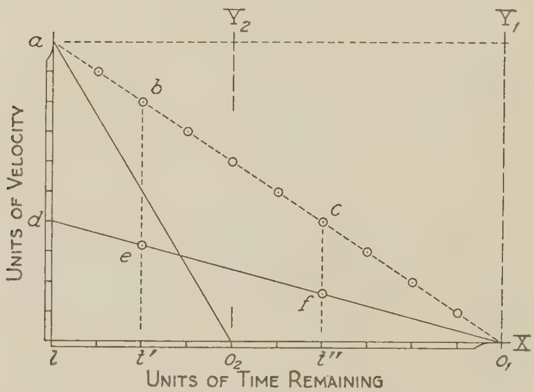


Fig. 98

$$\frac{ld}{la}=\frac{t'e}{t'b}=\frac{t''f}{t''c}, \text{ etc.}$$

*The actual rate is continually an invariable percentage of the rate to be real-*

[25] See also § 163.

*ized by making a definite alteration in either the size or physical condition of the orifice.* It is in fact immaterial whether the alteration would result in a larger or smaller rate than the actual one, for surely if the actual rate were described by $aO_2$, our observations on partially closed flow would describe a line $dO_2$ (not drawn), where similar fractional values, now greater than unity, would be invariable. If this proposition were to be tested in an experiment, the fluid withdrawn during the time of each observation should be returned to the reservoir, and the time occupied by the observations should be separately counted out, in order that the curves may pertain to ideal performance in spite of alterations that really belong to Case 1 in theoretic performance. The return of the fluid would be particularly advisable in case the volume withdrawn during the observations is a considerable proportion of the volume remaining within the reservoir at the time.

*Percentage rate production, regardless of the size and physical condition of the orifice—so long as these are invariable in the course of production—is that form of production which fulfills the relative curves of Figures 92 and 93.*

Constant rate production is frequently dictated by our own needs or by restrictions placed upon us by others. We may require a given amount of fluid per day, per week, or per any other unit of time; and others may only be able to transport, sell, or consume a given amount of fluid per unit of time.[26] We permit our reservoir to produce accordingly. Velocity is constant with time, as shown in Figure 99; it is necessarily less than the maximum percentage rate for a given period of time. Let us suppose our reservoir can begin production at 100 gallons per hour, and decline to zero in 20 hours. Clearly a constant rate cannot equal or exceed 100 gallons for any length of time, for it can in fact produce at that rate for one instant only. At 90 gallons per hour it can produce constantly for a certain length of time, for 50 gallons per hour it can produce constantly for a longer period, and for 10 gallons per hour a still longer period. *It is evident that for any established constant rate there comes a time when the constant rate overtakes*

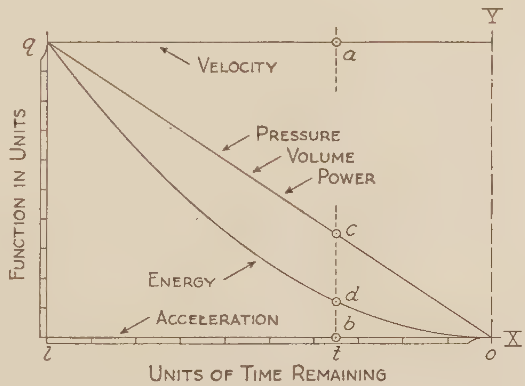

Fig. 99

[26] Percentage rate production and constant rate production are identical in Hydraulic Control; consequently we did not have to discuss the present problem in that control. If, in Volumetric Control, the restrictions on output are based on a "percentage run," of course the problem is merely one in percentage rate production.

*the percentage rate.* This we shall say takes place in the instant $t$, when the velocity curve breaks at $a$ and thereafter travels the usual straight inclined line of decline for normal production.

When the velocity is forced to travel along the straight horizontal line $qa$, as of ideal production at a constant rate, what paths must the other functions of performance travel?

We may at once say that

$$Ac = \text{zero} \quad (244)$$

for there is no change in the velocity. Velocity-time relations themselves are expressed by the equation

$$Ve = K \quad (245)$$

therefore by integration we find that

$$Vo = KT \quad (246)$$

the straight inclined line $qO$ in the figure. $O$ is approached, but not reached. Nevertheless, the fact that it is approached is important, for it permits us to say that the constant of integration in Equation 246 is zero, inasmuch as $Vo$ = zero, when $T$ = zero.

The relation between pressure and volume is not altered in any way; consequently

$$P = KT \quad (247)$$

Its path is made to coincide with that for volume by selecting the proper scales.

Energy is equal to pressure multiplied by volume; therefore

$$E = KT^2 \quad (248)$$

Its path is the parabola. Power is the rate of displacement of energy, and therefore by differentiation we have

$$Po = KT \quad (249)$$

likewise a straight inclined line.[27] In the same manner its path is made to coincide with that for volume.

Now we have the equations for all the functions. All break at the instant $t$, where the vertical line cuts their paths. *Interrelations between pressure, volume, and energy are not affected by this mode of production; their values*

[27] In a particular case, where we have

$$Po = K_1 T$$

by integration we obtain

$$E = \tfrac{1}{2} K_1 T^2$$

The constant $K$ of Equation 248 includes the necessary 1/2.

*are therefore continually in accord with the relative curves.* Since the other functions involve time, the relations between them, as expressed by the relative curves, no longer hold. They are, however, definitely related in a mathematical way, and their corresponding values can be easily determined by computation.[28]

*Given a set of curves in percentage rate production, with dimensions in units, a set of curves for any possible constant rate production, with dimensions likewise in units, can be readily constructed, including the time-location of their break.* The break is located by finding an area subtended by $Ve = K$, for the constant rate production, equal to the area subtended by $Ve = KT$, for the percentage rate production, between the maximum velocity, on the left, and the velocity on decline equal to the constant rate, on the right.

*Conversely, given a set of curves for any possible constant rate production, together with the time-location of their break, the set of curves in percentage rate production can be readily constructed.*

**118.** *Paths on producing into reservoirs.*—A solution tank, or a gas tank, normally of Volumetric Control when in the process of production, may be placed into communication with a reservoir that contains fluid under a greater pressure than that within the tank. Under the circumstances the tank may be said to fill with fluid, or it may be said that production takes place into the reservoir. What paths are traveled by the various functions of performance in such a process?

In section 44 we derived the equation between pressure and time for the problem entitled, "the time required to empty a vessel," and the present problem is precisely the reverse; namely, "the time required to fill a vessel." We understand, of course, that in both of these processes Nature is allowed to take her course, without restriction or interference on our part. On the assumption that the reservoir of higher pressure is in Hydraulic Control we might proceed with the mathematical analysis in the same manner as before. We would, however, derive exactly the same equation between the two functions; therefore let us accept the following equation,

$$P = KT^2 \qquad (250)$$

as set up in section 101, for the relation between pressure and time. As a matter of fact it is immaterial whether the reservoir of higher pressure is in Hydraulic, Volumetric, or Capillary Control. The analysis is simpler if the first is assumed, for no account need be taken of a decline in pressure because of the fluid that leaves the one reservoir to enter the other.

The curve for pressure is shown in Figure 100 (p. 304). It is a parabola with its vertex at the upper right-hand corner of the plat. The point here locates equilibrium for the reservoir, in Volumetric Control, in the process of

[28] The computations are in accord with the equations for the functions.

being filled. The $X$ and $Y$ axes have the positions indicated. Instead of a decline in accordance with percentage rate production we now have an "accline." In both of these processes Nature travels a path toward a state of equilibrium.

From Equation 250, combined with pressure-volume relations for this control, we may at once write down the following:

$$Vo = KT^2 \quad \text{(251)}$$

$$Ve = KT \quad \text{(252)}$$

$$Ac = K \quad \text{(253)}$$

$$E = KT^4 \quad \text{(254)}$$

and

$$Po = KT^3 \quad \text{(255)}$$

These are our normal relations between functions. The curves of accline are the inverted curves of decline. By revolving Figure 100 about the $X$ axis, through 180 degrees, we return to our former curves.

If the receiving tank is of the closed type, the point of equilibrium is attained, provided only that its capacity for withstanding the strain is not surpassed. A receiving tank of the open type, when being filled with liquid, must be of sufficient height in order that the point of equilibrium may be attained. If it is not, it overflows; the curves break as indicated at some instant $t$ in the figure.

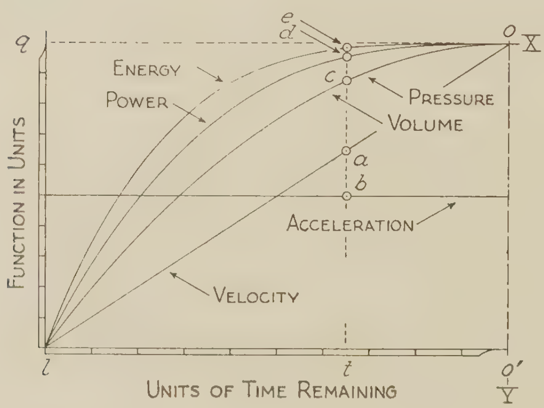


Fig. 100

The relative curves cannot be used for a process of filling in this manner, unless the position of the $X$ axis is known in advance; that is, unless the final pressure to be attained is known.

Let us suppose now that an operator fills a reservoir of this control. He can do so by means of some sort of a pump. We shall at first imagine that the pump runs at a constant speed; consequently we may say that

$$Ve = K \quad \text{(256)}$$

a straight horizontal line when plotted, as shown in Figure 101. By integration we obtain

$$Vo = KT \quad \text{(257)}$$

the constant of integration being zero, inasmuch as $Vo =$ zero when $T =$ zero.

By differentiating Equation 256 we have

$$Ac = \text{zero} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (258)$$

Pressure-volume relations are as usual; consequently we may write

$$P = KT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (259)$$

Since energy is the product of pressure and volume,

$$E = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (260)$$

This equation by differentiation becomes[29]

$$Po = KT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (261)$$

These equations are precisely the same as the ones we found for constant rate production. Figure 101 is in fact the image of Figure 99 in a mirror. The $Y$ axis formerly at the right is now at the left. It is interesting to note that in this process of filling at a constant velocity the functions take on values that are farther and farther away from the point denoting equilibrium. *When Nature's way prevails, equilibrium is approached, and when the operator's way prevails, equilibrium is left behind.* The vertices of the curves always remain at the point of equilibrium.

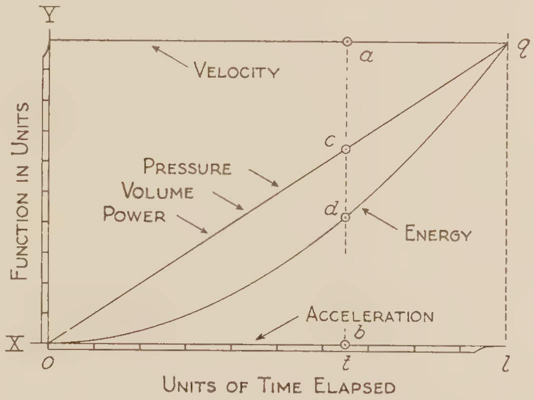


FIG. 101

If the reservoir being filled in this manner is a solution tank, there will of course come a time when the liquid begins to run over. In this case the curves break in the manner indicated, at values determined by a vertical line through $t$. It may be that with either an open solution tank or a closed gas tank the capacity of the plant furnishing energy to the pump is reached. This capacity cannot be exceeded, and therefore the curves at some instant $t$ in Figure 101 break into those of Figure 102 (p. 306). The starting-points on these new paths which pertain to production into a reservoir at constant power may rest on a vertical line anywhere between zero at the left and, let us say, $l$ on the right, the latter being at an undefined distance from zero. We may imagine $t_1$ to determine such a vertical line, and say that $t_1c'$, according

[29] In agreement with the following equation there is a constant 1/2 included in $K$ of Equation 260.

to the scale for Figure 102, represents the same values for pressure and volume as $tc$ on its scale in Figure 101.

The equations for the paths traveled by the functions under the new conditions are readily obtained. First we have

$$Po = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (262)$$

By integration this becomes

$$E = KT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (263)$$

where once more the constant of integration is zero.[30] Now this equation must be separated into its constituents of pressure and volume. The two exponents $T$ must be alike, and their sum must be one; obviously each has a value of one-half.[31] Now we may write

$$P = KT^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (264)$$

and

$$Vo = KT^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (265)$$

By differentiating the latter we obtain

$$Ve = KT^{-1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (266)$$

the equation of a curve belonging to the family of hyperbolas whose asymptotes coincide with the axes for co-ordinates. The fact that the velocity is infinitely great when $T$ is zero, as we see by the equation, is in agreement with the system of mechanics we are constructing. *The expenditure of a finite amount of energy, at any specified rate, against a zero load, should, according to our fundamental principles and definitions of the functions, result in an infinite velocity.* In practice we are, of course, only concerned with the portions of the curves to the right of a vertical line through $t_1$.

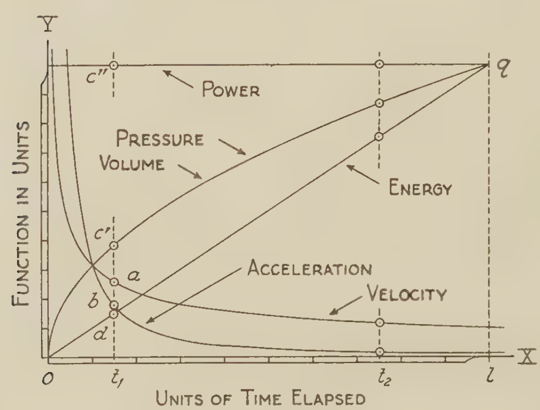


Fig. 102

[30] Equation 263 has no constant 1/2 in its $K$. Energy is represented by a rectangle subtended by the power-time curve, this curve being a straight horizontal line.

[31] The exponents must be alike because of pressure-volume relations in this control. In the equation $P = KVo$, $P$ and $Vo$ under any circumstances must have the same exponent with $T$, so long as this equation itself is correct. It is always correct in Volumetric Control.

Acceleration-time relations are obtained in the usual manner. By differentiating Equation 266 we have

$$Ac = -KT^{-3/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (267)$$

Obviously we may also write

$$-Ac = KT^{-3/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (268)$$

As with velocity the exponent is negative; the curve belongs to the family of hyperbolas.

When an operator fills a reservoir, time elapsed is the function of performance. Now we see in the figure that values of velocity decrease while values of $T$ increase. This is the meaning of the negative sign before $K$ in Equation 267. Values of pressure, volume, and energy increase when $T$ increases; consequently equations obtained by differentiating the equations for these functions do not have a negative sign before $K$. Values of power neither increase nor decrease; the derivative of its relation with $T$ is zero, without positive or negative sign.

Equation 268 shows us that acceleration is negative. Its value increases, and approaches zero value, while $T$ increases.[32] This follows from the fact that the derivative of its equation with time bears a positive sign. To be mathematically exact the acceleration-time curve should be placed by itself, below the $X$ axis. I have put it in the quadrant with the other curves merely as a matter of convenience.

The hyperbolic curves in the figure are exaggerated in the vertical direction for the purpose of more clearly displaying the properties of variation in these functions. All ordinates for velocity have been multiplied by 20, and all for acceleration have been multiplied by 400.[33]

Production into a reservoir at constant power will continue indefinitely, so long as energy is supplied at the pump, and so long as no part of the pumping system ruptures. In case of the latter all curves break, as indicated at $t_2$.

*Given a set of curves as in Figure 101, with facts necessary to locate the vertical position of either* c *or* d, *from which the time-location of* t *may be known, the set of curves as in Figure 102 may be readily constructed, and the figure will include the definite time-location of* $t_1$ *with respect to zero at its left.* The converse procedure likewise may be carried out.

**119.** *Determination of pressure.*—We have seen that when Nature fills a reservoir, the pressure within this reservoir takes on successive values that describe the parabolic path $lcO$ in Figure 100. Now the casing within a well,

[32] That is to say, the velocity approaches a constant value as $T$ increases.

[33] The curves showing the paths of the functions in this text are constructed by means of computed points, and those curves appearing in any one figure are ordinarily consistent, being in accord with a definite set of assumed values for the constants $K$ in the velocity-time and pressure-time equations associated with the functions.

when closed at the top, acts precisely as a rigidly constructed gas tank, and when it is left open, it acts as a solution tank, both of which are type reservoirs for this control. *If we consider it inadvisable to stop flow completely from a well, or if we do not wish to wait the necessary length of time for the final pressure to be registered, we may make use of the fact that pressure-time relations on filling are parabolic by taking observations on pressure and time for the first portion of the curve and computing the final pressure, having opened the well immediately after these observations are completed.* It is evident that the final pressure to be attained within the casing is precisely the pressure of the reservoir feeding the casing.

The final pressure to be computed should preferably be that for $S$, the static pressure of the reservoir. From this value we may then subtract the value of the constant back pressure, already known or calculated according to the conditions of production, thereby obtaining the value of the potential pressure, for $P = S - C$.

When we know the position of the $X$ axis, which contains the vertex of the parabola, and when we know that the axis of the parabola is perpendicular to $X$, two points are sufficient to determine the parabola. This fact we take advantage of in constructing and using the relative curves. In the present process of filling, however, we do not know the position of the $X$ axis; in fact it is precisely the position of this axis that we wish to determine. We do know that the axis of the parabola is perpendicular to the $X$ axis. Three points are necessary and sufficient to determine the parabola.

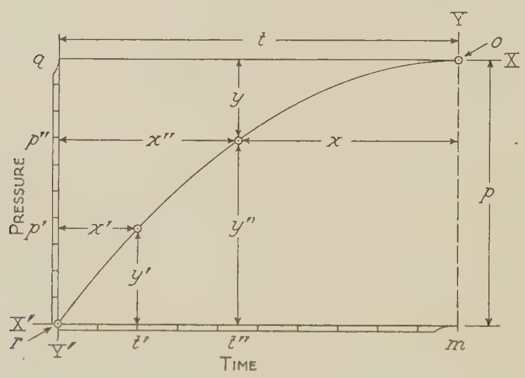

Fig. 103

Let us consider the pressure-time curve as shown in Figure 103. With the origin at the vertex $O$ its equation is

$$y = kx^2 \quad \dots\dots (269)$$

If we transform co-ordinates so that the origin is at $q$, we have

$$y = k(t - x'')^2 \quad \dots\dots (270)$$

where $t$ is the distance between $q$ and $O$. And if we again transform co-ordinates so that the origin is at $r$, we have

$$p - y'' = k(t - x'')^2 \quad \dots\dots (271)$$

where $p$ is the distance between $q$ and $r$. There are three quantities unknown in this equation: namely, $p$, $k$, and $t$. With the data for three observations the quantities $p$ and $t$ may be determined.

Let us proceed from here with an illustrative problem.[34] For convenience we shall simplify Equation 271 thus:

$$p - y = k(t - x)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (272)$$

where $y$ and $x$ signify successive values, such as $y'$ and $x'$, $y''$ and $x''$. To avoid possibility of confusion we should assume the quantities $y$ and $x$, as shown in the figure, to be erased.

A well is closed in at 2:55 P.M.;

At 3:00 P.M. the gauge shows 40 pounds
At 11:00 P.M. the gauge shows 220 pounds
At 11:00 A.M. the gauge shows 415 pounds

First compute the intervals in $x$ and $y$, taking 3:00 P.M. and 40 pounds as the starting-point $r$:

| | |
|---|---|
| For the first observation | $x = 0$, and $y = 0$ |
| For the second observation | $x = 8$, and $y = 180$ |
| For the third observation | $x = 20$, and $y = 375$ |

Now write the three following equations in accordance with Equation 272:

$$p = kt^2$$

$$p - 180 = k(t - 8)^2$$

$$p - 375 = k(t - 20)^2$$

These when solved simultaneously give

$$k = 5/16$$

$$t = 40 \text{ hours}$$

$$p = 500 \text{ pounds per square inch}$$

At the time of the first reading the gauge indicated a pressure of 40 pounds; this must be added to the computed value of $p$; thus

$$RS = 540 \text{ pounds per square inch}$$

This is the pressure which will be finally indicated by the gauge if the well

[34] The parabolic path for pressure is easily verified in the laboratory with vessels of the closed and open types; that is, with gas and with liquid.

is maintained closed for at least 40 hours. For $S$ we must add the pressure of the atmosphere that is not recorded by the gauge:

$$540 + 14.7 = 554.7 \text{ pounds per square inch}$$

We are now in a position to compute the value of the potential pressure of the natural reservoir.

Where liquid rises in the casing, its height with time travels the same sort of parabolic path. The weight of any liquid in the casing at the time of the observations on the gauge must be added to these observed pressures. Any vapors present are automatically accounted for; no special allowance need be made for them.[35]

It is to be noted that the final point to be attained by accumulated production, as shown in the upper portion of Figure 84, may be computed in the same manner. This method clearly involves the function of time in its computations, and it is therefore not as satisfactory as the method based upon pressure and volume. Of course the present method of determining the pressure likewise involves the function of time, but we have no other method, unless it be to close the well until the maximum pressure is attained.

[35] See § 50.

CHAPTER XIX

# Logarithmic System of Co-ordinates

"Our Euclidean geometry is itself only a sort of convention of language; mechanical facts might be enunciated with reference to a non-Euclidean space which would be a guide less convenient than, but just as legitimate as, our ordinary space; the enunciation would thus become much more complicated, but it would remain possible."—HENRI POINCARÉ

**120.** *Introduction.*—In the course of our investigation we have continually dealt with quantities which vary in their values according to some simple experimental law concerned with the physical properties of fluids, or to some mathematical derivation with respect to the mechanics of fluid production. Very frequently we have met the variation expressed by the "power-function" equation, $y = kx^n$, where the exponent $n$ is either negative, zero, or positive, integral or fractional. We already know that if $n$ is negative, the curve that graphically represents the variation is hyperbolic, and if $n$ is positive, such a curve is parabolic. The curve for $n$ of zero value separates these two general families of curves.

Thus far all our graphical representations of variations have been drawn according to Cartesian co-ordinates. Axes perpendicular to each other bear scales either according to units of the functions or according to percentage values that are based on an adopted standard for comparison. Equal distances along these axes denote either equal values or equal increments and decrements in values, depending upon the position of the point on the scale from which measurements are made, and upon the direction of measurement. The Cartesian plat, containing either two or three perpendicular axes, represents our idea of Euclidean space. To our minds this is the simplest space imaginable; we prefer it wherever it is the most convenient and advantageous. But if we should see a greater utility in imagining a non-Euclidean space, we would not hesitate to adopt it.

The power-function equation is one of a few well-known expressions that can be easily treated in logarithms. Thus

$$y = kx^n \quad \dots\dots\dots (273)$$

becomes

$$\log y = \log k + n \log x \quad \dots\dots\dots (274)$$

Now if we have a series of experimental values for $x$ and $y$, and if we know in advance the values of $k$ and $n$, we may consult logarithmic tables for cor-

responding values of $x$, $y$, and $k$, and plot the logarithm of $y$ as ordinates, and the sum of the logarithm of $k$, and $n$ times the logarithm of $x$ as abscissas, on a Cartesian plat. In this manner the curve of Equation 273 becomes the straight line of Equation 274. This line is easily extended in either direction with a straight edge; consequently we may determine corresponding values of $x$ and $y$ beyond the limits of our experimental data.

For us this procedure will usually be unsatisfactory. While we may know the value of $n$ from our investigations, a knowledge of the value of $k$, in advance of plotting, is inconveniently obtained. The situation may be improved, however, by using a system of logarithmic co-ordinates in place of the Cartesian system.[1] The vertical and horizontal scales, instead of being marked off by equal intervals, are spaced according to the logarithms of numbers, and the data in values of $x$ and $y$ may be plotted directly in accordance with them. A plat defined by these logarithmic scales has the property of canceling the term "log" in Equation 274, and in consequence thereof the following equation remains:

$$y = k + nx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (275)$$

an equation that defines the straight line in the Cartesian system of co-ordinates. The values of $k$ and $n$ need not be known in advance of plotting. As two points determine a straight line in the Cartesian system, the "curve" on the logarithmic plat is determined by two observations in $x$ and $y$. We can plot these values, draw the line with a straight edge, and immediately note the value of $n$ by the slope of the line.[2] *The number standing as an exponent in Equation 273, whatever its value, becomes the slope of the line, or the tangent of the angle which the line makes with the* X *axis.*[3] The quantity $k$ is the intercept of the line on the $Y$ axis. Actually we need not consider the numerical value of $k$ in our drawing; it will take care of itself.

What sort of two-dimensional space does this logarithmic plat represent? It is strictly a non-Euclidean space; equal distances along the axes do not

[1] The logarithmic plat was first designed and introduced by William F. Durand, Professor Emeritus of Mechanical Engineering, Stanford University.

[2] Not only does the curve of any power-function equation become a line that may be drawn with a straight edge, but, conversely, any line that may be drawn with a straight edge is the curve of some power-function equation.

[3] The slope of the line determined by the exponent $n$ may be easily learned from a table of natural tangents, for $n$ is the tangent of the required angle. As examples, we have

| $n = \tan\theta$ | $\theta$ |
|---|---|
| 0.5 | 26°34′ |
| 1.0 | 45°00′ |
| 1.5 | 56°19′ |
| 2.0 | 63°26′ |
| 3.0 | 71°34′ |
| etc. | etc. |

represent equal values in any manner. Here indeed is a convenient and advantageous non-Euclidean space for us to use in experimental investigations that involve the power-function equation.[4] With an ordinary straight edge we are able to draw easily any possible curve that has specific values of the constant quantities $k$ and $n$. Of course it will be to our advantage to understand thoroughly the nature of these quantities in advance of our studies concerning particular reservoirs in their performance, for this understanding will guide us in recognizing the possible and plausible characteristics of fluid delivery.[5]

We must note a curious circumstance in connection with the use of the logarithmic system of co-ordinates: we imagine ourselves in Euclidean space, and we think in terms of this space in two dimensions at the very time we look upon the plat in logarithmic space in its two dimensions. We say that the line drawn with a straight edge on the plat is a straight line. It is not; it only appears so because we hold ourselves in Euclidean space while we look at it. *The line is in fact either a hyperbola or a parabola.* If it were "lifted" from its position and, without deformation, placed upon a Cartesian plat, it would certainly be a straight line, as we may infer from Equation 275. We may call it a straight line if we wish, provided we hold its true mathematical status in mind.

**121.** *Design of the logarithmic plat.*—The logarithmic plat is easily constructed by means of a scale conveniently divided according to the decimal system of numbers and a table of logarithms to the base 10. Mark off squares the sides of which measure 10 on the scale[6] and divide these sides according to the following table, the first column showing the position of the points from one of the corners of the squares and the second showing the scale number to be attached to these points:

| $x$ and $y$ | Number |
|---|---|
| 0.000 | 1, 10, 100, etc. |
| 3.010 | 2, 20, 200, etc. |
| 4.771 | 3, 30, 300, etc. |
| 6.021 | 4, 40, 400, etc. |
| 6.999 | 5, 50, 500, etc. |
| 7.782 | 6, 60, 600, etc. |
| 8.451 | 7, 70, 700, etc. |
| 9.031 | 8, 80, 800, etc. |
| 9.542 | 9, 90, 900, etc. |
| 10.000 | 10, 100, 1,000, etc. |

[4] Investigations involving the exponential equation are aided by the use of the so-called semi-logarithmic plat.

[5] In particular, it will guide us in recognizing the characteristics of delivery in Capillary Control, as distinguished from delivery in Volumetric Control. (See chap. xxiv.)

[6] The square, as measured in centimeters or inches, may have any dimensions whatever. It is only necessary that the side be imagined divided into its ten equal parts according to the scaled ruler placed against it.

Lines parallel to the sides of the squares may now be drawn through the points. Each complete square constitutes what we shall term a "frame." There is no point representing zero or negative numbers on the plat. Down the scale of numbers the squares may be extended indefinitely; each successive corner represents a value that is one-tenth of its predecessor. Thus from 1.0 we reach 0.1, then successively 0.01, 0.001, 0.0001, and so on. Up the scale of numbers the succession is similar; each corner represents a value that is ten times its predecessor. From 1,000 we reach 10,000, 100,000, 1,000,000, and so on. Obviously these numbers are all powers of 10; thus 10 to a negative power represents a number less than 1, 10 to the zero power equals 1, and 10 to a positive power represents a number greater than 1.[7]

Distances between the lines on the logarithmic plat are themselves logarithmic; therefore they are not amenable to the usual methods of interpolation. Points between those given above should be drawn, in order that the distances between lines may be made shorter. In this way the error by ordinary interpolation is diminished. To locate these points we simply plot the logarithms of

1.1, 11, 110, etc.
1.2, 12, 120, etc.
1.3, 13, 130, etc.
. . . . . . . .
1.9, 19, 190, etc.

Subdividing may be carried as far as we like, for we may place the lines corresponding to 1.15, 11.5, 115, and so on, between those just located, if the dimensions of the frame warrant our doing so.

Figure 104 illustrates in detail the situation with respect to a plat that is ruled in logarithmic co-ordinates. But one frame is shown, and this contains two lines $A$ and $B$ to represent the hyperbola and parabola, respectively. For convenience these lines are given a slope of $-1$ and $+1$ to 1; that is, the equation for $A$ is $y = kx^{-1}$, while that for $B$ is $y = kx$. The first is the equation of the rectangular hyperbola, the expression of which has heretofore been given in the form $xy = k$, and the second is the equation of the inclined straight line that we have previously agreed to include in the family of parabolas. Since the horizontal scale extends from left to right, the tangent of the angle $\theta_2$ must be considered negative, and the tangent of the angle $\theta_1$ positive.

Each of these two lines extends in both directions without finite limit.[8] The hyperbola must satisfy the condition that

$$x = \text{infinity when } y = \text{zero}$$

and

$$y = \text{infinity when } x = \text{zero}$$

[7] The logarithmic divisions, with their numbers attached, are exactly like those of the logarithmic slide-rule. (See Appendix G.) On the rule, however, we "picture" the necessary ciphers or decimal points ascribed to the numbers.

[8] In logarithmic space zero and infinity are represented by points equally distant from unity, if we imagine an attempt to measure these distances by Euclidean instruments.

Similarly the parabola must satisfy the condition that

$$x = \text{zero when } y = \text{zero}$$

and

$$y = \text{infinity when } x = \text{infinity}$$

In the figure finite space has been given an imaginary border, beyond which lies infinity. This is done for the purpose of bringing the $X$ and $Y$ axes to the plat of our drawing, so that we may visualize the relationship between $A$ and $B$ and the axes in terms of our usual conception of Euclidean space. $A$ on extension downward to the right meets the $X$ axis at the point $a_2$, located at an infinitely great distance to the right of the origin of co-ordinates $a_1$. $B$ on extension downward to the left passes through the origin. This arrangement obviously agrees with the requirements associated with $y =$ zero in the two equations.

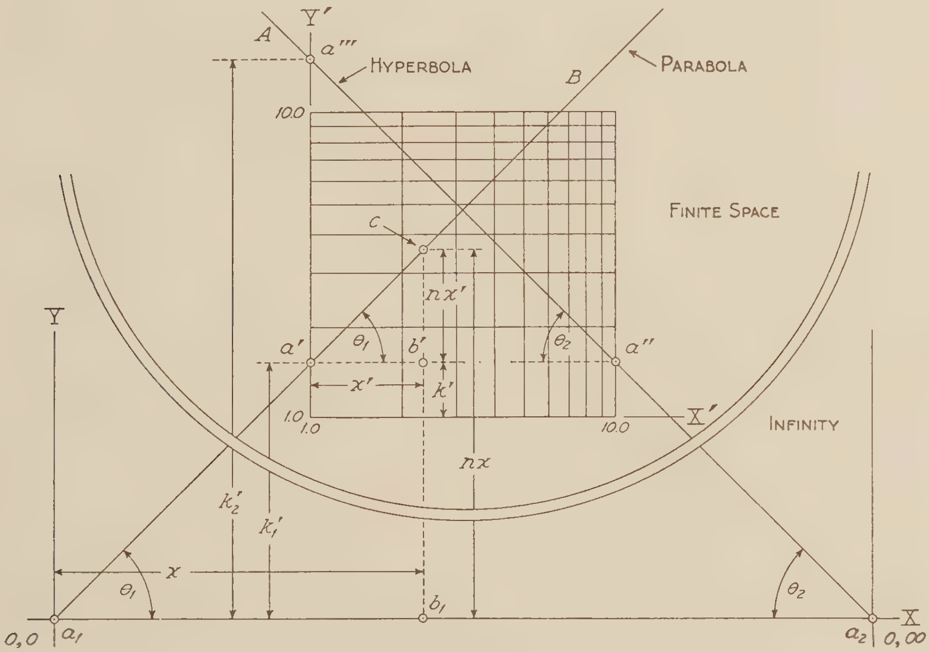


Fig. 104

While the $X$ and $Y$ axes are the true axes of co-ordinates, we may for convenience agree to use the intersecting sides of the frame as auxiliary $X'$ and $Y'$ axes, and plot data in accordance with them. If our plat should contain many frames, any such pair of lines which separate individual frames may be used in this manner, although for practical purposes we would choose extreme lines in order that all quantities may be taken positively in single directions for $x$ and $y$. Axes placed anywhere between the sides of the individual frames would be meaningless; likewise the shifting of integral powers of ten to positions anywhere between such sides would be meaningless. *The*

*three features—the auxiliary axes, the sides of the frames, and the integral powers of ten—must coincide in any case.*[9]

A plotted point $c$, say on $B$, is located at a distance $x$ from $Y$, and at a distance $y = nx$ from $X$. It is also located at a distance $x'$ from $Y'$, and at a distance $y' = nx' + k'$ from $X'$. The distance $k'$ is analogous to the intercept of a line on the $Y$ axis in Cartesian co-ordinates. Here it is obvious that its value depends upon the positions we have selected for the auxiliary axes. So long as $B$ in its equation is satisfied by the co-ordinates of $a'$, it is also satisfied by the co-ordinates of $a_1$. A lateral or vertical shifting of $B$ on the plat throws it off both points $a'$ and $a_1$ at the same time. Thus having once located $X'$ and $Y'$ at will, we may treat with $a'$ as we would ordinarily with $a_1$ in the Cartesian system. Likewise the line $A$ intercepts $Y'$ at $a'''$, and with this point we may deal as if it were $a_2$ in the Cartesian system.

122. *The declining line.*—In Figure 104 we observe that the line $A$ extends from the upper left-hand portion of the plat to the lower right-hand portion. We agree that the line is a hyperbola, one which may well be that of Figure 5 for Boyle's Law in absolute phase. It is evident that the earlier curve extends in the same general direction as the present one, and that the horizontal scales in the two are numbered from left to right. On the other hand, the line $B$ extends from the upper right-hand portion of the plat to the lower left-hand portion. It is a parabola of the first power. Whereas with $A$ smaller values in $y'$ were accompanied by larger values in $x'$, with $B$ smaller values in $y'$ are accompanied by smaller values in $x'$. $B$ may well be the inclined straight line of Figure 14 for Boyle's Law in potential phase, in so far as the two are situated with respect to the origins of co-ordinates, and consequently with respect to the numbers on the horizontal scales. However, the origin, the slope of the line, and the positive direction for measuring $x$ are now horizontally reversed in their positions on the plat.

$B$ may as well be the velocity-time curve for reservoirs in Volumetric Control, except for the fact that it is also horizontally reversed in position. We are ordinarily accustomed to plot the data of observations on the performance of reservoirs with increasing values of $x$, representing units of time elapsed, from left to right. As a consequence we look upon a line which extends from the upper left-hand portion of the plat to the lower right-hand portion as a properly located "declining line." The line $A$ already has this

[9] This is in consideration of the fact that we have chosen 10 as our logarithmic base. If we had selected the Naperian base $e$ instead, we would necessarily say that the auxiliary axes, the sides of the frames, and the integral powers of $e$ must coincide. With frames in $e$ the intermediate lines, as drawn in Fig. 104, represent powers of

$$e = 2.30258\,50930$$

The scales for the given frame would therefore extend from $e^0 = 1$ to $e^1 = e$. It is to be noted that the position and slope of the lines $A$ and $B$ would not be altered with this change of frames.

position, as reproduced in Figure 105($a$). $B$ may be put into this position by turning the plat over, as we turn the page of a book. It then appears as in Figure 105($b$). It is to be noted that the vertical lines are now spaced from the opposite side, and the scales are reversed accordingly. *For the rate of production curve time elapsed is replaced by time remaining in response to the positive direction of* x *along the horizontal axis.*[10]

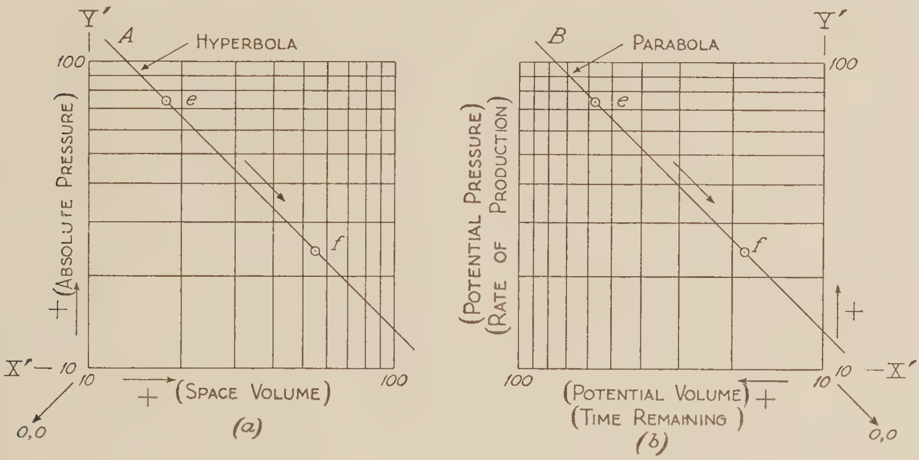


FIG. 105

It is not to be inferred that we can shift $B$, with all its points as $e$ and $f$, from its parabolic plat and place it without deformation upon $A$'s hyperbolic plat, for the purpose of transforming data on pressure-volume relations in Volumetric Control to data on Boyle's Law in absolute phase, granting that the particular values of the constants involved might be agreeable to such a transformation. Nor can we in like manner transform data on velocity-time relations in Volumetric Control, as expressed in time remaining, to data on the same relations as expressed in time elapsed. *If we should have exact data on two observations pertaining to a perfectly fulfilled hyperbolic law, such as determine the points* e *and* f *on* A, *all other points on the perfect line through these points represent data in ideal performance. But if the same exact data on the two observations were inadvertently plotted as* e *and* f *on* B, *not one other point on the line determined by them would represent a datum in ideal performance. The converse, as between* B *and* A, *is likewise true for exact data on two observations pertaining to a perfectly fulfilled parabolic law.*

[10] There is no reason why we cannot plan our parabolic curves so that they may lie as shown in Fig. 104; that is, from the upper right to the lower left corners of the plat. The only advantage with their positions in Fig. 105 is that from habit we may interpret them and retain their images more readily. They appear to us as familiar declining lines. The logarithmic paper now available on the market has its vertical lines spaced from left to right. Where the sheet is semitransparent the reverse side may be used, if we insist on the familiar declining line.

Given the data for such points as *e* and *f* on *A*, any correct procedure on our part in deriving a specific equation that is satisfied by them must result in an expression which denotes a hyperbola. Again, given the data for such points as *e* and *f* on *B*, a like procedure must result in an expression which denotes a parabola. As an example in particular, if *e* and *f* on *A* represent data on rate of production and time elapsed, the derived specific equation is necessarily that of a hyperbola, whereas if *e* and *f* on *B* represent data on rate of production and time remaining, the derived specific equation is necessarily that of a parabola. The correct equation—that of the parabola—has of course been selected on the basis of fluid mechanics, and not on the basis of pure geometry.[11]

If we know the slope of either *A* or *B*, that is, if we know the value of the exponent *n* in the equation of the hyperbolic or parabolic law, a further knowledge of the location of one point only, such as *e*, is necessary and sufficient for the purpose of drawing the line on its logarithmic plat. When the quantities represented by ordinates and abscissas both possess intensive properties, a single observation is sufficient to locate the one point, whether the apparatus—for us a reservoir of some sort—is entirely accessible or only partially so. But when either one or both of the quantities thus represented possesses extensive properties, two observations are required to locate one and two points,[12] unless the apparatus is entirely accessible.

If we are uncertain as to the slope of the line in advance of plotting, the location of two points is necessary and sufficient, provided the data are competent. If the data are not competent, a series of points must be located. We shall consider the latter circumstance in section 124.

**123.** *The relative curves.*—The relative curves for the fundamental primary function relations in Volumetric Control are shown on a parabolic logarithmic plat in Figure 106. But four frames appear in the figure; thus the percentages extend from 1 to 100 in both directions.

The slopes of the lines, being determined by the value of the exponent in the equations, are as follows:

| Relation | Slope |
|---|---|
| Pressure-Time | 2 to 1 |
| Volume-Time | 2 to 1 |
| Velocity-Time | 1 to 1 |
| Acceleration-Time | 0 to 1 |
| Energy-Time | 4 to 1 |
| Power-Time | 3 to 1 |

The fact that these parabolas of various powers, now drawn with a straight

[11] The line *A*, being erroneous, cannot satisfy velocity-time relations except at the two points *e* and *f*, whereas the line *B* satisfies these relations at all its points, including *e* and *f*.

[12] One point alone cannot be located. See § 84 for a discussion of intensive and extensive properties of reservoir functions.

edge, give the same percentages as those drawn on the Cartesian plat, Figure 92, is readily observed on comparing the two figures. All lines converge upward to the left and meet at the point 100-100. They continually diverge downward to the right. In this they differ from the corresponding curves on the Cartesian plat, where those for declining functions meet at the point 0-0.[13]

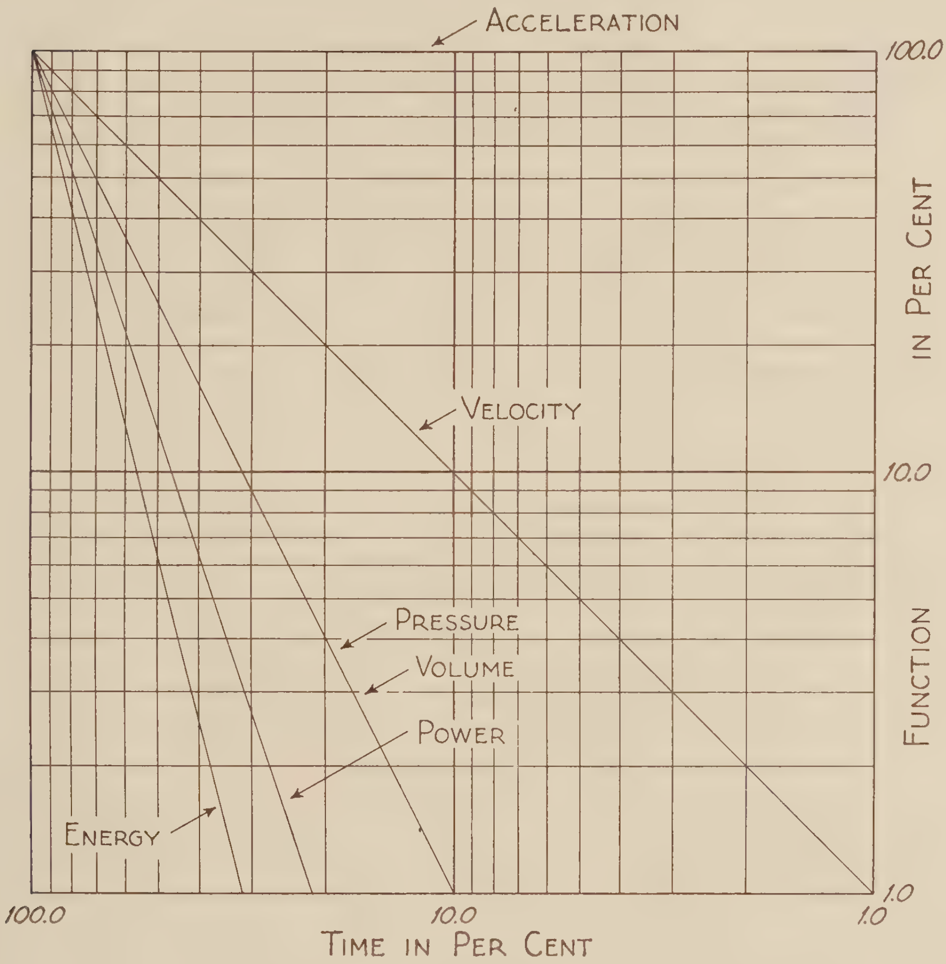


FIG. 106

More frames may be added to the figure at will. Those permitting an extension of the lines upward to the left represent percentages greater than 100, while those permitting an extension downward to the right represent percentages less than 1. Throughout the full length of the lines on extension the relative error in reading the values of individual points is always the same. In this respect the logarithmic plat differs from the Cartesian plat, for

[13] The value of the relative constant $K$ need not be known in constructing these lines.

in the latter we know that throughout the full length of the curves on extension the absolute error in reading the values of individual points is always the same.[14]

The relative curves for the derived primary function relations may also be shown on a parabolic logarithmic plat. All possible lines required for these

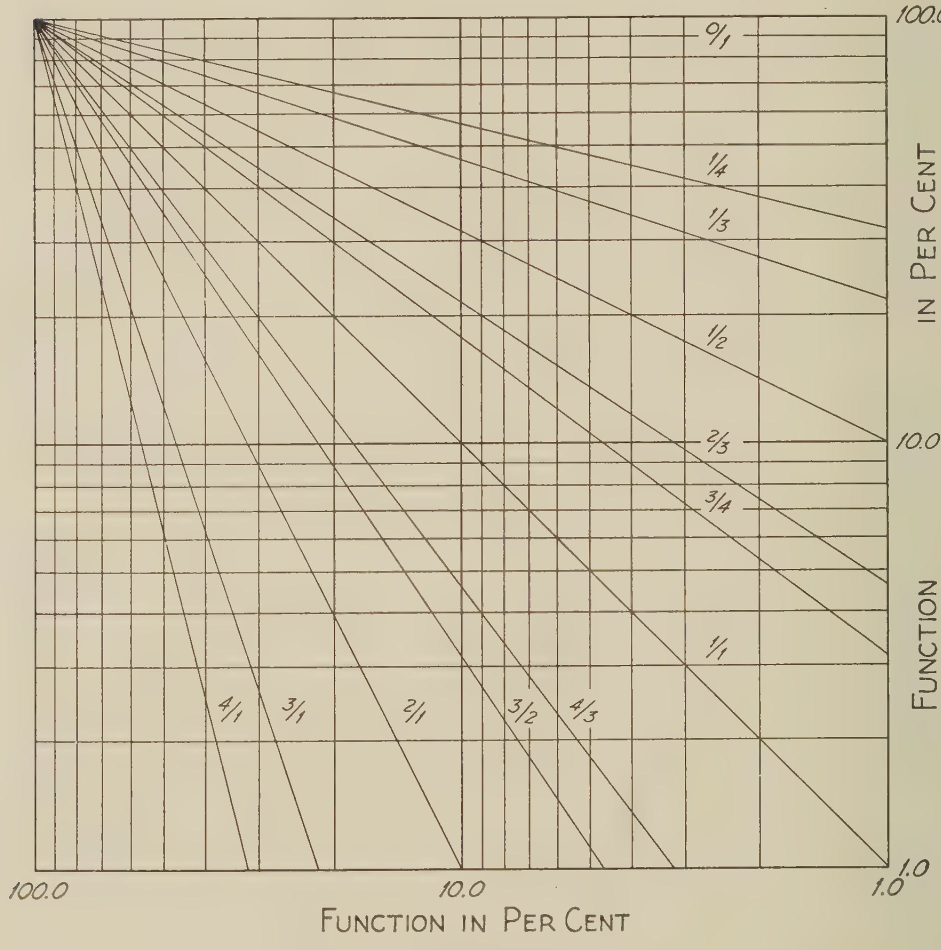


Fig. 107

are shown in Figure 107, where each is designated in accordance with the slope and exponent to be obtained by forming the proper couplet between

[14] Between Cartesian and logarithmic plats the situation with respect to errors that correspond to equal small intervals $\Delta x$ and $\Delta y$ (as measured in centimeters or inches) is as follows:

On the Cartesian plat, the relative error is variable and the absolute error is constant; while on the logarithmic plat, the relative error is constant and the absolute error is variable. Clearly the situation is reversed on the two plats. (See Appendix G.)

the functions. To reverse the order in the couplet inverts the slope in the same manner as it inverts the exponent. At least one couplet can be found to suit any of the lines that appear in the figure. For example, to find a couplet to suit a slope of 1/2 we select velocity for the first function because its exponent with $T$ is 1, the numerator of the present fraction, and we select either pressure or volume for the second because these have exponent 2 with $T$, the denominator of the fraction. For the slope of 2/1 the order will be the reverse. Incidentally it is to be noted that 1/2 is the same as 2/4, and so on. The fractions 0/1, 0/2, 0/3, and 0/4 are equal.

The slope 1/1 divides the plat into two sections; the lower one contains the lines for slopes greater than unity, and the upper one contains the lines for those less than unity. The same principle holds in Figure 93 with regard to exponents greater or less than unity.

**124.** *Straightening the curve.*—If the data for plotting the lines on the logarithmic plat refer to the fundamental and derived relations in ideal performance, and are thus properly expressed in potential units, the individual points will lie on lines that may be drawn with a straight edge. If the performance is ideal, and the functions are wrongly expressed, the points lie on a curved line. As an example of the latter case, let us suppose a reservoir to produce against a constant back pressure differing from the atmospheric pressure, yet the data for pressure are taken directly from the gauge that records pressures above the atmosphere. Now if no correction is made for the fact that the zero for potential pressure does not coincide with the atmospheric zero, the data furnish a curved line.

Again, if the data refer to the fundamental and derived relations in theoretic performance, and no adjustments are made for alterations in this performance, the individual points furnished by the data will appear to lie on a broken and curved line. When the data are expressed in correct units, we are to find these adjustments to be properly cared for. There will be one or more segments of lines to be drawn with a straight edge, depending upon the nature and the number of the alterations.

Let us investigate the first proposition, one which concerns ideal performance.[15]

In Figure 108 (p. 322) the line $B$ is a correct line between velocity and time remaining. It is properly directed toward the origin of co-ordinates at infinity, downward to the right. We shall now consider four possible cases of error in observing and plotting data:

*a*) If at the time of observations we consistently reduce the constant back pressure by a fixed amount, the velocity is thereby temporarily increased, and in place of plotting such points as $e$ on $B$ we plot such points as $e'$ on $B'$.

[15] For the second proposition see the following sections on theoretic performance: § 129, for Case 1; § 130, for Case 2; and § 132, for Case 3.

The resulting curve has a convex side facing the lower left-hand corner. A correction is needed; we simply shift $B'$ downward by successive amounts until it appears straight. To accomplish this shifting an arbitrarily selected constant amount is subtracted from the ordinates of points along the curve, and the resulting new points are connected by a new curve. If the subtracted

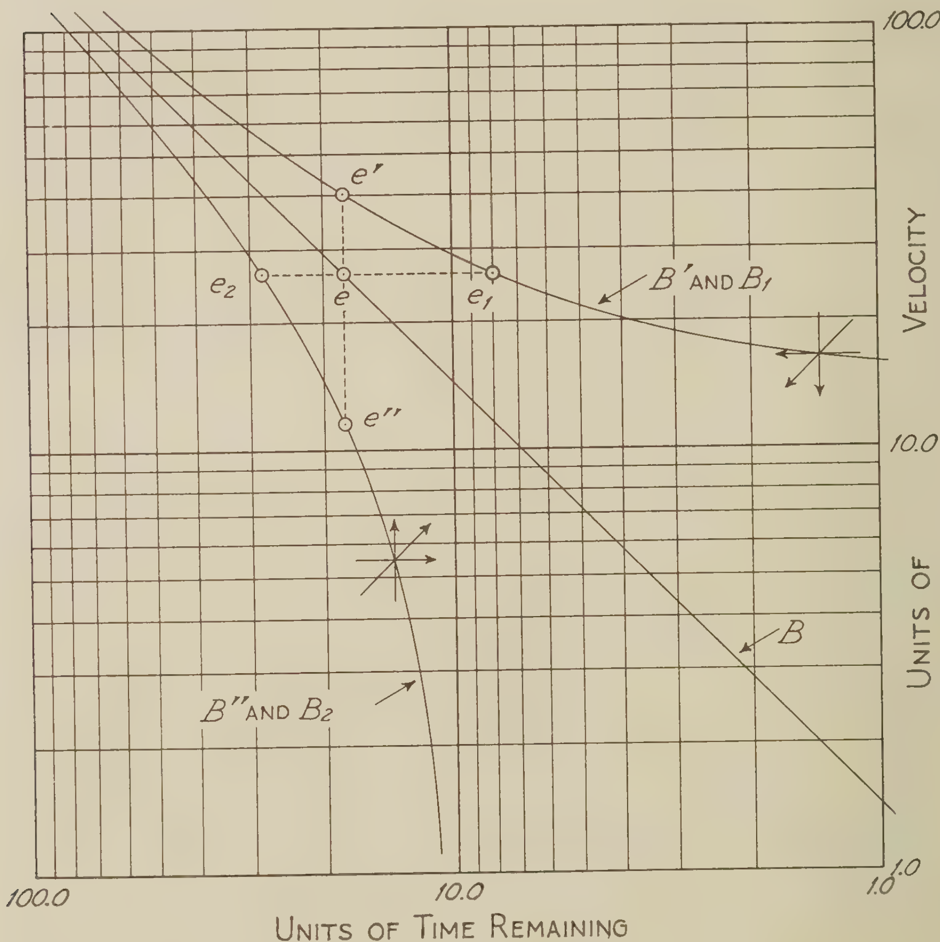


FIG. 108

amount is too little, the curve remains convex on the same side, but if it is too great the curvature is reversed. By a method of "cut and try" in this manner of shifting we finally obtain the correct line at $B$.

*b*) If at the time of observations we consistently increase the constant back pressure by a fixed amount, the velocity is thereby temporarily decreased, and in place of plotting such points as $e$ on $B$ we plot such points as $e''$ on $B''$. The convex side of the curve now faces the upper right-hand corner. By

adding the necessary constant amounts to the ordinates of points along the curve we finally obtain the correct line at $B$. If the amount added becomes too great, the curvature is reversed as in the preceding case.

*c*) If our previously determined value of time remaining is too short, for such points as $e$ we plot such points as $e_1$. The resulting curve is $B_1$ with its convex side facing the lower left-hand corner. To shift the curve we now must add the proper amount to the abscissas of points upon it, until we obtain the correct line $B$.

*d*) If our previously determined value of time remaining is too long, for such points as $e$ we plot such points as $e_2$. The resulting curve is $B_2$ with its convex side facing the upper right-hand corner. Shifting now requires the subtraction of the proper amount from abscissas of its points. As before, we finally obtain the correct line $B$.

I have assumed errors in such a manner as to cause the curved lines to coincide by pairs. Thus we have $B'$ and $B_1$ above $B$, and $B''$ and $B_2$ below $B$. That errors in ordinates and abscissas can give the same curved lines is a fact which is easily verified by testing any line, such as $B$, with its slope of 1 to 1, on this type of plat. The coincidence of these lines is of practical interest, for it means that corrections can be made by shifting either horizontally or vertically, or both horizontally and vertically, regardless of the source of the error, or errors, in the data of observations.

Where the data might be given in terms of time elapsed, we can alter them so as to read in terms of time remaining by roughly approximating the life of the reservoir at the time of a particular observation, converting all data to such life, plotting the points, and shifting the resulting curve until it appears as a straight line. We must of course use a parabolic plat for parabolic data, if we wish to obtain the most accurate results possible. We cannot expect to obtain the same accuracy by plotting data in time elapsed on a hyperbolic plat. Certainly a resulting curve can likewise be shifted on this plat, but the final line cannot be straight, inasmuch as the data are fundamentally parabolic.

To explain the fact that errors in ordinates and abscissas have the same effect upon the curve we should consider the equations of the lines actually plotted, as if they were "lifted" from the logarithmic plat and placed upon the Cartesian plat. The four equations corresponding to the above cases are as follows:

For $B'$: $$(y - c_1) = kx \qquad (276)$$

For $B''$: $$(y + c_2) = kx \qquad (277)$$

For $B_1$: $$y = k(x + c_3) \qquad (278)$$

For $B_2$: $$y = k(x - c_4) \qquad (279)$$

These agree with the specified conditions of error, the constants $c$ being the amounts of error; that is, the amounts which must be subtracted or added in order to reduce the equation to

$$y = kx \quad \ldots\ldots (280)$$

Performance is actually taking place in accordance with this last equation, while our observations are made in accordance with the first four. It is easy to see that Equation 276 is the same as Equation 278, and that Equation 277 is the same as Equation 279, by placing the constants in the first two on the right-hand side. Thus for $B'$ we have

$$y = kx + c_1$$

and for $B''$ we have

$$y = kx - c_2$$

which are evidently of the same form as Equations 278 and 279, respectively. In the figure,

$$c_1 = kc_3$$

and

$$c_2 = kc_4$$

The titles in this figure may be changed to read "Units of Pressure" for ordinates, and "Units of Volume Remaining" for abscissas, without affecting the conditions of error and correction. $B'$ results, say, from plotting gauge pressures where production takes place against a constant back pressure greater than that of the atmosphere; $B''$ results from plotting the gauge pressures where production takes place against a constant back pressure less than that of the atmosphere; while $B_1$ and $B_2$ result, say, from consistent errors in the determination of the volume remaining within the reservoir at the time of the observations. This latter condition might easily arise as a result of having erred in the observations or computations for the original volume in the reservoir.

Figure 109 illustrates a pressure-time curve on its parabolic plat. The slope of the line we know to be 2 to 1. We have, as before, four possible cases of error in observing and plotting data, and as a consequence we meet with the curves $B'$, $B''$, $B_1$, and $B_2$. They cannot coincide by pairs, as we may readily see from their equations. These are as follows:

For $B'$: $$(y - c_1) = kx^2 \quad \ldots\ldots (281)$$

For $B''$: $$(y + c_2) = kx^2 \quad \ldots\ldots (282)$$

For $B_1$: $$y = k(x + c_3)^2 \quad \ldots\ldots (283)$$

For $B_2$: $$y = k(x - c_4)^2 \quad \ldots\ldots (284)$$

all of which on correction become

$$y = kx^2 \quad \ldots\ldots (285)$$

Equation 281 by transposition becomes

$$y = kx^2 + c_1$$

and Equation 283 by expansion becomes

$$y = kx^2 + 2kc_3x + kc_3^2$$

wherein it is evident that the curves $B'$ and $B_1$ cannot under any circumstances coincide, since the latter contains a term in $x$ that does not occur in

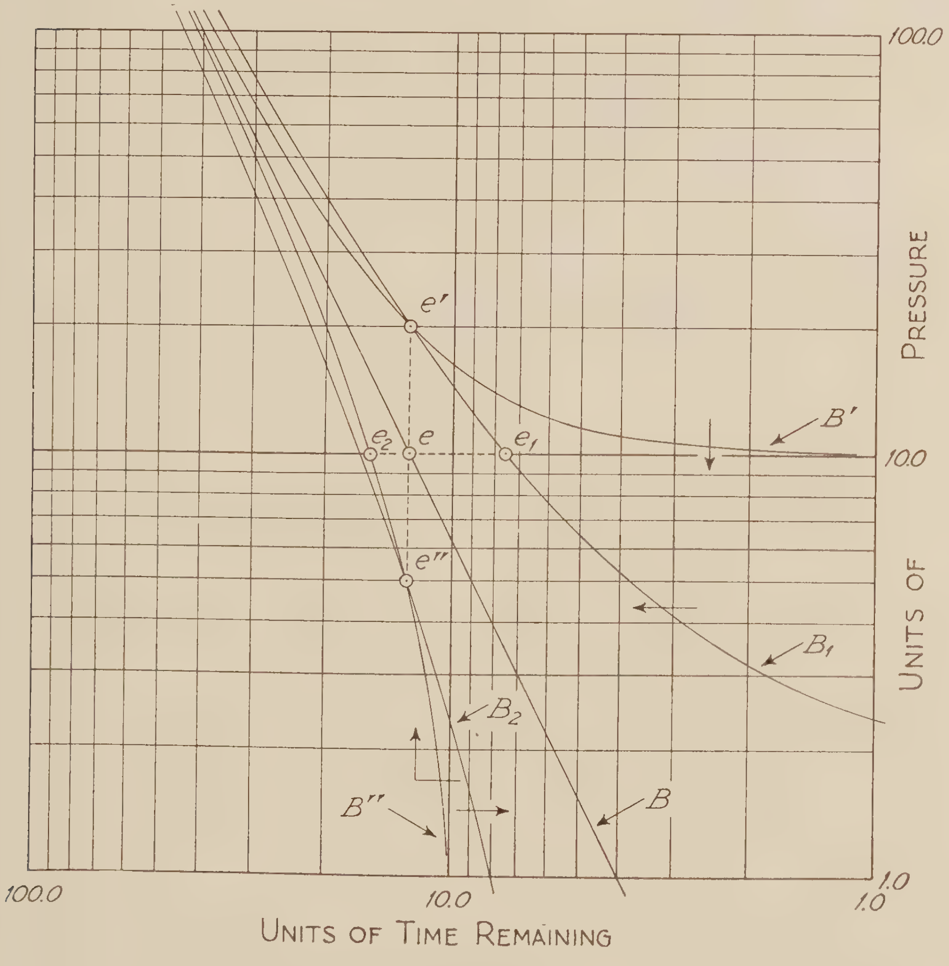


FIG. 109

the former. The same situation exists as between Equations 282 and 284. In making the necessary corrections by shifting the curves on the plat we cannot shift at will; $B'$ must be moved downward so that $e'$ coincides with $e$, $B_1$ must be moved to the left so that $e_1$ coincides with $e$, and so on. There must not be any lateral shifting of the former, or vertical shifting of the latter.

If we were to make consistent errors of observation in a test of Boyle's Law in absolute phase, and plot our points on a hyperbolic plat as in Figure 110, we would obtain the four curves with the following equations:

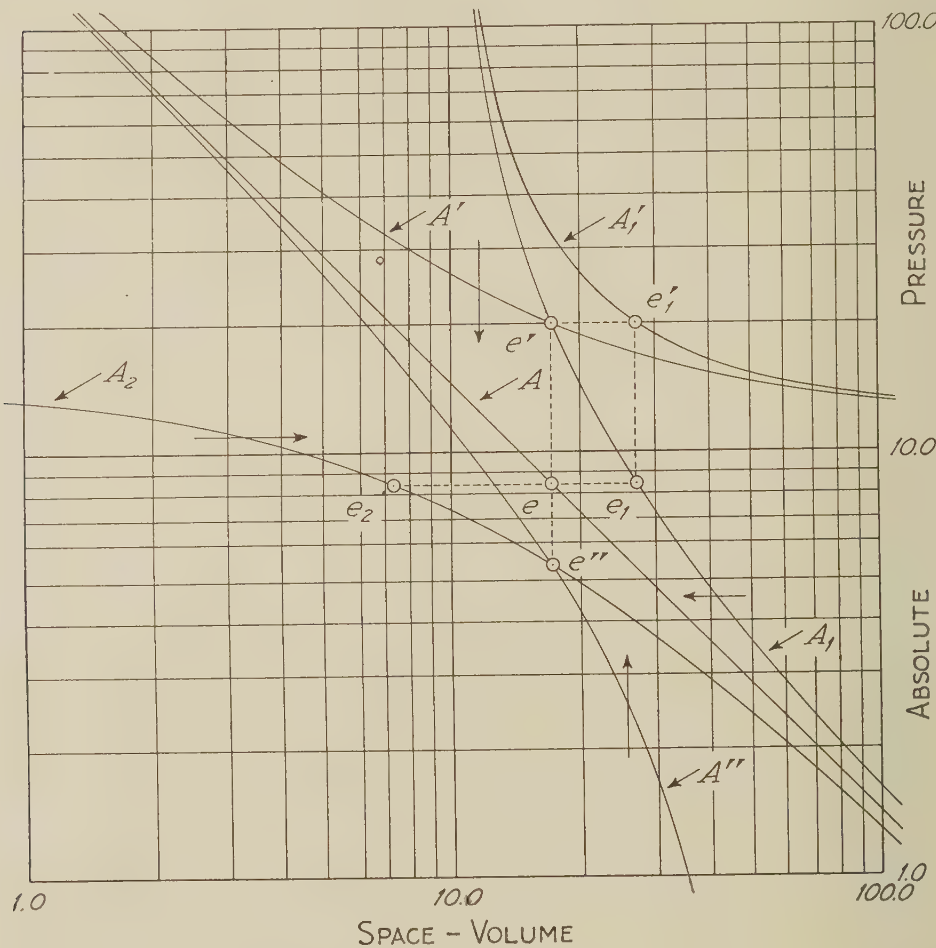


FIG. 110

For $A'$: $\quad x(y - c_1) = k \quad \ldots\ldots\ldots\ldots\ldots\ldots (286)$

For $A''$: $\quad x(y + c_2) = k \quad \ldots\ldots\ldots\ldots\ldots\ldots (287)$

For $A_1$: $\quad (x - c_3)y = k \quad \ldots\ldots\ldots\ldots\ldots\ldots (288)$

For $A_2$: $\quad (x + c_4)y = k \quad \ldots\ldots\ldots\ldots\ldots\ldots (289)$

all of which on correction become

$$xy = k \quad \ldots\ldots\ldots\ldots\ldots\ldots (290)$$

Now Equation 286 on expansion becomes

$$xy - xc_1 = k$$

and Equation 288 on expansion becomes

$$xy - yc_3 = k$$

wherein it is evident that the curves $A'$ and $A_1$ cannot coincide, since the second terms on the left side cannot be identical except for one observation in $x$ and $y$. Although the correct line $A$ possesses a 1 to 1 slope, the necessary corrections in $A'$ and $A_1$ must be made by shifting in the proper direction; $A'$ must be moved downward, and $A_1$ must be moved to the left. The same argument holds as between $A''$ and $A_2$.

It may be possible that errors exist in both $x$ and $y$ for the curves in Figures 108, 109, and 110. To illustrate such a circumstance we have the curve $A_1'$ for observations on Boyle's Law. For the correction it is essential that we shift both downward and to the left in order that $e_1'$ coincides with $e$. The equation for the curve is

$$(x - c_3)(y - c_1) = k$$

It is immaterial which one of the corrections is made first; in fact we can shift $e_1'$ to $e$ by any path we choose.

125. *Parabolic curves of the first power.*—It appears that only the curve for an equation of the first power on a parabolic plat can be shifted regardless of the error or errors in observations. Where in Figure 108 an error in $y$ can be corrected by a shifting in $x$, or vice versa, in Figure 109 this cannot be done. Now all of our fundamental and derived primary function curves for production from reservoirs in Volumetric Control are parabolas of various powers. If we wish, we may plot any of them as parabolas of the first power, with the exception of those relations which involve acceleration. Here we have the straight horizontal line which is already in a sufficiently simple form. There are two convenient methods by which the transformation may be made. Let us consider the equation between pressure or volume, and time, as an example in the first method. Since the equation is of the form

$$y = kx^2 \quad \text{......................... (291)}$$

we may simply substitute for $x^2$ a variable $z$, such that

$$z = x^2 \quad \text{......................... (292)}$$

The equation reduces to

$$y = kz \quad \text{......................... (293)}$$

an equation of the first power. Thus we can plot the observed values of pressure or volume as ordinates, and the square of time remaining as

abscissas, in order to obtain a line with a slope of 1 to 1 on the parabolic plat. Or we might take the square root of Equation 291:

$$y^{1/2} = kx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (294)$$

and substitute a variable $z$ for the term on the left. Thus

$$z = kx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (295)$$

likewise an equation of the first power. Now we can plot the square root of pressure or volume as ordinates with time remaining as abscissas, and obtain the line with a slope of 1 to 1.

For power we can similarly plot the values of this function with the cube of time remaining, or the cube root of the function with time remaining. And so on for all other equations between functions of performance. Where the exponent in the equation is an odd fraction, such as in the relation between power and pressure, we may write

$$y = kx^{3/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (296)$$

$$y^2 = kx^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (297)$$

$$y^{1/3} = kx^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (298)$$

according to which we have a choice in plotting power against the three-halves power of pressure,[16] the square of power against the cube of pressure, and the cube root of power against the square root of pressure.

As to the second method we observed in section 109 the relation between the exponents borne by the functions of performance with time. Accordingly we may plot as ordinates the pressure divided by velocity, the volume divided by velocity, the power divided by pressure or volume, and the energy divided by power, with time as abscissas, thereby obtaining a line with a slope of 1 to 1.[17]

Consistent errors in measuring the values of the functions reduce the final lines to be obtained by shifting to mere approximations in using the first method, for the difference between $(x + c)^n$ and $x^n$ is not a constant quantity except in the case where $n$ is equal to 1. A variable difference between these

[16] Fractional exponents can be handled either by logarithmic tables or by the log-log slide-rule. A description of this rule is given in Appendix G.

[17] Incidentally we may note that the two methods of transformation give us inclined straight lines of various slopes on the Cartesian plat. This is frequently worthy of consideration in treating the data of performance. If we may be assured that the values possessed by the functions are measured in potential units, any deviation from the straight line indicates, qualitatively and quantitatively, the departure from the ideal, either with respect to the reservoir itself as a container, or with respect to the physical properties of the fluid being produced. In these schemes of transforming curves to straight lines, either on Cartesian or on logarithmic plats, the areas subtended by the lines cannot be taken to represent units of the function having the next exponent greater by one.

two quantities cannot be corrected by shifting the points of the curves through a constant distance, as measured on the scales, on the logarithmic plat. In using the second method this difficulty is not encountered, for the division of one constant quantity by another results in still another constant quantity, and this may be corrected by shifting.

The practice of shifting the curves on logarithmic plats is well established in connection with the data of reservoir performance. *While we shift to make a straight line, we are in fact seeking the origin of co-ordinates, the point* $a_1$ *in Figure 104, at infinity, in the case of the parabolic plat, and the point* $a_2$, *likewise at infinity, in the case of the hyperbolic plat.* For the data of reservoir performance we are seeking the time-location of equilibrium under the given conditions of production. In the light of these principles we should be able to appreciate the argument of the first two paragraphs in section 102.

## CHAPTER XX

# Theoretic Performance

"Experiment furnishes us with the values of our arbitrary constants, but only suggests the form of the functions. Afterwards, when the form is not only recognized but understood scientifically, we find that it rests on precisely the same foundation as Euclid does."—James Clerk Maxwell

**126.** *Introduction.*—We have hitherto established the laws for the delivery of fluid from reservoirs in Volumetric Control. These laws, as we know, are based upon an ideal; not only is the reservoir, as a physical container, perfectly ideal, but also are the conditions imposed upon its performance. No act originating either with Nature or with us disturbs the mathematically defined paths of the primary functions. Let us now consider theoretic performance. With the same ideal reservoir, be it either a type solution tank or a type gas tank, the latter producing gas alone or gas with liquid, we shall deliberately make alterations in the conditions of production, as provided for in section 22.

Theoretic performance in Hydraulic Control received our attention in chapters x and xi. There we included the ideal natural reservoir with the artificial ones. This was possible for the reason that we had previously defined such a natural reservoir. As yet we cannot do this for Volumetric Control, for the definition must take into consideration certain principles that are to be investigated in chapter xxii. Our present study may nevertheless be complete. The effects of theoretic performance are dependent only upon the control of the reservoir; they are not dependent upon any other classification of reservoirs.

We may expect theoretic performance in Hydraulic and Volumetric controls to have much in common. Why should this not be so? The reservoirs differ in ideal performance merely in the fact that such functions as pressure and velocity are either mathematically constant or variable with time, according to a balanced or an unbalanced condition between the rates of replenishment and production. *At any instant, and for any instant, a reservoir in Volumetric Control is equivalent to one in Hydraulic Control. It is in the succession of instants where we observe a difference between them.* Now the variation during this succession of instants alone requires us to study further the effects of alterations in the conditions of production. Our earlier investigation must necessarily be supplemented.

We have the three cases in this performance as before. It will be remembered that these are the following:

CASE 1. Alterations in the external friction

CASE 2. Alterations in the static pressure

CASE 3. Alterations in the constant back pressure

Perhaps in our analysis of these cases with respect to the performance of the type reservoirs we can consider the internal friction a negligible function, inasmuch as the interior spaces of these reservoirs are hollow. However, let us preferably consider these tanks to be filled with a porous medium, such as sand, and assume the internal friction to be of consequence in its effect upon production. In doing this our analysis becomes more comprehensive, and it approaches more closely the situation to be encountered in the natural reservoir. But before we take up the subject of theoretic performance with such porous-filled tanks we should pause to consider their ideal performance, for it would be well to know if differences might arise as a result of the presence of the medium.

127. *Tanks with a porous medium.—A solution tank, filled with sand or other porous, insoluble material of homogeneous texture, and containing liquid in the absence of occluded bubbles of gas, performs in accordance with the same laws as one without the porous material.*[1] Figure 111 represents such a tank. We have the usual four horizontal lines that define the various intensities of pressure, including $P$, $C$, and $S$. According to the lines at the right, $h$, the head of the liquid above $O'$, determines $P$, the potential pressure of the system as constructed. In the process of production $P$ declines parabolically with time in the usual manner.

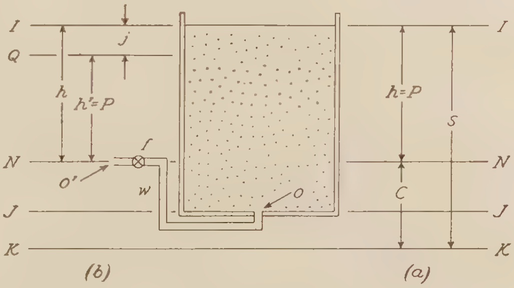


FIG. 111

Where occluded bubbles of gas are present throughout the mass of material in the tank, the tank performs in accordance with the same laws as one without the porous material, so long as $h$ exceeds a certain definite amount. The occluded bubbles partially offset the pressure due to the weight of the liquid on the orifice $O$. This is Jamin action, as briefly mentioned in section 29, and as yet to be fully investigated in connection with the study of reservoirs in Capillary Control. On closing the valve $f$ for the purpose of measuring the head we

[1] This statement is based solely upon experiment in the laboratory.

find its amount to be $h'$, less than $h$ by an amount designated as $j$. Now we say that so long as $h$ exceeds $j$, the tank performs in Volumetric Control. Whereas at the right the line $I$ lowers in its position until it coincides with $N$ on the establishment of equilibrium, at the left a fifth line $Q$, distant $j$ below $I$, lowers in its position until it coincides with $N$, always carrying $I$ downward with it at the same distance. *But when Q coincides with* N *equilibrium is not established; the reservoir at this very instant is converted from Volumetric Control to Capillary Control.*[2] This conversion is a matter to be studied in detail at a later time; the important point for us at the present moment is the fact that the reservoir can perform as one in Volumetric Control.

*A gas tank, filled with sand or other porous, insoluble material of homogeneous texture, and containing gas in the absence of liquid in small or great amount, performs in accordance with the same laws as one without the porous material.*[3] Where liquid is present to any extent, we again meet with Jamin action. Globules of the liquid separate the gas into bubbles, and the pressure on the orifice $O$ is partially offset. As before, we have lines at the right and left in Figure 112, wherein such a porous-filled gas tank is shown. In order that the reservoir may be in Volumetric Control, either the tank must be devoid of liquid or the pressure at $g_1$ must exceed any amount $j$ that may be present on account of the presence of liquid. $I$ coincides with $N$ for equilibrium in one case, while $Q$ coincides with $N$ for conversion in the other.

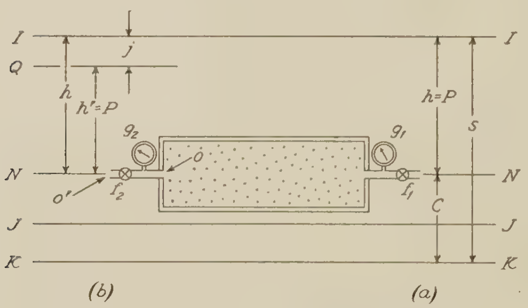

FIG. 112

With the solution tank the value of $j$ is not easily determined, for the position of the free surface of the liquid below the top of the porous material cannot be readily observed, even if such a tank were to be equipped with a vertical glass window. Only by observing $h$, and subtracting from its value that of $h'$, as measured by a manometer or gauge at $O'$, may the amount of $j$ be known. On the other hand, the value of $j$ is easily determined in the case of the gas tank, for we need only close the valve $f_2$ and note the difference between the pressures at $g_1$ and $g_2$.

For a given tank with its porous medium, be it of the open type as in Figure 111 or of the closed type as in Figure 112, the value of $j$ depends upon

[2] The point approached as the time-location of equilibrium in Volumetric Control in reality proves to be the time-location of the conversion of control. That is to say, the zero-zero point calculated in accordance with the primary function equations represents the instant of conversion in the case of a combination reservoir filled with a porous medium. This can be verified by experiment in the laboratory.

[3] This statement is likewise based solely upon experiment in the laboratory.

the surface tension of the liquid and the number of bubbles present in the porous mass.[4] Of course, with regard to the value of $j$ in general, more remains to be said at a later time.

We are to exclude any consideration of the possibility that these reservoirs may be producing in Capillary Control. In the absence of bubbles they are necessarily in Volumetric Control, and in the presence of bubbles they will be assumed to be in this control by virtue of a closed-in pressure of sufficient intensity. We may be certain of the control by observing that all the curves of Figures 92 and 93 are fulfilled in ideal performance. In particular we can always make a rapid determination by means of observations upon velocity and pressure relations. To do this we simply alter the value of either $S$ or $C$ and proceed to tabulate or plot $Ve$ against $P$. Where $Ve = KP^{1/2}$, we may be assured that the reservoir is in Volumetric Control, but should $Ve = KP^{3/2}$, the reservoir is in Capillary Control. We should observe $P$ at the outlet, say at $O'$; otherwise we may be taking $P = h$ where in fact $P = h'$. However, the error introduced by taking $h$ instead of $h'$ is not difficult to correct, for the curve is simply misplaced laterally on the plat. According to the curve $P$ is not zero when $Ve$ is zero, but it has a positive value equal to $j$.[5] The shifting of the curve through a distance representing $j$ places the curve in its proper position such that $P$ is zero when $Ve$ is zero.

The porous medium naturally modifies the volume of fluid which a given container may produce. Solid material occupies space that might otherwise be filled with the fluid. Furthermore, that fluid which is present cannot flow so easily toward the orifice as in the case of the hollow reservoir. We simply say, then, that while the curves for Volumetric Control are fulfilled regardless of the porous medium, the values of the constants $K$ in the equations for the functions, as expressed in units, are dependent upon the presence or absence of the porous medium. In the presence of the medium these values depend upon the porosity of the medium, its coarseness or fineness, the viscosity of the fluid, and the kinetic effect of Jamin action—as distinguished from the static effect considered above—in case bubbles of gas exist throughout the medium.[6] Obviously the values of the relative constants $K$ are independent of all these factors.

The absolute volume contents of a porous-filled solution tank may be apportioned in the following manner:

*a*) *The potential volume.* This quantity of liquid, as we know, is subject to delivery at the constant back pressure maintained at the orifice. It is the

[4] We are here assuming a tank with a given sand. If different sands are to be successively placed in the tank, the value of $j$ will be different in each instance.

[5] This is indeed a satisfactory way to determine if there is Jamin action and what its value might be in our artificial reservoirs which we use for illustrating the behavior of natural reservoirs in Hydraulic and Volumetric controls.

[6] We shall find that the kinetic effect of Jamin action is quite similar to the effect of viscosity. For the discussion of static and kinetic effects of Jamin action see § 154.

only part of the absolute contents to be recorded exclusively by means of observations upon performance.

*b) The volume corresponding to the registered constant back pressure.*[7] This quantity of liquid is retained by the tank after equilibrium is established. As the volume $vo$ of section 116 corresponds to $C$, the present volume, say $vo'$, corresponds to $RC$. Its value may be expressed by either of the following equations:

$$vo' = \frac{RC\,Vo}{P}, \quad \text{and} \quad vo' = \frac{RC\,vo}{C}.$$

That portion of $vo$ corresponding to $A$, the pressure of the atmosphere, vanishes.[8]

*c) The adsorbed volume.* When the liquid first comes in contact with the porous material, a small portion of it becomes adsorbed upon the surface of the solid matter. This liquid is no longer a fluid substance, for it is incapable of flowing.[9] Obviously the records of performance do not include this amount, nor can it be determined by any calculation based upon performance.

*d) The absorbed volume.* The porous medium constitutes an absorbent mass; a portion of the liquid remains clinging to the surfaces, particularly in the angular corners of the pores and in the channels connecting them. This quantity is truly liquid. If the porous mass is homogeneous in texture, the amount thus clinging is constant per unit of space within the tank, being identically situated in the vertical and all horizontal directions. In so far as the records of performance are concerned, the actual space occupied by the absorbed volume might as well be filled with solid matter.[10] It functions as a liquid so long as the free surface of the main mass of liquid lies above it, but its function is reduced to one of mere space occupancy at the instant the free surface passes below it in the course of production. The records of performance will not reveal its presence; the primary function curves are complete and mathematically correct, as if it were not present.

When bubbles of occluded gas are present, to the above must be added the following:

[7] Specifically, the back pressure here referred to is due to a vertical section of flow-line exterior to the tank, as at $w$ in Fig. 111, or to the vertical component of an inclined section in a like position.

[8] This is in virtue of the fact that air enters at the top of the tank, replacing liquid produced. This is the only difference between the mechanics of reservoirs of the open and closed types. It is clearly a minor difference, in that it simply causes $RC$ to replace $C$ in our computations. In the present case, any changes in barometric pressure do not affect $vo'$.

[9] See § 26.

[10] If we were to attempt to replace this liquid with solid matter, we would only cause other liquid to occupy a similar position in the reconstructed medium, and the statement would necessarily have to be repeated, as applied to this volume.

*e*) *The volume* (*liquid and gas*) *retained by Jamin action.* While a portion of this volume is to be produced in Capillary Control, most of it, as we shall find, is to be retained within the medium by the bubbles after final equilibrium is established. This quantity is held in a manner somewhat, although not strictly, analogous to the way in which the absorbed volume is held.[11]

A corresponding analysis can be made with respect to the absolute volume contents of a porous-filled gas tank. With regard to the gas we now have the following:

*a*) *The potential volume,* as before.

*b*) *The volume corresponding to the entire constant back pressure.* It is the volume $vo$ of section 116, an amount corresponding to $C$. As we know, its value may be expressed by the following equation:

$$vo = \frac{CVo}{P}$$

Thus it includes the amounts corresponding to $RC$ and $A$.[12]

*c*) *The adsorbed volume.* Gas is adsorbed on the surface of solid matter in the same way as liquid. After adsorption it is no longer fluid.

*d*) *There is no absorbed volume of gas strictly analogous to absorbed liquid in the solution tank.* That which remains within the porous mass after production has ceased is already cared for in item (*b*).

If liquid is also present, we must add the following:

*e*) *The volume* (*gas and liquid*) *retained by Jamin action.*

Either tank is properly a combination reservoir in case both classes of fluid are produced.

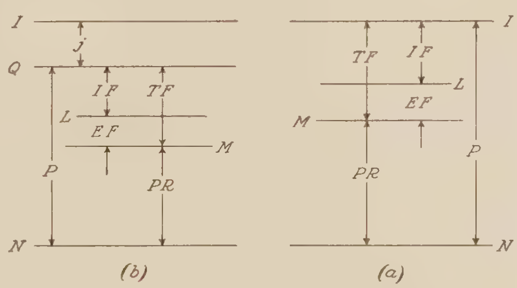


Fig. 113

**128.** *Harmonious percentage variation.* — Between the lines $N$ and $I$, or between the lines $N$ and $Q$, in Figures 111 and 112 we may insert horizontal lines $L$ and $M$ in the manner of Figure 47. These appear in Figure 113. Now we have above $N$ the following pressures:

[11] This volume is to receive full consideration in our analysis of Capillary Control. (See in particular § 174.)

[12] We shall learn that all natural reservoirs in Volumetric Control are of the open type; consequently in these we deal with $RC$, and not with $C$. As these quantities of fluid are in any case retained by the reservoir, the potential volume $Vo$, where either liquid or gas alone is produced, is independent of them. In the combination reservoir the volume $G_{Vo}$ is likewise independent of them, while $G'_{Vo}$ and $G''_{Vo}$ are dependent upon them. (See § 116.)

$P$, the potential pressure
$EF$, the external friction head
$IF$, the internal friction head
$TF$, the total friction head
$PR$, the residual pressure

We already know the significance of these "five pressures $F$" and their relations in Hydraulic Control; consequently we know about them at and for any instant in Volumetric Control. Let us see what relations they bear toward one another during a succession of instants in the latter control.

From our earlier investigations we have

$$Ve = K_1 P^{\frac{1}{2}} \qquad (299)$$

and

$$Ve = K_2 (PR)^{\frac{1}{2}} \qquad (300)$$

In the beginning these originated from experience. It is clear that we may write from these equations the following:

$$PR = KP \qquad (301)$$

If we will express these functions in percentages we may replace this $K$ by the relative constant equal to unity. Thus we have

$$PR = P \qquad (302)$$

as the relative equation between the residual pressure and the potential pressure. It simply states that $PR$ in percentages is always equal to $P$ in percentages. There is *harmonious percentage variation* between them in ideal performance. In fact throughout this performance

$$\frac{PR}{P} \text{ is constant.}$$

We also have

$$P = K_3 T^2 \qquad (303)$$

and therefore it is evident that we may write

$$PR = K_4 T^2 \qquad (304)$$

where

$$K_4 = K_3 \frac{PR}{P}, \text{ a constant.}$$

By subtracting the last equation from the preceding one:

$$P - PR = (K_3 - K_4) T^2 \qquad (305)$$

or

$$TF = K_5 T^2 \qquad (306)$$

wherein the value of $K_5$ is obvious.

From Equations 303 and 306 we have

$$TF = KP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (307)$$

This converted into a relative equation becomes

$$TF = P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (308)$$

There is harmonious percentage variation between the total friction head and the potential pressure in ideal performance. Furthermore, it follows that

$$\frac{TF}{P} \text{ is constant}$$

throughout this performance.

If we consider a given tank with its orifice, containing an imaginary non-viscous fluid, of course we can write from Equation 307 the following:

$$TF = EF = KP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (309)$$

since $IF =$ zero. But we are to assume that the value of the internal friction head is not zero. It is of consequence in the delivery of fluid. Given a porous-filled container with a defined orifice, the value of the external friction head which originates at the orifice will depend upon the rate of flow through the orifice. This rate in turn will depend upon the viscosity of the fluid and the permeability of the porous medium. However, from experience we can say that the variation in the value of the external friction head is not dependent upon these factors. $IF$ may be either zero or real and consequential, yet we may write

$$EF = KP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (310)$$

This expression converted into a relative equation shows that there is harmonious percentage variation between the external friction head and the potential pressure in ideal performance. And

$$\frac{EF}{P} \text{ is constant}$$

throughout this performance.

Let us write Equation 307 thus:

$$TF = K_6P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (311)$$

and Equation 310 thus:

$$EF = K_7P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (312)$$

where

$$K_7 = K_6 \frac{EF}{TF}, \text{ a constant,}$$

as may easily be proved.

By subtracting the last equation from the preceding one:

$$TF - EF = (K_6 - K_7)P \quad \dots\dots\dots\dots\dots (313)$$

or

$$IF = KP \quad \dots\dots\dots\dots\dots\dots\dots\dots (314)$$

Between the internal friction head and the potential pressure there is harmonious percentage variation in ideal performance.

$$\frac{IF}{P} \text{ is constant.}$$

Thus we see that between any two of the five pressures $F$ in ideal performance the following relative equation may be written:

$$F_1 = F_2 \quad \dots\dots\dots\dots\dots\dots\dots\dots (315)$$

an expression which shows a harmonious percentage variation between them. Furthermore, between any one of the five and time remaining the following relative equation may be written:

$$F = \frac{1}{100} T^2 \dots\dots\dots\dots\dots\dots\dots\dots (316)$$

This expression shows a parabolic percentage variation in $F$.[18]

All distances between $N$ and $I$, or between $N$ and $Q$, in Figure 113 vary in identical proportional amounts, and all vary as the square of time remaining, in ideal performance. With the exception of $I$ in the presence of $Q$, all lines lower in their positions, closing the distances between them in a regular manner such that they reach $N$ simultaneously. In the presence of $Q$, as previously noted, $I$ lowers with it, continually maintaining its constant distance above.

Let us now consider the three cases in theoretic performance of these porous-filled reservoirs, in so far as they remain in Volumetric Control. It is unnecessary to refer alternately to one and the other of the two tanks we have been dealing with, for their performance is identical. In the main let us refer only to the solution tank, equipped as shown in Figure 111.

**129.** *Case 1. Alterations in external friction.*—The conditions pertaining to this case have been described in section 76. There we considered the same sort of a physical container, although we took it as one maintained in Hydraulic Control by providing for a rate of replenishment equal to the rate of production. Now we are to assume that the rate of replenishment is zero; the line $I$, or $Q$, as we have already noted, lowers in its position during the process of production, approaches the line $N$, and finally coincides with it.

[18] See § 76. It should be clearly understood that the only justification we have for stating the principle of harmonious percentage variation is experience itself.

We shall say that during the course of production the external friction head is altered by changing the position of the valve *f*, that is, by partially closing it or by opening it more widely, or that it is altered by a more or less permanent partial choking or clearing at the orifice $O$. The situation with respect to seven pressures agrees with the table given in the section just cited, and there the shifting of the horizontal lines was fully described.

The potential pressure is not altered at the instant of the change in the external friction head, but it proceeds to decline on a different path from the one it was previously set upon. In Figure 114 this pressure has traveled from *a* to *b*, when the external friction is increased. It therefore travels the path $bO_2$ instead of $bO_1$. The constant $K$ in the equation between potential pressure and time remaining is changed in value. We might say for convenience that alterations in accordance with this case, and likewise those in accordance with the other two cases, set up a new potential reservoir, this having been defined in section 19.

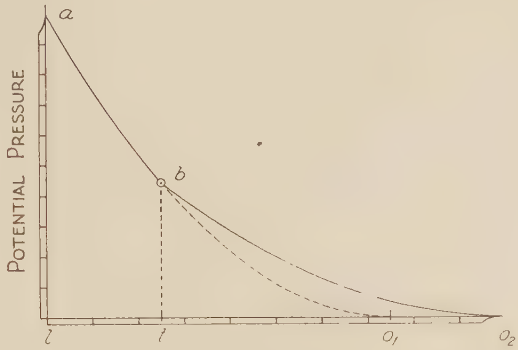


Fig. 114

The residual pressure, for the same alteration as above, travels the path *ab*, Figure 115, breaks to a point *c*, and thereafter proceeds to $O_2$.

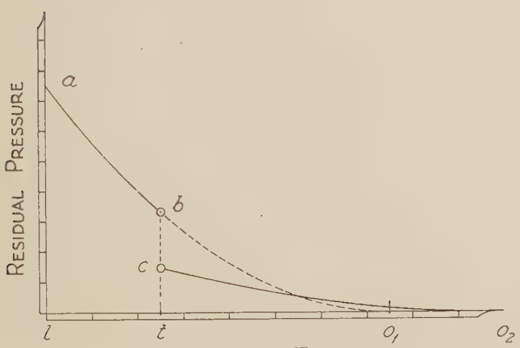


Fig. 115

The potential pressure, as we know, determines the fact that there shall be flow from the orifice, while the residual pressure determines the velocity or rate of flow. Corresponding to Figure 115 we have the velocity-time curve in Figure 116 (p. 340). Velocity travels the path *ab*, breaks to a point *c*, and thereafter proceeds to $O_2$. The alteration causes the constant $K$ in the equation between this function and time remaining to assume a new value.

In the three present figures but one horizontal axis, the potential axis $X$, is shown. The absolute and atmospheric axes might be added. On the assumption that subject-matter concerning these axes is here unnecessary,

in view of the consideration given them in the study of Hydraulic Control, they are omitted. We should merely remark that the horizontal line $N$ now, as formerly, determines the position of the potential axis for all functions,[14] and the other axes take their places in accordance with the lines $J$ and $K$, as usual.

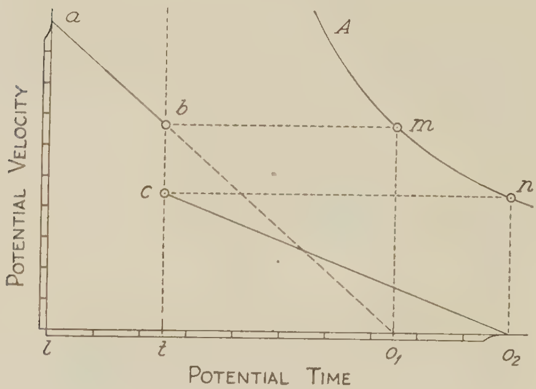


FIG. 116

The harmonious percentage variation between the internal friction head and the residual pressure, as explained in section 77, holds identically in the present case. Also there is a quantitative exchange in units between the external friction head, on the one hand, and the sum of the internal friction head and the residual pressure, on the other. Practically the entire discussion in that section applies here with equal force. The data of the problem need only be modified to accommodate the artificial reservoirs that are now under consideration.

The statement was made in section 105 to the effect that the potential volume of a reservoir in this control, at any instant during its life, is equal to one-half the potential velocity at the instant, multiplied by the time in life remaining. Then in Figure 116 the potential volume in the reservoir at the instant $l$ is one-half of $la$ multiplied by $O_1l$. At the instant $t$ this amount of fluid is diminished by that represented by the area $abtl$, leaving an amount represented by the area $tbO_1$ yet to be produced. Now if we refer to Figure 111 we see clearly that the volume of liquid to be delivered from the tank under given conditions in $S$, $P$, and $C$ is in no way dependent upon the setting of the valve $f$, or upon the physical condition of the orifice $O$; that is, the potential volume is in no way dependent upon the value of the external friction head.[15] An alteration in this head only affects the velocity of flow, and consequently the time that denotes the life of the reservoir. The triangle $tbO_1$ is in the present case altered in its linear dimensions to a triangle of equal area, the triangle $tcO_2$. It is only necessary to calculate the dimensions of a rectangle $tcnO_2$ equal in area to the rectangle $tbmO_1$. The side $tc$ is known by observing the new velocity; therefore the base $tO_2$ is easily determined.

[14] See § 78, next to the last paragraph.

[15] We are obviously assuming that the orifice is not completely closed. The statement holds for all possible degrees of opening at the orifice. We take this to exclude complete closing.

For a series of points on the vertical line at $t$, denoting various values of $Ve$ to be obtained by altering the external friction head, the point $m$, defining the time-location of equilibrium, that is, defining $L$ as measured from $t$, lies on a curve $A$. We know that

$$VeL = 2Vo \qquad (317)$$

where the quantity on the right is a constant. Clearly this is the equation of a rectangular hyperbola of the form $yx = k$. $A$, the locus of $m$, is then such a curve. Its asymptotes are the vertical line at $t$ and the $X$ axis. For a given reservoir the upward limit for the location of $m$ is determined by the physical properties of the orifice, and the downward limit is at infinity to the right of the figure, a location conforming to the complete closing of the orifice.[16]

We have found that the value of the constant $K$ in the velocity-time equation is altered. $K$ is actually the value of acceleration; thus we see that alterations in accordance with this case affect the decline in the velocity of flow. In Figure 117 are shown three of an infinite number of possible velocity-time curves for a particular reservoir. All subtend equal areas representing a potential volume of 1,000 gallons of liquid. Let us determine the three values of acceleration corresponding to these velocities:

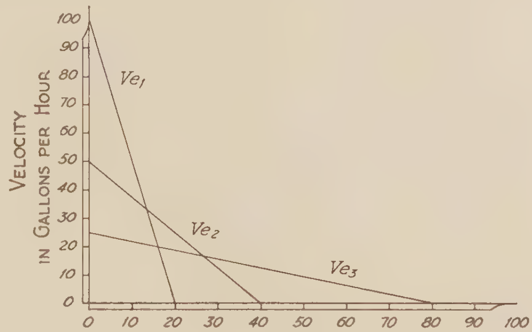


Fig. 117

$Ac_1 = 100/10 = 10$ gallons per hour per hour for the curve $Ve_1$,
$Ac_2 = 50/20 = 2.5$ gallons per hour per hour for the curve $Ve_2$, and
$Ac_3 = 25/40 = 0.625$ gallons per hour per hour for the curve $Ve_3$.

Now by inspection we see that

$$\frac{Ac_1}{Ac_2} = \frac{Ve_1^2}{Ve_2^2}, \quad \text{and} \quad \frac{Ac_1}{Ac_3} = \frac{Ve_1^2}{Ve_3^2}.$$

In general then

$$Ac = KVe^2 \qquad (318)$$

where $K$ is another constant, obviously 1/1,000 in the present case. *The value of acceleration for the remainder of time in life depends upon the*

[16] $A$ is one of our hyperbolic curves of loci. (See footnote 5, § 116, p. 291.) In this particular case the hyperbolic relation reminds us somewhat of Boyle's Law in absolute phase. Both involve a constant mass-volume of fluid, one necessarily a gas and the other either a gas or a liquid.

*square of the velocity at the instant of an alteration in the external friction head.*[17]

Since the potential pressure and the potential volume of a given reservoir are not altered by changes in the external friction head, the potential energy of the reservoir is not altered. The curves for these three functions separately plotted with time of course vary with the alteration, inasmuch as time itself is altered. Their mutual curves, however, being independent of time in every way, are identically the same, regardless of these alterations.[18]

Potential power, being the product of potential pressure and potential velocity, must alter in direct proportion to the latter function, since the former one remains the same. This power relates only to the displacement of potential energy. The suppressed energy being displaced from the reservoir is not affected in amount by alterations in the external friction head. The suppressed power is altered only because the potential velocity is altered.[19] The situation is illustrated in Figure 118, where the potential velocity has been reduced to one-half its value by increasing the external friction head.

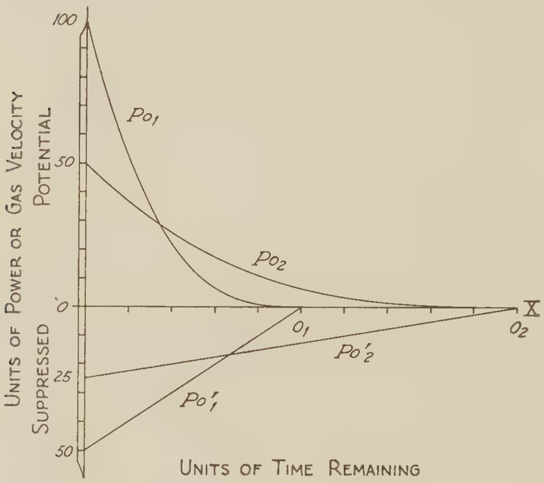


FIG. 118

This figure is an adaptation of Figure 97. The areas subtended by the curves above the horizontal axis, representing potential energy, are equal, each being one-fourth of the altitude times the base. Likewise those subtended by the curves below the axis, representing suppressed energy, are equal, each being one-half the altitude times the base.[20]

If the tank produces both gas and liquid, no reapportionment of the gas is

[17] Thus an increase in the external friction head has the following effects: (*a*) decrease in the rate of production, (*b*) prolongation of the life of the reservoir, and (*c*) decrease in the decline of the rate of production. A decrease in the external friction head has the opposite effects. In neither instance is there any effect on the volume to be produced. We deceive ourselves if we think that an increase in the external friction head increases the volume $Vo$.

[18] See § 41.

[19] The areas subtended by the curves for potential and suppressed power are altered in their lateral dimensions, but not in the amount of surface which they cover.

[20] In this figure the two terms of Equation 239 (p. 295) are united, since both terms have the function $T$ with the same exponent.

necessary. The proportional production of gas is not altered,[21] and the respective amounts of potential and suppressed gas remain the same. The gas-time curves are altered for the same reason that the corresponding curves for potential pressure and potential velocity are altered. In Figure 118 the full ordinates between the upper and lower curves differ as between the pair for $O_1$ and that for $O_2$, yet the areas subtended by the pairs, representing the volume of the two portions of gas, are equal in their parts, and therefore in their entirety.

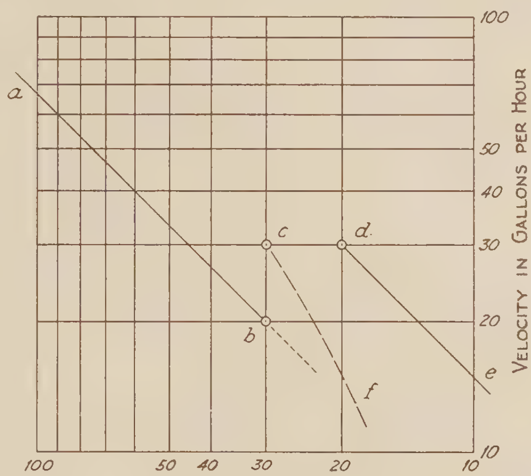


Fig. 119

On the logarithmic plat an alteration in the external friction head causes a shift in the position of the "straight line," upward to the right if the head is decreased, and oppositely if the head is increased. A simple case is shown in Figure 119. A velocity-time curve describes the path *ab* when, at the latter point, external friction is decreased by opening the valve sufficiently to permit an increase in the velocity from 20 to 30 gallons per hour, an amount denoted by *bc*. But if the rate of flow is increased to three-halves its value life, or time remaining, must be reduced to two-thirds its value in order that the area subtended by the curve on the Cartesian plat may remain the same. This reduction is denoted by *cd*. Clearly the line *ab* breaks at *b*, and the path thereafter traveled is *de*. The sections are parallel, for the slope is only dependent upon the exponent in the equation between the functions. This exponent is not altered. The constant $K$ is altered, and this fact is indicated by the different intercept of *de* on either of the vertical sides of the frame.

It is important to note that if the velocity is plotted without taking into account the shift in time remaining the path appears to be *cf*, a curved line which requires the usual shift indicated by the direction of its bow.

[21] We are here considering a reservoir in which by-passing is impossible because of the submergence of the orifice.

## CHAPTER XXI

# Theoretic Performance *(Continued)*

We are to seek the "certain principles of material things . . . . not by the prejudices of the senses, but by the light of reason, and which thus possess so great evidence that we cannot doubt of their truth."—René Descartes

130. *Case 2. Alterations in static pressure.*—This case involves, as we know, both real and apparent alterations in the value of the static pressure, alterations which are erratic with respect to time; that is, they do not take place naturally as a result of decline in accordance with production. These alterations may, of course, pertain to either an increase or a decrease in this pressure.

The conditions pertaining to this case have been described in section 78. Now we are to refer to our artificial porous-filled tanks of Figures 111 and 112, favoring the former one where the separate consideration of the latter is not essential. The situation with respect to seven pressures agrees with the table given in the section just cited, and there the shifting of the horizontal lines was fully described.

It is a simple matter to alter the real static pressure of artificial reservoirs, for fluid need only be withdrawn or introduced independently of delivery at the orifice. The quantity of liquid in the solution tank is easily altered by taking out some, or adding more, at the open top, and the quantity of gas in the gas tank may be likewise altered by means of the valve at $f_1$. If we provide the tanks with additional orifices, adjoining the one at $O$, the opening or closing of these will cause apparent changes in the static pressure as measured at the original one.

It is clear that if we were to have several orifices to be treated simply as a multiple one there would be no meaning to the expression, "apparent changes in the static pressure," unless there should happen to exist others that are not included in the group. When all orifices are grouped, the situation with regard to opening or closing some of them reverts to Case 1, for such an act merely constitutes an alteration in the value of the external friction head offered by the multiple orifice. For Case 2, then, we are constrained to consider individual or multiple orifices where the delivery is affected by the presence of still further orifices in the same reservoir—the latter being subject to changes in size or condition—when we deal with apparent alterations in the static pressure.

Let us first investigate the effects of alterations in the real static pressure. The orifice is continually of the same size and condition, while dèliberate changes are made in the pressure head of the fluid on this orifice.

We may say at once that these alterations merely affect the history of performance. *On the addition of fluid the reservoir immediately assumes a status that existed earlier in its life.* By decline in Volumetric Control it had performed in a regular manner, only to be set back in its path toward the state of equilibrium. *Contrarily, on the withdrawal of fluid the reservoir immediately assumes a status that otherwise would exist later in its life.* The reservoir is set ahead in its path toward the state of equilibrium. The situations are illustrated in Figure 120. We shall assume that at the beginning of production pressure and velocity, plotted according to suitable scales, are denoted by the point $a$. In ideal performance these functions would travel their respective paths toward equilibrium at $O$. Now if at some instant $t_2$ a quantity of fluid were added to the contents of the reservoir, say a quantity represented by the area $t_2c_2c_1t_1$, the curves are re-established at $b_1$ and $c_1$, as of an earlier instant $t_1$ in its history of performance. Travel on the paths $b_1b_2$ and $c_1c_2$ must be repeated, and, as time remaining is altered from $t_2$ to $t_1$, the duration of time represented by the distance between these points is lost by the reservoir in its attainment of equilibrium. Again, if at this instant $t_2$ a quantity of fluid were withdrawn from the contents of the reservoir, say a quantity represented by the area $t_2c_2c_3t_3$, the curves are immediately established at $b_3$ and $c_3$. There is a gap in the paths between $b_2b_3$ and $c_2c_3$, and the reservoir gains the time represented by the distance between $t_2$ and $t_3$.

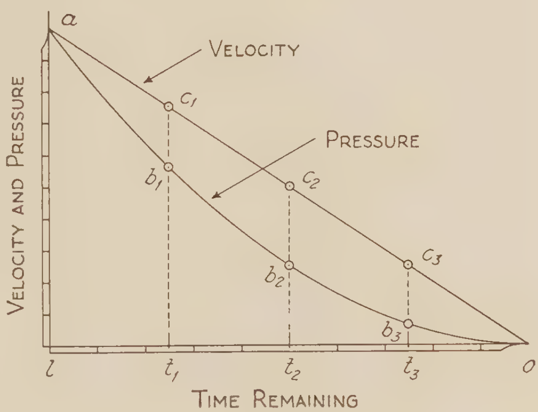


Fig. 120

The paths themselves are not altered in any manner. The constants $K$ in the primary function equations are the same before and after the alteration. Since we found one of these $K$'s to be the value of acceleration, we see now that this function does not alter in its value in this case.

Given the values of the various primary functions at an instant immediately preceding an alteration in accordance with this case, these being known either by direct observation at the instant or by calculation from earlier values with the use of the relative curves in ideal performance, new values to be assumed by the functions upon the alteration are easily determined from the relative curves by making the proper *vertical cut*. Thus if we know the new

and old values of the static pressure, and consequently the percentage ratio between the new and old values of the potential pressure, we need but locate this ratio on the relative pressure-time curve of Figure 92, or on the horizontal axis in Figure 93, pass a vertical line through the point, and read off the percentage ratios between the new and old values of the remaining functions at the intersections of the line with their curves. Now these two figures cover only the ratios between 0 and 100 per cent. For general use we may conveniently prepare a chart as outlined in Figure 121, about to be described.

The chart includes three frames to which more may be added if desired. That at the lower right-hand corner is a plat in Cartesian co-ordinates, thus duplicating Figure 92. This plat, as we know, has the advantage of representing the path of the functions in familiar Euclidean space. It furthermore has the advantage of including the final unit space whose lower right-hand corner denotes zero-zero, the point of equilibrium. The other frames are parabolic logarithmic plats. If in such a chart these frames were in Cartesian co-ordinates, it is clear that their extremities in the $X$ and $Y$ directions would be 200 and 300 per cent, respectively, but as we have them their extremities are 1,000 and 10,000, respectively. Ratios frequently run high in practice, and these would require a chart that might prove to be inconvenient in size, were the ordinary co-ordinates to be used throughout. In so far as the appearance of the curves in logarithmic space is concerned, we may note that for values greater than 100 per cent the difference between their present appearance and that which they would present in Euclidean space is not great. In the latter space they set out with a slight curvature, but become more nearly straight as they reach the higher values.[1]

All the curves pass through the point 100-100 for reasons which were explained in chapter xvi. They cross at this point. While this fact may indeed seem strange at first, it is certainly in accord with the mechanics of fluid production. Perhaps the best way to gain confidence in the correctness of the crossing is to test the curves by using them in the solution of a simple problem that requires passing and repassing through the 100-100 point.[2]

It is interesting to note that the zero and first power curves in the upper frames are extensions of straight lines in the lower frame, while the curves of higher power in the upper frames are extensions of the tangents to the curves in the lower frame at the point of crossing. Let us see why this is so.

The general equation for these curves is

$$y = kx^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (319)$$

where the exponent $n$ is zero or a positive number. The point 100-100 is actually the point 1-1 in the natural system of numbers, for 100 per cent is

[1] An examination of Fig. 121 should dispel any remaining doubts concerning the propriety of parabolic equations and curves in our analysis of production.

[2] This situation is to be illustrated subsequently in this section.

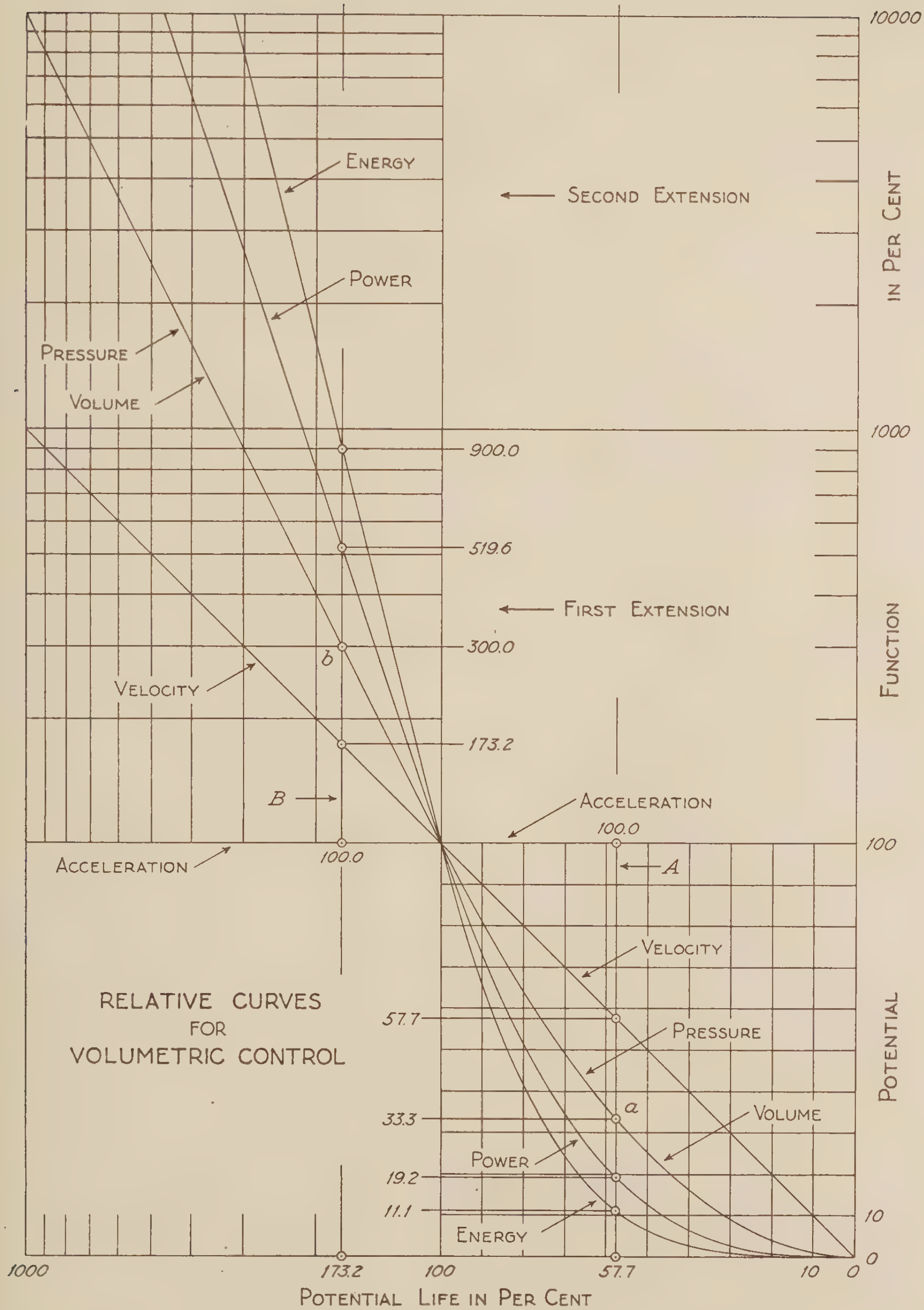
10000
Energy
Second Extension
Power
Pressure
Volume
1000
900.0
519.6
First Extension
300.0
b
Velocity
173.2
B
Acceleration
100.0
Acceleration
100.0
100
A
Velocity
Relative Curves
for
Volumetric Control
57.7
Pressure
a
33.3
Volume
Power
19.2
11.1
Energy
10
0
1000
173.2
100
57.7
10
0
Potential Life in Per Cent
Potential Function in Per Cent

Fig. 121

unity. Consequently the value of $k$ in the equation is 1, if natural numbers be used. Thus the equation reduces to

$$y = x^n \qquad (320)$$

The general equation for the tangent to the curve of this equation is found by taking the first derivative of the expression:

$$y' = nx^{n-1} \qquad (321)$$

and the tangent at the point 1-1 is

$$y' = n \qquad (322)$$

for $x = 1$, since 1 raised to any power is 1. So much for the slope of the curve immediately at the point of crossing; now let us consider the same general equation with respect to the slope of the line on the logarithmic plat at the risk of repeating what we have done in an earlier section.

Again Equation 319 reduces to Equation 320 in the natural system of numbers. The logarithm of the latter is

$$\log y = n \log x \qquad (323)$$

Now we know that the logarithmic scales for $x$ and $y$ cancel the word "log" in this equation; therefore we may transform it into the following equation:

$$y'' = nx \qquad (324)$$

where the slope of the line is

$$y' = n \qquad (325)$$

Equation 325 agrees with Equation 322, and this fact is true of course regardless of the value of $n$. The curves in the different frames are therefore related in the manner stated.

To illustrate the vertical cut we shall suppose that the alteration in the static pressure is such as to reduce the potential pressure to one-third, or say 33.3 per cent, of its value. Find this ratio on the pressure curve at the point $a$ and draw through it the vertical line $A$. At the intersections made by this line we find the following ratios for the remaining functions:

| | |
|---|---|
| Volume | 33.3 per cent |
| Velocity | 57.7 per cent |
| Acceleration | 100.0 per cent |
| Energy | 11.1 per cent |
| Power | 19.2 per cent |
| Life | 57.7 per cent |

all these being the ratios between the new and old values.

We shall next suppose that the alteration in the static pressure is such as to increase the potential pressure to 300.0 per cent of its value. This ratio

on the pressure curve is found at the point $b$, and through it the vertical line $B$ is drawn. Now we find the following ratios for the remaining functions:

| | |
|---|---|
| Volume | 300.0 per cent |
| Velocity | 173.2 per cent |
| Acceleration | 100.0 per cent |
| Energy | 900.0 per cent |
| Power | 519.6 per cent |
| Life | 173.2 per cent |

As a simple problem let us consider a solution tank that immediately at the time of the alteration has a potential pressure of 32 feet of liquid, a velocity of 80 gallons per hour, and a life of 16 hours. This potential pressure is reduced to $10\frac{2}{3}$ feet by withdrawing liquid from the top. According to the cut the velocity is reduced to 46.2 gallons per hour, and life is reduced to 9.2 hours. But suppose we had waited until the tank had these values as a result of delivery. Now these values may be taken as 100 per cent, and supposing the potential pressure to be increased to 300 per cent by the addition of liquid at this instant we would find by the cut that the velocity increases to 80 gallons per hour, and life increases to 16 hours.[3] The crossing of the curves must be correct; otherwise we would be unable to return to values with which we started.

The two cuts illustrated have reciprocal relations. Corresponding ratios indicated by them, when multiplied together, give 100 per cent. This fact substantiates the crossing of the curves at 100 per cent.

The effect of the resisting pressure $j$, Figures 111 and 112, due to the action of gas bubbles throughout the mass of liquid and to liquid globules throughout the mass of gas, respectively, is equivalent to a vertical cut on the present chart, to the right of life at 100 per cent, at any and every instant during the process of production. The resistance $j$ is constant in amount; therefore its relative value in terms of $P$ increases without limit as $P$ approaches zero.

The potential pressure curve serves also for the external friction head, the internal friction head, the total friction head, and the residual pressure. While these all vary in harmonious percentage variation in ideal performance, they for the same reason act concordantly in assuming new values upon alterations in the static pressure.

131. *Case 2 (continued)*.—Is the proportional production of gas to liquid at a single or multiple orifice affected by alterations in the value of the real static pressure? In the simple ideal system shown in Figure 72 it is obvious that alterations in the pressure bearing upon the liquid in the annular tank do affect the amount of gas dissolved per unit volume of liquid, and conse-

[3] The vertical cut, as shown in Fig. 121, is of course applicable to all reservoirs, artificial and natural, in this control.

quently they must affect the proportional amount of gas entering with the liquid at $e$. For an increase in the static pressure extra gas is available from the supply at $g$, and less of it accumulates in the pocket above the liquid. For a decrease in the static pressure less gas is picked up by the liquid, and the excess accumulates in the pocket.[4] The change in the proportion of gas will not be noted immediately at the orifice, but at some later instant depending upon the distance between the standpipe and $e$, and also upon the lineal velocity of flow between the plates. *If there were no supply from which to draw, nor an available space for the accumulation of a pocket, these alterations would not affect the proportional production of gas.*[5]

A standpipe in the position of $W_{\frac{1}{2}}$, aside from influence at $e$, is subject to a decrease or an increase in the size of the gas pocket on the top of the structure, inasmuch as this gas is a source of supply in case the pressure is increased, and the space is available for a pocket in case the pressure is decreased. We shall say that in the ideal reservoir, producing both gas and liquid, the proportional production of gas alters in direct proportion to the alteration in the static pressure. $S$ per cent of the gas is some amount greater or less than 100 per cent after the change, $C$ per cent remains constant, and $P$ per cent is changed quantitatively in accordance with $S$ per cent. *The performance of the reservoir with respect to the gas is simply set back or set ahead in its history.*

Because of the fact that the constant back pressure remains unchanged in this case no redistribution of potential and suppressed power and energy is necessary. To picture the effects of these alterations upon the power-time and gas velocity-time curves we may imagine the curves for 50 per cent removed from Figure 118, leaving the appropriate curves for 100 per cent. Now extend these outward from $O_1$, and imagine the vertical line at the left to be shifted toward the left or right in accordance with an increase or a decrease in the static pressure, respectively.

The situation with regard to alterations in the apparent static pressure possesses particular features, because it is involved with simultaneous alterations in accordance with Case 1. Let us suppose the tank of Figure 111 to be equipped with additional openings at the bottom. All taken as a group constitute a multiple orifice. It is clear that the external friction head offered at the "orifice" is reduced with additional openings, and consequently the rate of delivery from the tank is increased, volume is unaffected, and life is shortened.[6] If we confine our attention to the original orifice, we find that on the

[4] That gas freed from solution upon the decrease in $S$ is an excess for the new value of $S$.

[5] This situation is to be met in natural reservoirs of this control, where oil and gas are restricted to a pool, and where the process of production will be terminated by the operator upon the depletion of the pool. The effects of the present phenomenon are entirely independent of any effects of by-passing.

[6] See § 84.

addition of openings its rate of delivery is decreased, volume is decreased, and its life is shortened. All openings as single orifices have the same life, if we assume them to be similarly located; that is, if we assume that all have the same closed-in pressure at any instant in the life of the reservoir. We shall proceed on the basis of this assumption and omit the consideration of the necessary slight adjustments that are demanded by contrary conditions.[7]

An observation on the apparent static pressure at the original orifice allows us to calculate the value of the apparent potential pressure. The latter locates a vertical cut on Figure 121, but this cut now serves only for the functions of velocity and power. Life is to be determined as in Case 1 by using the data on the velocity for all orifices as one, and that on volume remaining. The new volume for the original orifice must be calculated apart from the vertical cut. To illustrate this let us take a simple case as follows:

The tank with an orifice $O$ began production at 100 gallons per hour and 20 hours in life. When at eight hours later velocity is at 60 gallons per hour and life is obviously 12 hours, a smaller orifice $O'$ is opened. This by observation is found to produce at 25 gallons per hour, and it is noted at the same time that $O$ lowers in its rate to 55 gallons per hour. What volume will each orifice produce separately, and how may their separate and combined velocity-time curves be constructed from the data already possessed?

For $O$ alone at the time of the alteration:

$$\tfrac{1}{2} \times 60 \times 12 = 360 \text{ gallons}$$

This is the volume remaining in the tank at the time. It is clearly not altered by the opening at $O'$; the two orifices will produce together this amount of liquid.

The total rate of production is 80 gallons per hour at the time of the alteration; therefore

$$\tfrac{1}{2} \times 80 \times L = 360 \text{ gallons}$$

or

$$L = 9 \text{ hours}$$

This is the life of the reservoir; flow will take place at both orifices for this length of time.

For the orifice $O$

$$Vo = \tfrac{1}{2} \times 55 \times 9 = 247\tfrac{1}{2} \text{ gallons}$$

and for the orifice $O'$

$$Vo = \tfrac{1}{2} \times 25 \times 9 = 112\tfrac{1}{2} \text{ gallons}$$

[7] All wells located on the same structural contour will show the same closed-in pressure, provided all are closed in at the time of the observation. Those on lower or higher contours will show greater or less closed-in pressure, respectively. By "closed-in pressure" we refer to amounts pertaining to the pressure at the bottom of the wells. Due allowance must be made for any liquid standing in the wells. This relation between the location on contours and the closed-in pressure holds only for reservoirs in Hydraulic and Volumetric controls.

To construct the curves scale off a distance to represent 9 hours on a horizontal axis, draw a vertical line at the left extremity, and upon this mark off scaled distances to represent 25, 55, and 80 gallons per hour; connect the three points to the right extremity of the horizontal scale by straight lines. These are the desired decline curves of production.

For all orifices taken together as a multiple one there is no change in the proportional production of gas from the ideal combination reservoir, since the value, the true value, of $S$ is unaltered for the group. In Figure 72, as the fluids enter at $e$, so must they be produced from the orifice, whether this be an individual or a multiple one. If there is a modification in the proportional production of gas at the original standpipe, due to the addition of others as producers, this is only because they are not located at the same level on the structure. Or in equivalent terms we might say that this is only because they are not located similarly with respect to a space that is capable of serving as a pocket for some of the gas.

132. *Case 3. Alterations in constant back pressure.*—This case involves alterations in the value of the constant back pressure. The situation with respect to seven pressures agrees with the table given in section 79. In that section the shifting of the horizontal lines was fully described.

To alter the constant back pressure at the orifice $O$ in the solution tank of Figure 111 it is sufficient to raise or lower the position of the outlet $O'$ by changing the length of the vertical tube $w$. To do the same at the orifice $O$ in the gas tank of Figure 112 we need only increase or decrease the pressure at $O'$ by changing the pressure in a flow-line exterior to it. The quantity of fluid which either tank is able to produce is changed by these alterations. Herein lies the fundamental difference between Cases 1 and 3.[8]

The static pressure remains unaltered, while the constant back pressure is given different values; clearly the value of the potential pressure depends upon these alterations. In fact a decrease in the constant back pressure is equivalent in most respects to an increase in the static pressure, and likewise an increase in the constant back pressure is equivalent to a decrease in the static pressure. Cases 2 and 3 are therefore very closely related in their mathematical aspects. Figure 120 will serve in identically the same manner in the present case, if we say now that the areas subtended by the velocity curve represent quantities of fluid added to or withdrawn from the volume subject to production by means of the suggested alterations in the constant back pressure. Thus for a reduction in the constant back pressure at an instant $t_2$ the curves are re-established at values that were possessed at an

[8] In order that the external friction head offered by the orifice will not be disturbed by changes in the flow-line exterior to the orifice, we can imagine a plate at $O$ with a hole small in comparison with the cross-section of the flow-line, as in Fig. 56 ($a$).

earlier instant $t_1$ in the history of performance, and for an increase in this back pressure the curves are immediately established at values that were otherwise to be attained later at some instant $t_3$. Either travel on the paths is forced to be repeated, or there are gaps in the paths. Obviously the paths themselves are not altered in any manner. The constants $K$ in the primary function equations are the same before and after the alteration.

Inasmuch as the alterations in potential pressure might be made equally in accordance with Cases 2 and 3, the vertical cut illustrated in Figure 121 serves in the present case. The functions take on new values as indicated at the intersections of the vertical lines and the respective curves.[9]

From our discussion in Hydraulic Control we know that alterations in the constant back pressure cause the potential axis $X$ to shift upward or downward with respect to the absolute axis $X'$.

To consider a particular case let us assume that a reservoir at some instant during its history of performance possesses the following values of functions:

$$S = 125 \text{ pounds per square inch}$$

$$C = 25 \text{ pounds per square inch}$$

and therefore

$$P = 100 \text{ pounds per square inch}$$

Also

$$Ve = 100 \text{ gallons per hour}$$

$$L = 100 \text{ hours}$$

Now the constant back pressure is altered. A new value for $P$ is established, and from the relative curves the new values for the remaining func-

[9] In brief, a decrease in the constant back pressure has the following effects: (*a*) increase in the rate of production, (*b*) prolongation of the life of the reservoir, and (*c*) increase in the volume to be produced. An increase in the constant back pressure has the opposite effects. In neither instance is there any effect on the decline in the rate of production. This is a practical method of increasing the volume to be produced from a natural reservoir in case $C$ is greater than $A$. (Compare with footnote 17, § 129, p. 342.) In application, the vertical cut for an increase in $P$ will show the following:

1. The new value of $Vo$. The difference between the new and the old values is the increase to be attributed to the decrease in $C$ at the well, a decrease which we may presume to be made by means of a pump.

2. The new value of $Ve$. With this we can compute the capacity and speed necessary for the pump. It will be useless to attempt to pump at a faster rate than that indicated by $Ve$.

3. The new value of life. By this we may construct the new velocity-time curve to represent the theoretically correct rate of production during the continued process of pumping. As $Ve$ decreases we shall be able to diminish the speed of the pump, and we might, if we wish, substitute pumps of smaller capacity from time to time, leaving us the larger ones for younger or better wells.

This procedure differs from the one described in § 117, namely, constant rate production.

tions are determined. Let us say that we have the following values after the alteration:

$$S = 125 \text{ pounds per square inch, as before}$$

$$C = 62 \text{ pounds per square inch}$$

and therefore

$$P = 63 \text{ pounds per square inch}$$

Also

$$Ve = 79.4 \text{ gallons per hour}$$

$$L = 79.4 \text{ hours}$$

The situation is shown in Figure 122, wherein the two values of $C$ determine the positions of $X_1$ and $X_2$ with respect to $X'$. Had no alteration been made, the potential pressure would have traveled the path $P_1$, but inasmuch as the alteration was made, it travels the path $P_2$. Life, as we see, becomes shortened by an amount represented by the distance, measured horizontally, between $O_1$ and $O_2$.

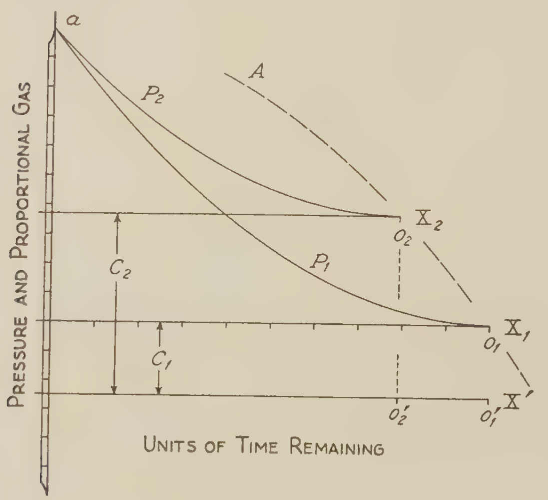


Fig. 122

It is clear that the constant back pressure might have been given any value between 0 and 125 pounds at this instant. For all values throughout this range the locus of the point $O$, denoting equilibrium, lies on a parabola with a vertex at $a$, and an axis pointing vertically downward from this vertex. Thus the parabola $A$ includes $O_1$ and $O_2$. We may take Figure 92 and revolve it in the plane of the plat through 180 degrees about its center, and see at a glance the loci of the various points $O$ for the various functions of performance, as these are caused to shift in their positions by alterations in the constant back pressure. If we were to construct curves that correspond to the present figure, for the other functions, all would have $O_2$ at the same horizontal distance from $O_1$, for life, or time remaining, is the same for all. Then in accordance with the loci of $O$, as indicated by the revolved plat, the various axes $X$ must rest at various distances above an absolute axis $X'$ for the respective functions. This feature is aside from any consideration of selected vertical scales to represent values of the various functions.

The lower limit for any point $O$ in Figure 122 is of course at the intersec-

tion of $A$ and the axis $X'$. $A$ extended below $X'$ is meaningless, for pressures below absolute zero are impossible, and the constant back pressure cannot be less than zero. That $L$ has a definite maximum limit is, of course, compatible with the idea that Volumetric Control is finite in its dimensions.

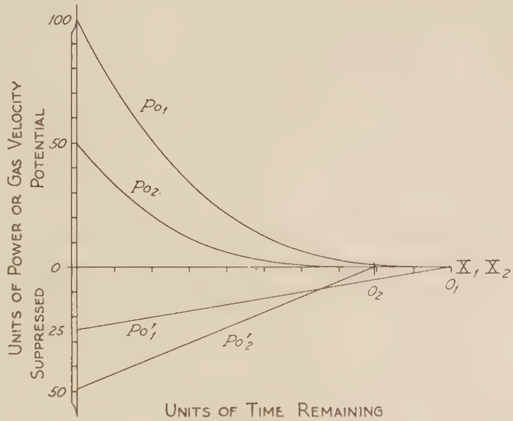

Fig. 123

To calculate the absolute contents of a reservoir of the open type we need only imagine $RC$, the registered constant back pressure, reduced to zero, and cut the relative curves accordingly, noting the value for volume. For a reservoir of the closed type we need only imagine $C$ reduced to zero, and follow the same procedure.

The vertical cut for the above data, where $P$ is 63 per cent of its original value, shows the potential power to be reduced to 50 per cent. Thus we may construct Figure 123 to correspond to Figure 118 in Case 1. Here the two axes $X_1$ and $X_2$ are superimposed in order to facilitate a comparison between the two cases. In the present case there must be a redistribution of the gas according to $S$ per cent still 100 per cent, but with $P$ per cent and $C$ per cent different on account of the alteration. According to the subtended areas, potential energy and potential gas are decreased, while suppressed energy and suppressed gas to be delivered are increased. The difference between the entire areas subtended by the pairs of curves represents the decrease in the energy and the gas to be produced, and therefore the increase in these to be retained by the reservoir in virtue of the increase in the constant back pressure.

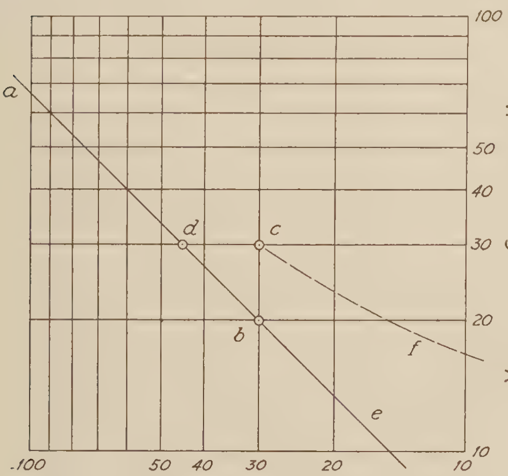

Fig. 124

To correspond with the logarithmic plat for Case 1 in Figure 119 we have Figure 124. The velocity travels from $a$ to $b$, when the constant back pressure

is decreased sufficiently to permit an increase in the velocity from 20 to 30 gallons per hour, an amount denoted by *bc*. But if the rate of flow is increased to three-halves its value life, or time remaining, must likewise be increased to three-halves its value, as shown by a vertical cut on Figure 121 at the point $Ve = 150$ per cent. This increase is denoted by *cd*. Clearly travel is interrupted at *b*, to be immediately re-established at *d*, and the path thereafter is *de*. The result is a continuous line *ae*, a portion of which is covered twice. This alteration does not affect the value of the constant $K$ in the fundamental primary function equation.

It is important to note that if the velocity is plotted without taking into account the shift in time remaining, the path appears to be *cf*, a curved line which requires the usual shift indicated by the direction of its bow. Increases in the velocity, according to Cases 1 and 3, call for shifts in the opposite direction.

133. *The three cases, concluded.*—Figure 124 serves to illustrate an alteration in accordance with Case 2, where the value of the true static pressure is altered abruptly. No modification in the figure is required. We simply say now that at the point *b* the velocity is increased from 20 to 30 gallons per hour by increasing the static pressure, while the constant back pressure remains unchanged.

The two Figures 119 and 124 are designed in conformity with increases in velocity. It should not be difficult to visualize the situation that arises when the velocity is decreased. Nor should it be difficult to visualize the situations that arise in connection with lines which represent primary function relations other than those between velocity and time. The shifts in all of these, as occasioned by any specific alteration, will be found to be in perfect agreement with vertical cuts on the chart of relative curves.[10]

Whereas in our study of Hydraulic Control it may have seemed unnecessary to distinguish between Cases 1 and 3, that is, to distinguish between the two back pressures which might be offered against production, our reason for doing so must surely have been made clear by this time. We found in section 21 that the external friction head acts as a mathematical variable, while the constant back pressure acts as a mathematical constant. Perhaps the distinction cannot be appreciated apart from the study of reservoirs in finite control.

We might know from the discussion in section 129 that vertical cuts in Figure 121 have a restricted application in Case 1. The figure can be modi-

[10] The relative constants $K$ are not affected by alterations in accordance with any of these cases, while the $K$'s of the fundamental primary function equations respond as follows:

| | | |
|---|---|---|
| Case 1 | | Altered |
| Case 2 | Real | Not altered |
| | Apparent | Altered |
| Case 3 | | Not altered |

fied, however, in a way that will enlarge its scope. To do this the following alterations are necessary:

*a*) Where acceleration appears, erase it, and insert potential pressure, volume, and energy in its place.

*b*) Where velocity appears, place power with it.

*c*) Where pressure and volume appear, for them substitute residual pressure and acceleration.

*d*) Replace "Potential Life in Per Cent" by "Reciprocal of Potential Life in Per Cent."[11]

*e*) Cancel the energy and power curves.

If desired, the internal friction head may be included with the residual pressure. The external and total friction heads should be treated apart from the relative curves in the manner outlined in section 77.

Percentage rate production is altered by a single change in the setting of a valve at the orifice. To accomplish the same alteration by means of Case 3 the constant back pressure must be continually changed. *A constant rate production by means of either Case 1 or Case 3 requires a continuous regulation of the respective back pressure.*[12]

The illustrative problems of sections 77, 78, and 79, as given in connection with the study of the three cases in Hydraulic Control, and the argument in section 80, concerning the dynamic relations between the cases, apply equally well in Volumetric Control. Where the problems were formerly solved by means of equations, the same results may now be obtained by cutting the relative curves. *The laws of theoretic performance in both controls are dependent upon the laws of ideal performance in Volumetric Control.*

Our discussion concerning the proportional production of gas in the theoretic performance of combination reservoirs does not include a consideration of the by-passing of the gas. We may judge from our earlier investigations that by-passing is merely what may be called "an accident in performance." It depends entirely upon the "lay" of the reservoir and the number and positions of its orifices. That it may have more than a momentary existence the reservoir must have lateral extension, and flow toward the orifice, or orifices, must in the main take place laterally. By-passing is impossible in the solution tank of Figure 111, if we assume gas to be present with the liquid, for flow toward the orifice is principally downward. We say that this orifice

[11] As a matter of fact the new value of life in per cent is the reciprocal of the new value of velocity in per cent. This follows in virtue of the hyperbolic loci of points in Fig. 116. "Hyperbolic variation" and "reciprocal variation" are synonymous terms, as we may readily see from the expression,

$$x \text{ varies as } \frac{1}{y}$$

[12] Nature performs this regulation under the conditions of Case 3, where a well is pumped for a constant rate production. She does this by continually lowering the level of the liquid standing in the well.

is extremely well submerged. By-passing is possible for very short intervals of time in the gas tank of Figure 112. Here the flow is lateral, but the extension of the reservoir proper is conceivably so small that we would have difficulty in detecting any undue proportion of gas. In the system of Figure 72 we assumed the possibility of by-passing. We need only imagine this system to have a great lateral extension, approaching that of a natural reservoir, in order to realize the effects of by-passing during a considerable length of time. In all lateral systems the orifices are necessarily imperfectly submerged. Alterations in theoretic performance should be expected to affect the proportional production of gas.

There is a feature concerning production from combination reservoirs which must not be neglected. Some liquids, notably soapy water, solutions of saponin, and many crude petroleums, possess the property of foaming. If bubbles surrounded by thin films are formed from these liquids, they do not break instantly, but on the contrary they last a considerable length of time, particularly if they do not come into contact with dry solid matter. These liquids are said to form a durable foam. Now when foaming liquids are produced from combination reservoirs the gas that is freed from solution upon the diminution of pressure becomes surrounded by a durable film. It is difficult, if not impossible within the time allowed, for this free gas to separate from the liquid and escape into any available space there may be for the formation of a pocket. The gas is forcibly swept out of the reservoir with the liquid. A diminution in the static pressure within a reservoir of the closed type, whether due to natural decline on production or to real or apparent alterations in accordance with Case 2, might not display the expected decline in the proportional production of gas.[13] If the reservoir is of the open type shown in Figure 111, a diminution in the static pressure that is due to natural decline on production will in any event display the expected decline in the proportional production of gas, because of the original distribution of gas in solution. This distribution is strictly in accord with the increasing hydrostatic head from the top to the bottom of the tank. Just as $S$ declines in the process of production, so must the proportional amount of gas decline. While the free surface of the liquid lowers, the declining percentage of gas arrives at the orifice. There is no escape of gas throughout the mass of liquid, as there is in the reservoir of the closed type; consequently there is no cause for foaming.

The reservoir system of Figure 72 might not show, on account of foaming, the expected proportional production of gas.

In dealing with foam it is necessary to say that it might or might not have an effect on the amount of gas per unit volume of liquid produced, because the duration of foam involves the function of time. We have the duration

[13] This is a matter of consideration with artificial reservoirs which we use for illustrating the behavior of natural reservoirs.

of the foam, on the one hand, and the velocity of movement, or the velocity of production, on the other. We can say that, for a given reservoir system, the effects of foam, particularly with respect to theoretic performance, depend upon the rate of production.[14] Slow production allows the film to break within the reservoir, while rapid production prevents it.

[14] In ideal performance the adjustments within the reservoir are proceeding in a regular fashion, and these are slow in comparison with the adjustments in theoretic performance.

CHAPTER XXII

# Sub-Volumetric Control

"Laws of Nature are, after all, merely our own inventions. They possess no authority other than that derived from their agreement with the facts they are invented to explain; they are inferior, not superior to phenomena. It is not true, for instance, to say that the planets move in ellipses because the law of gravitation constrains them; what is true is that we believe the law of gravitation to be true, because the planets move in ellipses." —E. J. C. Morton

**134.** *Introduction.*—Volumetric Control is accurately defined by the ideal performance of specifically designated ideal reservoirs. To describe these reservoirs it was found necessary to ascribe to them certain particular features. The gas tank was said to be one that is rigidly constructed; it is not subject to expansion and contraction, when alterations are made in the volume of gas which it contains. As to form no limitations were set forth. Whether it be prismatic, cylindrical, or spherical in shape is immaterial; in fact it might possess any conceivable regular or irregular form. The free surface of the gas, being coincident with the interior surface of the tank, is constant throughout a process of production. With the solution tank, however, the conditions are different. To provide a free surface of constant area the tank must have sides that can be generated by moving a line, always parallel to itself, around the edge of any horizontal area selected as the base of the tank. The base itself, of course, may be either regular or irregular in shape.[1]

Such were our limitations for these tanks. Limitations were not demanded in our study of Hydraulic Control, for the problem there resolves itself into a mere matter of maintaining a constant pressure head on the orifice. The gas holder has the capacity to expand and contract, and the form of the solution tank is immaterial, since the production of liquid under no circumstances alters the position and the area of the free surface of the liquid.

It is quite apparent that our conception of performance that is accompanied by decline in the functions of performance will be a restricted one, unless we cover a field greater than that exemplified by the type reservoirs in Volumetric Control. How can we hope to understand the performance of natural reser-

[1] See §§ 40 and 44.

voirs, if we do not break down the barriers between the ideal and the actual? Truly such barriers exist; they are particularly evident within the greater class of reservoirs that includes the reservoirs in Volumetric Control, on the one hand, and all conceivable reservoirs that would perform in a like manner, except for the fact that they possess a non-mathematical form, on the other hand.

We should not expect a natural reservoir that is capable of producing large quantities of oil, gas, or water to be the analogue of the rigidly constructed gas tank in its physical make-up. Rather should we expect it to be the analogue of the solution tank, one that is porous-filled and open to the atmosphere "at the top." It would perform as a reservoir in Volumetric Control, if it were of the required mathematical form.[2]

But how unlikely it is that a natural reservoir can have the required form! How unlikely it is that the primary function curves of Volumetric Control can be fulfilled! Granted; but let us not decide at once to discard our ideas of performance in Volumetric Control, for possibly a natural reservoir can closely approximate the performance of the type reservoir. And how shall we know if this be so? Surely from experience.

Let us see what influence form has upon the primary function curves for the delivery of liquid from a solution tank. We shall consider four familiar mathematical forms, as follows:

*a*) The **V**-shaped tank with its axis horizontal and its apex pointed vertically downward.

*b*) The pyramidal or conical tank with its axis vertical and its vertex pointed downward.

*c*) The right circular cylindrical tank with its axis horizontal.

*d*) The spherical tank.

For the sake of simplicity these are to be hollow, with their orifices at the bottom.

Indeed we shall find fundamental relative curves for these tanks to differ remarkably from the curves of Figure 92. Pressure-time relations cannot be expressed by the equation $P = KT^2$, nor can pressure-volume relations be expressed by the equation $P = KVo$, inasmuch as both of these are dependent upon the form of the tank. There is, however, one derived primary relation common among all possible forms of tanks: namely, the relation between pressure and velocity. Potential velocity varies as the square root of potential pressure; that is, $Ve = KP^{1/2}$, the correctness of which we can perhaps see. If not, we can easily verify it by experiment.

In view of the differences and agreement between the performance of these tanks and that of the type solution tank in Volumetric Control I propose to allow these facts to define a general class of reservoirs which may be conveniently designated as *Sub-Volumetric Control.* This class shall include the

[2] I here ignore the possibility that the reservoir may be one in Capillary Control.

present four tanks and all other conceivable tanks of mathematical form.[3] Specifically it shall include forms of these tanks that result from cutting them horizontally, either from the top or from the bottom.[4] Furthermore, we shall include in this general class all possible reservoirs of non-mathematical form.

135. *The* V*-shaped tank.*—So far as we are concerned the difference between tanks of mathematical and non-mathematical forms lies in the fact that for the former the equations and curves expressing the relations between the various functions of performance can be determined by analytical methods, whereas for the latter the curves can be determined only by empirical methods; that is, by actual experiment with each separate tank. For these there are no fundamental and derived primary function relations to be expressed in the form of an equation, with the exception of the usual one between pressure and velocity.

Our analytical procedure in the case of mathematical forms is already established in general terms. In brief it is as follows:

*a*) Set up the proper differential equation in accordance with Equation 38 (p. 72), integrate, determine the value of the constant of integration, and replace the symbols by $P$, $K$, and $T$. Thus we obtain the relation between potential pressure and time remaining.

*b*) Determine the relation between potential pressure and potential volume. Write this in terms of $P$, $K$, and $Vo$.

*c*) Eliminate $P$ between the two equations just obtained, and thereby have the relation between potential volume and time remaining in $Vo$, $K$, and $T$.

*d*) Differentiate this last equation with respect to $T$ to obtain the relation between potential velocity and time remaining in terms of $Ve$, $K$, and $T$.

*e*) Differentiate once more to obtain the relation between potential acceleration and time remaining in terms of $Ac$, $K$, and $T$.

*f*) Multiply the equations of (*a*) and (*c*). This gives us the relation between potential energy and time remaining, as expressed in $E$, $K$, and $T$.

*g*) Either multiply the equations of (*a*) and (*d*), or differentiate the equation of (*f*) with respect to time, in order to obtain the relation between potential power and time remaining, as expressed in terms of $Po$, $K$, and $T$.

When the mathematical equations are complicated, it is sometimes convenient to modify the procedure after the equations of (*a*) and (*b*) are obtained, by taking advantage of the invariable relations between pressure and

[3] There are, of course, several other forms which might be included in a complete analysis. The first which come to mind are the following: (1) the inverted V-shaped tank; (2) the inverted pyramidal or conical tank; (3) the half circular cylindrical tank with its axis horizontal, the base being at the diameter of the circle; and (4) the half spherical tank with its base at the diameter of the sphere.

[4] These cuts are unlike, for one affects the position of the orifice, while the other does not.

velocity with this class of reservoirs. As a timesaver we might also avail ourselves of short cuts wherein the handling of equations is discontinued, and a sufficient number of points are calculated in accordance with the mathematical derivations that are outlined in the foregoing general procedure. We shall see how these particular methods are applied.

Now for the **V**-shaped tank. Its relative curves are shown in Figure 125. Let us derive their equations.

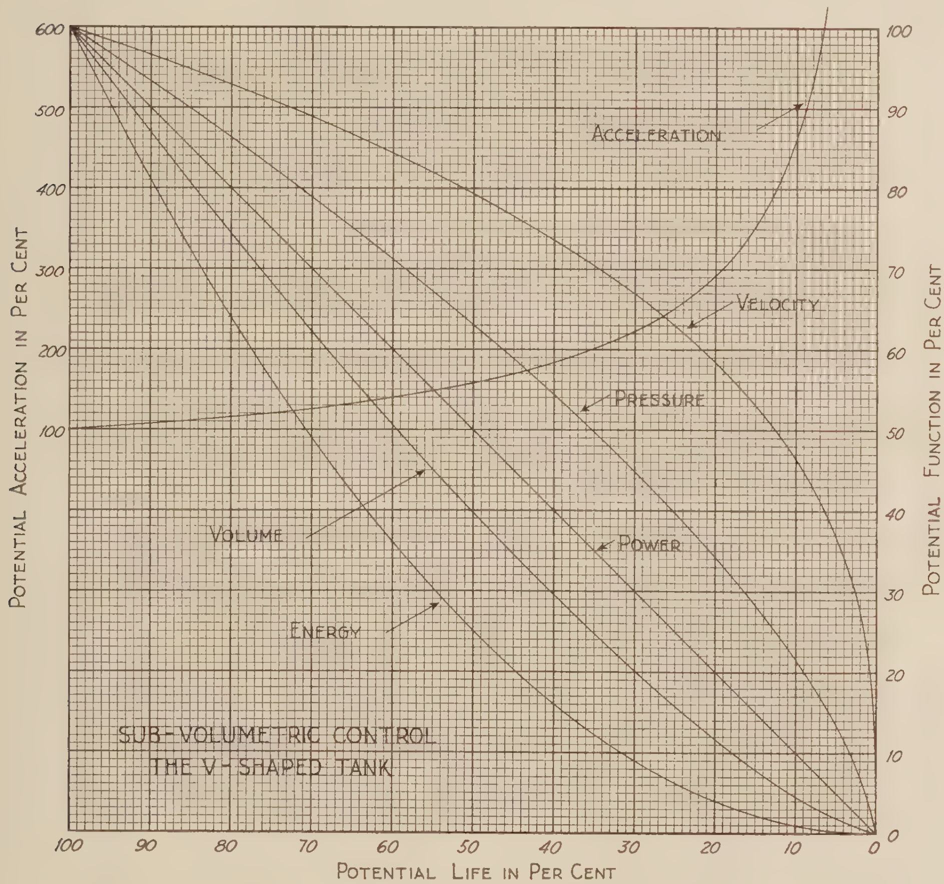


Fig. 125

Since we are not interested in the values of constants until we come to set up the relative equation, we can designate the preliminary ones by the symbols $k$ or $K$, without regard to the possibility of their being, or not being, related to each other. The equation referred to above can therefore be reduced to

$$dt = k \frac{A}{y^{1/2}} dy \qquad (326)$$

wherein $dt$ is the differential of the time $t$, $A$ is the area of the free surface of liquid, and $dy$ is the differential of the pressure $y$. As stated in section 44, $A$ must be expressed in terms of $y$ before the equation can be integrated. This is simple, for with the **V**-shaped tank

$$A = ky \qquad (327)$$

Upon substituting this value of $A$ into Equation 326 we have

$$dt = ky^{1/2}dy \qquad (328)$$

which by integration becomes

$$t = ky^{3/2} \qquad (329)$$

if we omit the constant of integration which is zero. By replacing symbols we have

$$T = KP^{3/2} \qquad (330)$$

or by transposition of the terms in this expression,

$$P = KT^{2/3} \qquad (331)$$

When $K$ is evaluated according to the rule given in section 110, the relative equation becomes

$$P = 4.642T^{2/3} \qquad (332)$$

To determine points for the curve it is only necessary to substitute a series of successive values for $T$ into this equation; for example, 0, 10, 20, 30, and so on.

According to the principles of solid geometry

$$v = ky^2 \qquad (333)$$

where $v$ is the volume of the liquid within the tank, and $y$ is the depth of the liquid, or the pressure head, as in Equation 327. By replacing the symbols we have

$$Vo = KP^2 \qquad (334)$$

Eliminate $P$ between Equations 331 and 334. Thus

$$Vo = KT^{4/3} \qquad (335)$$

The relative equation becomes

$$Vo = 0.2154T^{4/3} \qquad (336)$$

By differentiating Equation 335 we obtain

$$Ve = KT^{1/3} \qquad (337)$$

and this becomes

$$Ve = 21.54T^{1/3} \qquad (338)$$

One more differentiation gives us

$$Ac = KT^{-2/3} \quad \text{(339)}$$

which becomes

$$Ac = 2{,}154T^{-2/3} \quad \text{(340)}$$

The curve for this equation is hyperbolic, as indicated by the negative exponent. Acceleration increases while time remaining decreases. The relative constant provides an arbitrary value of 100 per cent acceleration when time remaining is 100 per cent.[5]

The multiplication of Equations 331 and 335 gives us

$$E = KT^2 \quad \text{(341)}$$

which becomes

$$E = 0.01T^2 \quad \text{(342)}$$

By multiplying Equations 331 and 337, or by differentiating Equation 342, we have

$$Po = KT \quad \text{(343)}$$

and this becomes

$$Po = T \quad \text{(344)}$$

Thus we have a complete set of equations for the fundamental primary function curves of performance for the **V**-shaped tank. All curves are parabolic with the one exception noted. Of the various derived primary function relations we already have one in Equation 334, as between pressure and volume. To derive pressure and velocity relations Equation 331 may be written

$$P^{3/2} = KT$$

and Equation 337 may be written

$$Ve^3 = KT$$

From these

$$Ve^3 = KP^{3/2}$$

therefore

$$Ve = KP^{1/2}$$

In this we return to our starting-point.

The equation for any couplet in derived relations can be written down by the rule given in section 110.

**136.** *The pyramidal or conical tank.*—The laws for the delivery of liquid from pyramidal and conical tanks are identical. All pyramids and cones are

[5] If there should be any reason for adopting some other point as 100 per cent acceleration, we could do so. (Acceleration at $T =$ zero excepted.)

generated geometrically in the same way; namely, by pivoting a straight line about a point in the line, returning to the position at the beginning. A closed figure is thus described, and this figure may have a cross-section of any desired form. It is immaterial whether this cross-section is square or circular, or in fact whether it is geometrically regular or irregular, in so far as those geometrical properties of the generated form that are of importance to us are concerned.

We proceed as before. $A$, the area of the free surface of the liquid, must be expressed in terms of $y$ before Equation 326 (p. 363) can be integrated. This is again simple, for with all pyramidal and conical tanks

$$A = ky^2 \qquad (345)$$

We therefore have

$$dt = ky^{3/2}dy \qquad (346)$$

which by integration becomes

$$t = ky^{5/2} \qquad (347)$$

The constant of integration is again zero. By replacing the symbols we have

$$T = KP^{5/2} \qquad (348)$$

or by transposition of the terms in this expression,

$$P = KT^{2/5} \qquad (349)$$

The relative equation is

$$P = 15.85T^{2/5} \qquad (350)$$

According to the principles of solid geometry[6]

$$v = ky^3 \qquad (351)$$

where $v$ is the volume of liquid within the tank, and $y$ is the depth of the liquid. When the symbols are replaced this becomes

$$Vo = KP^3 \qquad (352)$$

Eliminate $P$ between Equations 349 and 352. Thus

$$Vo = KT^{6/5} \qquad (353)$$

and the relative equation is

$$Vo = 0.3982T^{6/5} \qquad (354)$$

In view of the fact that the procedure from here presents no novel fea-

[6] The principles of geometry allow us to express $A$ and $v$ in terms of $y$ for all reservoirs of mathematical form. Herein lies the difference between mathematical and nonmathematical forms.

tures the remaining equations may be tabulated without further discussion. We have the following:

$$Ve = KT^{1/5} \quad \ldots\ldots\ldots\ldots \quad (355)$$

$$Ve = 39.82T^{1/5} \quad \ldots\ldots\ldots\ldots \quad (356)$$

$$Ac = KT^{-4/5} \quad \ldots\ldots\ldots\ldots \quad (357)$$

$$Ac = 3{,}982T^{-4/5} \quad \ldots\ldots\ldots\ldots \quad (358)$$

$$E = KT^{8/5} \quad \ldots\ldots\ldots\ldots \quad (359)$$

$$E = 0.0631T^{8/5} \quad \ldots\ldots\ldots\ldots \quad (360)$$

$$Po = KT^{3/5} \quad \ldots\ldots\ldots\ldots \quad (361)$$

and

$$Po = 6.31T^{3/5} \quad \ldots\ldots\ldots\ldots \quad (362)$$

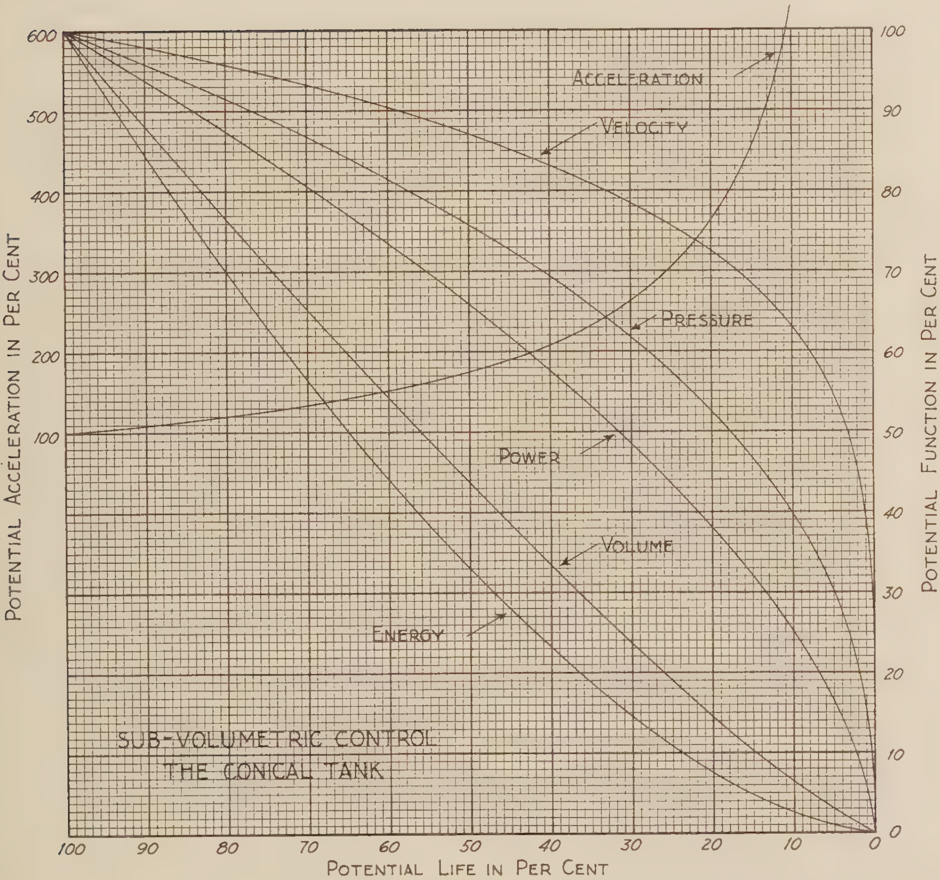


FIG. 126

The relative curves are shown in Figure 126. Their general characteristics are similar to the curves of Figure 125. Pressure-volume relations, as shown by Equation 352, are now represented by a cubic parabola instead of

the true parabola for the same relations in the V-shaped tank. Pressure-velocity relations, as may be determined by Equations 349 and 355, are as usual.

137. *The horizontal cylindrical tank.*—The analyses of the preceding tanks are simple in comparison with that of the horizontal cylindrical tank. All the curves of Figures 125 and 126 are power-function curves of a comparatively simple nature. Their equations are of the form $y = kx^n$, where the exponent $n$ is either positive or negative, and consequently where the curves are either parabolic or hyperbolic. In all equations the term on the right is a monomial expression. We are now to encounter equations that have terms which are more complex.

The determination of $A$, the free surface of the liquid within a cylindrical tank whose axis is horizontal, is not difficult. The result of such a determination, to be substituted into the equation

$$dt = \frac{A}{y^{1/2}}\, dy \ldots\ldots\ldots\ldots\ldots\ldots (363)$$

is as follows:

$$A = k(2Ry - y^2)^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots (364)$$

in which $R$ is the radius of the vertical circular cross-section of the tank, and $y$ is the depth of the liquid, as before.[7] Upon substituting this value into Equation 363 we have

$$dt = k(2R - y)^{1/2}\, dy \ldots\ldots\ldots\ldots\ldots\ldots (365)$$

This by integration becomes

$$t = -\, k(2R - y)^{3/2} + C \ldots\ldots\ldots\ldots\ldots\ldots (366)$$

where $C$ is the constant of integration. We have assumed $t$ to be time remaining in the usual manner, and now a negative sign appears before the constant in the equation.[8] This is significant indeed, for it means that *the negative of time remaining, that is, time elapsed, is the true function of performance for this tank.* Heretofore we have only said that, in general, time remaining is the true function of performance. Now we meet an exception which is unique in the present treatise.

We shall reckon, then, with time elapsed. Let

$$y = 2R \text{ when } t = \text{zero}$$

[7] Here we take the equation of the circle with its center at the origin of co-ordinates, and transfer the origin to a point on the circumference, this point representing the position of the orifice at the bottom of the tank. The steps to be taken thereafter are purely algebraic.

[8] It is preferable to assume all constants $k$ and $K$ as positive. If the assumption is incorrect, a negative sign will automatically appear before them.

and the constant of integration becomes equal to zero. Now we replace the symbols to obtain

$$T = -K(2R - P)^{3/2} \qquad (367)$$

or by transposition of the terms in this expression,

$$P = KT^{2/3} + 2R \qquad (368)$$

For the relative equation the value of $K$ must be determined. When

$$T = \text{zero}, \qquad P = 2R = 100\%,$$

and when

$$T = 100\%, \qquad P = \text{zero}.$$

The substitution of the latter set of values into Equation 368 gives

$$K = -\frac{2R}{100^{2/3}}$$

which reduces to

$$K = -100^{1/3} = -4.642$$

by taking $2R$ to be 100 per cent.[9] The relative equation is therefore

$$P = 100 - 4.642T^{2/3} \qquad (369)$$

Points for the curve are determined in the usual way; $P$ will be expressed in percentages of $2R$.

The geometrical principle involved in the determination of pressure-volume relations is simple, but the actual calculation necessitates the integration of a somewhat complex expression and thereafter a tedious series of substitutions into the resulting equation. Figure 127 represents a vertical profile of the horizontal cylindrical tank. Its center is at $O'$, and the orifice of the tank is at $O$. While the free surface of the liquid lowers in the process of production, what percentage of the original volume, say the full volume, remains within

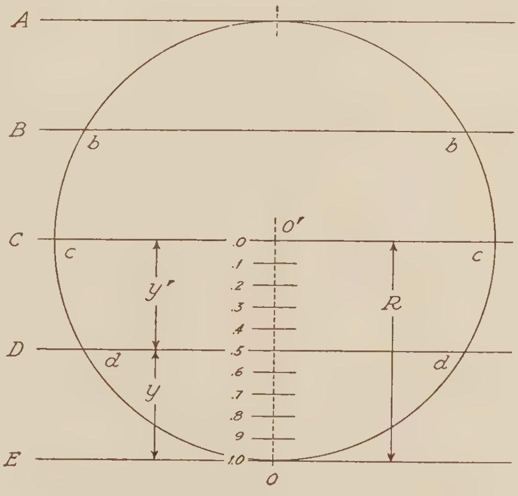

FIG. 127

[9] The number expressing the value of $K$ requires a negative sign before it. In mathematics there is no reason why a constant which we represent by the symbol $K$ cannot be negative, so that a negative sign before it renders it a positive number. It is less confusing, perhaps, always to consider the symbol $K$, without sign, a positive number.

[ § 137

the tank? In other words, while a horizontal line is passed from the top to the bottom of the circle in the figure, what percentage of the entire circle is subtended by the line? The line occupies successive positions as at $A$, $B$, $C$, $D$, and $E$, say, and when it is at $B$, what is the percentage ratio between the area $ObbO$ and the area of the complete circle? We need only make our computations for the bottom half of the circle, if we take advantage of the symmetry of the circle with respect to its diameter. We divide the vertical radius $R$ into ten equal parts, and determine such areas as $OccO$, $OddO$, and so on. Since the equation of a circle is simplest when expressed with regard to an origin at its center, it is convenient to first determine such areas as $dccd$, measured from $C$, then subtract this from the area of the half-circle to obtain such areas as $OddO$. That is to say, we use a distance $y' = R - y$, where $y$ is the pressure head on the orifice, and subsequently reduce the quantities to terms of $y$. It is readily seen that, where $B$ is above $C$ at the same distance $D$ is below, the area $ObbO$ is equal to the area of the half-circle plus the area $dccd$. Thus for all pressures $y$ between 0 and 100 per cent a corresponding volume $v$, likewise between 0 and 100 per cent, can be determined.

Let $v'$ be the volume that corresponds to $y'$, then the integral equation for direct substitutions is[10]

$$v' = \frac{y'}{2}(R^2 - y'^2)^{1/2} + \frac{R^2}{2} \text{ arcsine } \frac{y'}{R} \ldots\ldots\ldots\ldots (370)$$

For arcsine we read, "The angle in radians whose sine is $y'$ divided by $R$." The ratios between $y'$ and $R$ must be expressed in natural numbers, as 0, 0.10, 0.20, 0.30, and so on, and not in percentages. The other quantities may well be expressed in percentages.

Let us have the tabulated values of $y$ and $v$; that is, of $P$ and $Vo$.[11] These pertain to the lower half of the tank, on the assumption that $Vo$ and $P$ for the entire tank are each 100 per cent, when it is full.

| $P$ | $Vo$ |
|---|---|
| 0.0 | 0.00 |
| 5.0 | 1.89 |
| 10.0 | 5.24 |
| 15.0 | 9.41 |
| 20.0 | 14.24 |
| 25.0 | 19.55 |
| 30.0 | 25.25 |
| 35.0 | 31.15 |
| 40.0 | 37.30 |
| 45.0 | 43.65 |
| 50.0 | 50.00 |

[10] This is the integration of

$$(R^2 - y'^2)^{1/2}\, dy'$$

See Form 125 in Peirce, *A Short Table of Integrals*, Ginn & Company.

[11] A table of arcsines for values between 0 and 1 is given in Appendix E.

These points may be plotted for the lower right half of the curve. Then the plat may be revolved 180 degrees about its center, and the plotting repeated as of a like position. In this manner we have the entire curve.

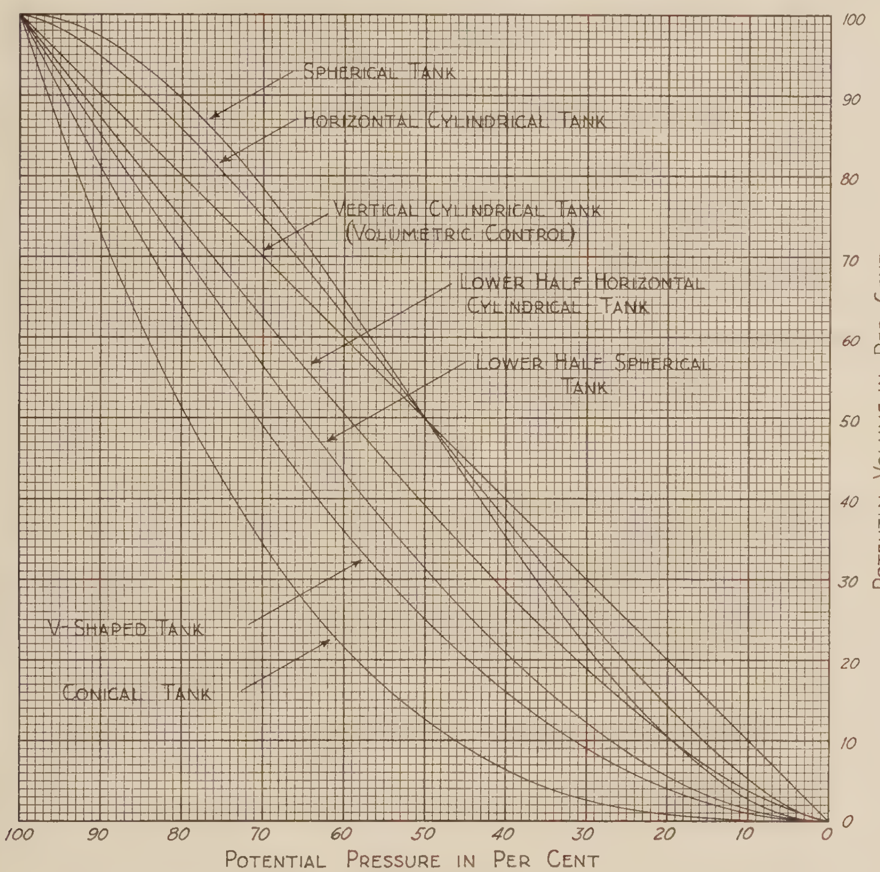


Fig. 128

Figure 128 shows a series of curves that represent relations between pressure and volume.[12] Those met in earlier sections are for the following:

[12] All areas subtended by the curves in this figure represent potential energy. The constants $K$ in the equation $E = KPVo$, necessary to express quantitative relations, can be determined from the equations of these curves. Where pressure-volume relations are parabolic, as in the first two of our four tanks, $K$ is truly a constant, the same as we found it to be in Hydraulic and Volumetric controls. In the horizontal cylindrical and spherical tanks $K$ assumes a series of values during the process of production. In other words, the $K$ commensurate with those for the other forms of tanks becomes a variable. For all similar tanks in identical stages of production, however, the variable $K$'s have identical values. *Under any circumstance whatever, whether* K *be constant or variable, its value is equal to the ratio between the area by the pressure-volume curve and the rectangle inclosing it.* We are reminded that all areas subtended by the power-time curves also represent energy.

(*a*) the vertical cylindrical tank in Volumetric Control, (*b*) the **V**-shaped tank, and (*c*) the pyramidal or conical tank. With these are included the relations for (*d*) the horizontal cylindrical tank, in accordance with the foregoing table of values.

In addition there are given the relations for the following which are yet to be discussed: (*e*) the spherical tank, (*f*) the lower half horizontal cylindrical tank, and (*g*) the lower half spherical tank.

Let us continue with the present problem. Our equation between pressure and volume is quite complex, and therefore it is not convenient for use in the manner of previous analyses. To obtain the curve for the relations between volume and time without the necessity of deriving the equation we may proceed as follows:

First. Complete the above table of relations between pressure and volume by the simple arithmetical methods that are suggested by the symmetry of the two half-circles.

Secondly. Change Equation 369 into the following form:

$$T = \left( \frac{100 - P}{4.642} \right)^{3/2} \dots\dots\dots\dots (371)$$

and compute values of $T$ that correspond with the values of $P$ in the table.

Now it is clear that we possess corresponding values of $P$, $Vo$, and $T$. The curve for volume and time can consequently be plotted from these.

The equation between velocity and time is easily obtained. Between velocity and pressure we have the relative equation

$$Ve = 10P^{1/2} \dots\dots\dots\dots (372)$$

which may be combined with Equation 369 in a way to eliminate $P$. Thus we obtain

$$Ve = (10{,}000 - 464.2T^{2/3})^{1/2} \dots\dots\dots\dots (373)$$

The advantage in having this equation lies in the fact that it can be differentiated with respect to $T$ for acceleration. Thus

$$Ac = \frac{-464.2}{3(10{,}000 - 464.2T^{2/3})^{1/2}T^{1/3}} \dots\dots\dots\dots (374)$$

an expression which may be reduced to

$$Ac = \frac{-154.7}{VcT^{1/3}} \dots\dots\dots\dots (375)$$

This is the absolute value of acceleration in terms of velocity. It is negative, and hyperbolic. Its proper location is in the quadrant below that containing the velocity. It is observed that when

$$T = 0, \text{ and } Ve = 100, \text{ then } Ac = -\text{ infinity},$$

and that when

$$T = 100, \text{ and } Ve = 0, \text{ then } Ac = -\text{ infinity}.$$

Let us change Equation 375 into a true relative equation. We shall say that when the pressure is 50 per cent, that is, when the tank is half full, acceleration is 100 per cent.[13] The negative sign may be ignored, since we are only interested in relative values based upon this 100 per cent value of the function. From Equation 369,

$$T = 35.35\%, \quad \text{when} \quad P = 50.00\%,$$

$$\text{and} \quad T^{1/3} = 3.282\%.$$

And from Equation 372,

$$Ve = 70.71\%, \quad \text{when} \quad P = 50.00\%.$$

In accordance with Equation 375 we may write

$$Ac = 100 = \frac{K}{3.282 \times 70.71} \ldots\ldots\ldots\ldots\ldots (376)$$

where $K$ is the desired relative constant, equal to 23,210. The relative equation is therefore

$$Ac = \frac{23{,}210}{VeT^{1/3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (377)$$

Corresponding values in pressure and volume may be multiplied to obtain energy, and similarly corresponding values in pressure and velocity may be multiplied to obtain power.

We are now able to construct the six fundamental primary function relative curves for the horizontal cylindrical tank, as shown in Figure 129 (p. 374). While all the relations pertain directly to time elapsed, nevertheless the hori-

[13] This is clearly an arbitrarily selected point for acceleration to be equal to 100 per cent.

zontal scale may be reversed so as to represent time remaining. This is, of course, permissible.[14]

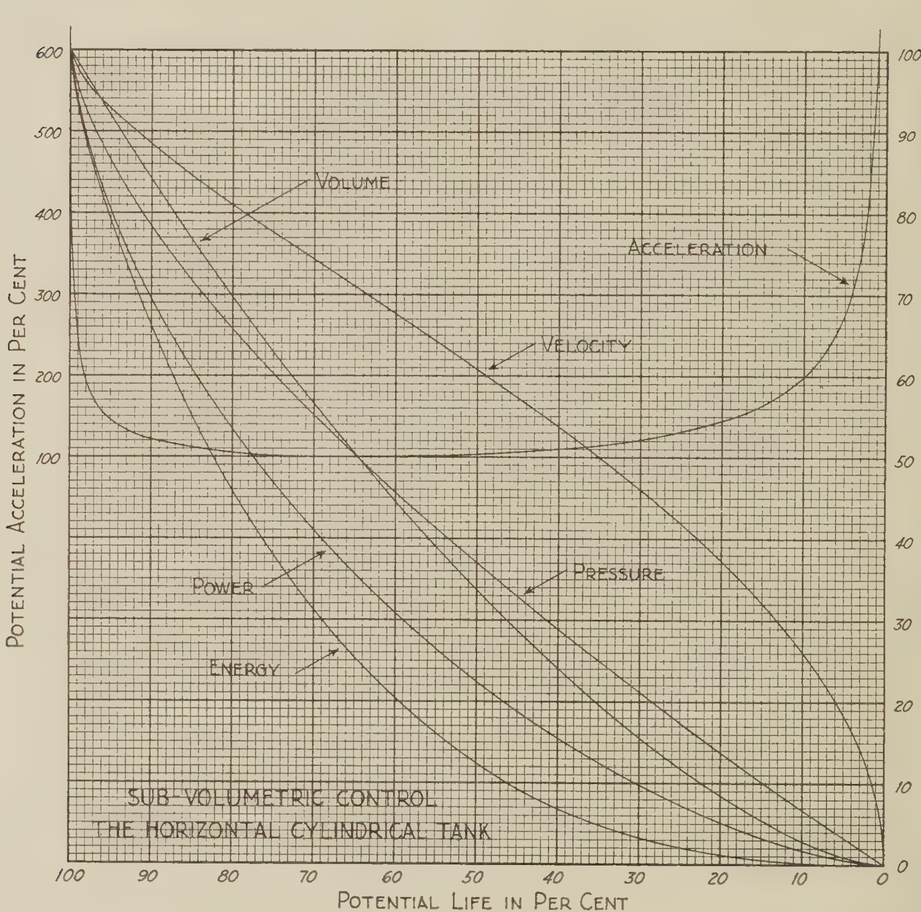


Fig. 129

138. *The spherical tank and other forms.*—The method employed in the preceding section is equally appropriate in the case of the spherical tank. For the free surface of the liquid we have[15]

$$A = k(2Ry - y^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (378)$$

[14] Whenever we know the position of the zero-zero point pertaining to the equation of a curve, the scales for the curve can be reversed. This reversal is entirely independent of the equation of the curve, whose true zero-zero point remains fixed with respect to its other points. For the sake of symmetry the acceleration-time curve is inverted.

[15] The expression following is derived from the equation of the sphere with its center at the origin of co-ordinates. The procedure in setting up the equation for $A$ is similar to that in the case of the horizontal cylindrical tank. It will be noted that integrations for the sphere are more easily performed than those for cylinders.

Thus the differential equation is

$$dt = k(2Ry^{1/2} - y^{3/2})\,dy \qquad (379)$$

which by integration becomes

$$t = k\left(\frac{4}{3}\,Ry^{3/2} - \frac{2}{5}\,y^{5/2}\right) + C \qquad (380)$$

If $t =$ zero when $y =$ zero, then $C$, the constant of integration, is zero. Time remaining is the function of performance for this tank.

By replacing the symbols we have

$$T = K\left(\frac{4}{3}\,RP^{3/2} - \frac{2}{5}\,P^{5/2}\right) \qquad (381)$$

When

$$P = 2R = 100\%, \quad \text{then} \quad T = 100\%.$$

The substitution of these values into the equation gives

$$K = 0.00375$$

consequently the relative equation becomes[16]

$$T = 0.25P^{3/2} - 0.0015P^{5/2} \qquad (382)$$

For the relations between pressure and volume we proceed in exactly the same manner as before, computing values of $v'$ in terms of $y'$, as shown in Figure 127, a figure which we may now imagine to represent a sphere. We find that

$$v' = \pi(R^2y' - \tfrac{1}{3}\,y'^3) \qquad (383)$$

The following are tabulated values of $y$ and $v$; that is, of $P$ and $Vo$. These pertain to the lower half of the tank, on the assumption that $Vo$ and $P$ for the entire tank are each 100 per cent, when it is full.

| $P$ | $Vo$ |
|---|---|
| 0.0 | 0.000 |
| 5.0 | 0.725 |
| 10.0 | 2.800 |
| 15.0 | 6.075 |
| 20.0 | 10.400 |
| 25.0 | 15.625 |
| 30.0 | 21.600 |
| 35.0 | 28.175 |
| 40.0 | 35.200 |
| 45.0 | 42.525 |
| 50.0 | 50.000 |

These points may be plotted in the same manner as before. The resulting curve is shown in Figure 128.

16 The transposition of terms, as in Equations 367 and 368, is impossible because of the fact that $P$ now occurs with two different exponents.

Corresponding values for pressure, volume, and time furnish us the means of plotting the volume-time curve. By substituting into Equation 382 the value

$$P = \frac{Ve^2}{100} \qquad (384)$$

we have

$$T = 0.25\,\frac{Ve^3}{1{,}000} - 0.0015\,\frac{Ve^5}{100{,}000} \qquad (385)$$

where the constants are in a convenient form for computations. To obtain points for the velocity-time curve it is obviously simpler to determine corresponding values for pressure, velocity, and time, in the way that the volume-time curve is handled, by using Equation 384.

The differentiation of Equation 385, and a subsequent simplification, results in the expression

$$Ac = \frac{250{,}000}{100P - P^2} \qquad (386)$$

Here again it has been necessary to adopt a relative constant such that the value of acceleration is 100 per cent when the tank is half full.

Energy and power are evaluated again by multiplying corresponding values of pressure and volume, and pressure and velocity, respectively.

The six fundamental primary function relative curves are shown in Figure 130.[17]

The derived primary function relations—those that are independent of $T$ as a function of performance—for the four tanks we have now analyzed may be obtained by cutting their respective charts with vertical lines, these lines being located by a particular value of one of the functions.

We may approach any of these tanks at any instant during their process of production and adopt the values of all the functions for the instant as 100 per cent standard conditions. To adapt the relative curves to such a situation we need to cut the curves as shown on the charts by a properly located vertical line, and stretch the remainder of the curves to the right of the line in such a manner that the points of intersection, where the vertical line cuts each curve, pass to the upper left-hand corner of the frame. For the first two tanks the relative curves continually repeat themselves upon stretching, but for the last two they do not. Here each cut gives a different curve. This is equivalent to the statement that with the first two tanks the sides may be either shortened or lengthened to any extent and the result is a tank of identically the same form, whereas with the last two a shortening alters the form of the tank, inasmuch as the radius remains unaltered, and a lengthening is impossible, unless there has previously been an equal or greater shortening, while even in such a case the lengthening alters the form of the tank.

[17] The acceleration-time curve is not inverted in this figure.

Two tanks of interesting form are the lower halves of the horizontal cylindrical and spherical tanks. Their pressure-volume relations are shown in Figure 128. These curves are obtained by simply stretching the portions of the complete curves which lie between 0-0 and 50-50, so that they extend from 0-0 to 100-100. If we wish to construct the fundamental relative curves for these half-tanks we need only cut Figures 129 and 130 at $P = 50$ per cent, and stretch the portions of curves at the right in the usual manner.

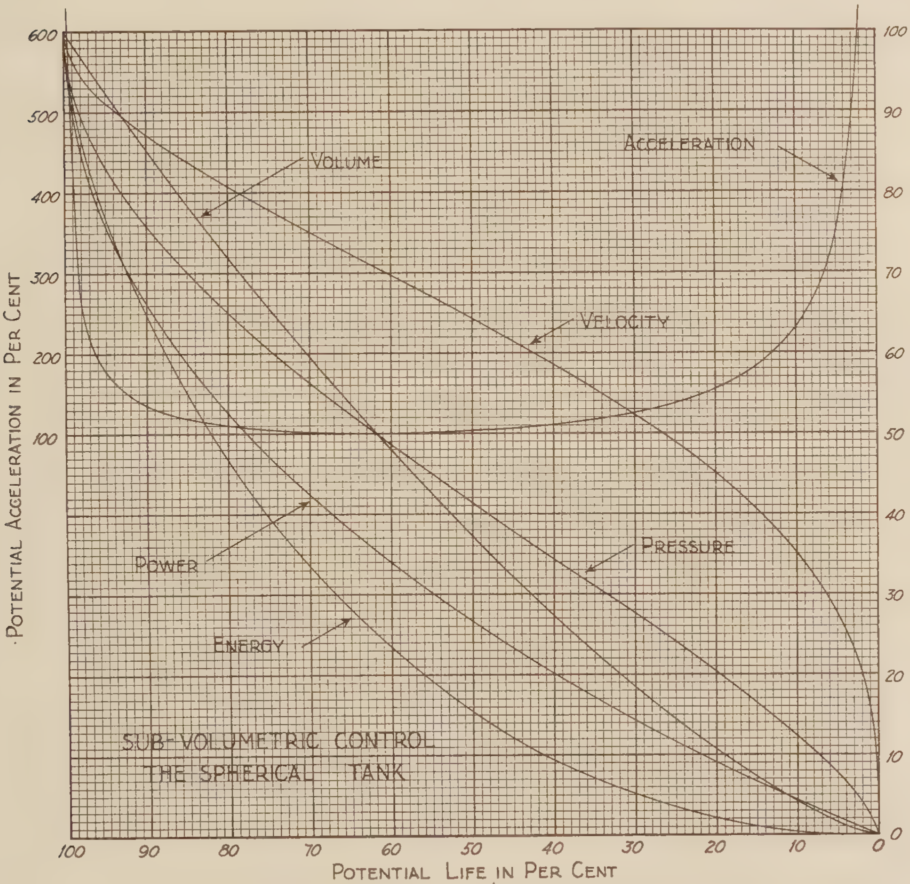


Fig. 130

Other tanks of mathematical form are readily imagined.[18] To analyze these we should proceed in the same way as above, reserving the right to modify the general method now and then to suit the occasion. I believe the present four suffice to show those characteristics in which we are deeply interested. We see how the performance of reservoirs may be analyzed. We see

[18] See footnote 3, § 134, page 362.

how the form of the reservoir affects the laws of delivery in case it produces liquid.

We have found the curves to be repeated or modified when the tanks are cut, or imagined cut, at the top. In this process the position of the orifice remains fixed, and in agreement with this fact the cutting and stretching of the relative curves pertained to a stationary zero-zero point on the charts. But if, as indicated in Figure 131, these tanks were cut off at the bottom, on any of the numbered lines as shown, and an orifice were supplied to a newly placed horizontal plate which serves to close the tank at the bottom, can we obtain correct relative curves by cutting and stretching the left-hand portions of curves to zero-zero? No; we cannot.

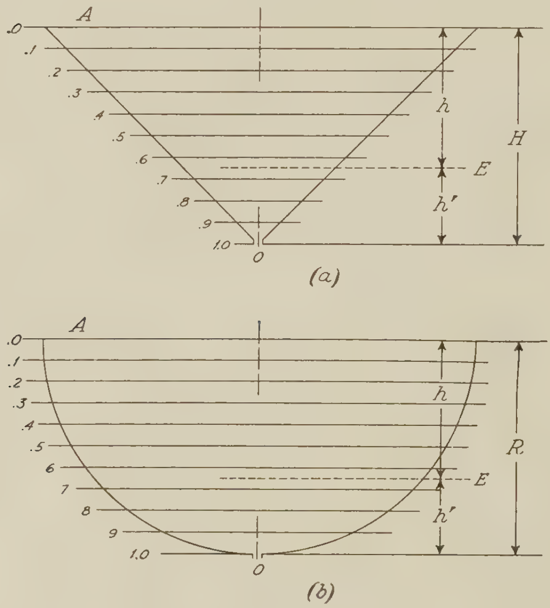


Fig. 131

Let us imagine the tanks to be cut successively in accordance with the diagrams in the figure. $R$ and $H$ remain unchanged, but $h'$ is increased from 0 to 1.0, while $h$ is diminished from 1.0 to 0. $E$ is simply the variable position of the bottom plate which contains the orifice $O$. These diagrams serve to represent the V-shaped tank, the pyramidal or conical tank, the half cylindrical tank and the half spherical tank. Now as $E$ is caused to vary from the bottom to the top, the relative curves progressively change from Figures 125 and 126 and from the charts for the half-tanks corresponding to Figures 129 and 130, to Figure 92, the last being the chart for Volumetric Control. Intermediate positions may be regarded as approximations to Volumetric Control; the smaller is the ratio $h/H$, or $h/R$, the closer is the approximation. The processes of cutting a tank at the top and at the bottom are mathematically dissimilar. Charts for the successive cuts may be constructed in accordance with the usual method.[19] At the outset the differential

[19] The scope of the present treatise does not permit the construction and presentation of a complete set of successive charts containing pressure-time, volume-time, or pressure-volume curves. I do not believe it is difficult to visualize the situation with respect to this gradual metamorphosis of the curves.

equations between pressure and time are different for each cut, while pressure-volume relations are yet determinable from the chart of Figure 128.

*In general, when the lateral extent of the reservoir is great in comparison with the vertical extent, the influence that the sides bear upon the laws of delivery is little, and this is of course true whether the sides are of mathematical or of non-mathematical form.*

## CHAPTER XXIII

# Natural Reservoirs

"The laws of Nature are the outcome of the characters of the entities which we find in Nature. The entities being what they are, the laws must be what they are; and conversely the entities follow from the laws. We are a long way from the attainment of . . . . an ideal; but it remains as the abiding goal of theoretical science."—A. N. WHITEHEAD

139. *Introduction.*—We can say from experience that there are natural reservoirs which produce oil, gas, and water, singly or in combination, in what is properly described as Volumetric Control. Like those in Hydraulic Control, the present reservoirs are of the open type. Where into the former class of reservoirs water is entering the formation from the surface at a sufficient rate to maintain the free surface of the liquid at a constant level, now air—possibly with some water—is entering instead. This air permits the lowering of the free surface during the process of production.

The ideal natural reservoir in Hydraulic Control was described in section 60. For the present control we need add but one item to those specifications: namely, *that the lateral dimensions of the reservoir are such as to provide a constant area of free surface to the column of liquid while the head is lowering.* In the field we might expect to find closer approximations to the ideal in Hydraulic Control than in Volumetric Control, for so long as the surface of liquid in the formation is maintained at a constant level its area is constant, and any variations in the lateral dimensions at successive elevations do not matter. We can only say that we frequently do meet close approximations in Volumetric Control, and this circumstance seems to be explained by the fact that at least one of the two lateral dimensions is great in comparison with the vertical dimension.[1]

How may we be certain of these approximations? Simply by noting the fact that when due allowances for alterations in the apparent static pressure of the reservoir are made, such alterations being caused by the drilling of

[1] This is in agreement with the last paragraph of § 138. In speaking of lateral dimensions of the reservoir we mean, as stated previously, the extension of the formation with its liquid, whatever may be the nature of this liquid. We may expect water to surround all pools of oil and gas, and this water is as much a part of the reservoir system as the oil and gas.

additional wells into the same formation, the primary function relations as shown in Figure 92 are closely fulfilled. The selection of the type solution tank as the representative of the class of reservoirs to which we now refer appears to be proper.

Now that we have our ideal natural reservoir in Volumetric Control clearly defined, we may proceed to discuss some of its pertinent features. At the outset it is evident that all we have studied in the preceding chapters concerning the fundamental and derived primary function relations in ideal and theoretic performance, as there applied to artificial reservoirs in this control, may be adopted as material which is properly applicable to the present reservoir. In regard to actual reservoirs in the field we shall prefer to let each one stand on its own merits with respect to its degree of accuracy in performance.

The secondary functions of performance which pertain to the events and conditions within the reservoir are dependent upon the ideal state only in so far as the gradient curves and surfaces are dependent upon it. In order that these curves and surfaces may possess relatively simple mathematical equations, and in order that these equations may possess the proper instantaneous succession of constants while the free surface of the liquid is lowering, the reservoir must be ideal. *The general analytical features of these curves and surfaces, however, really persist without regard to the existence or non-existence of the ideal state.* Where we find asymptotic relations between the kinetic and static pressure gradients that are mathematically perfect, we find them likewise between these gradients when they are warped. Where the radius or area of drainage extends to the physical limits of the ideal reservoir, it does likewise in the non-ideal reservoir of the same class. Where water continually encroaches upon an area occupied by a pool of oil or gas in one, it does likewise in the other. Thus it is with all other secondary functions. The differences between the ideal and the non-ideal involve questions that concern the mathematical constancy, or the mathematical variation, of features throughout the space occupied by the reservoir and throughout the time which constitutes the life of the reservoir; they do not involve questions that concern the basic characteristics of these features.[2]

**140.** *The pressure diagram.*—For the purpose of analyzing the situation within a natural reservoir in Volumetric Control the pressure diagram of Figure 47, although designed for Hydraulic Control, will serve. The discussion introduced in section 71 concerning this diagram, and continued in subsequent sections, is applicable in the present control, provided we recognize certain features which now become essentially relevant.

[2] See §§ 91 and 92. "At any instant, and for any instant, a reservoir in Volumetric Control is equivalent to one in Hydraulic Control. It is in the succession of instants where we observe a difference between them."—§ 126. This applies specifically to the primary functions in theoretic performance and to the secondary functions in ideal, theoretic, and actual performance.

The horizontal lines pertain to the well at $W$. During the process of production the line $I$ now approaches the line $N$. In view of the fact that a porous medium is present, a horizontal line $Q$, somewhere between $I$ and $N$, must be added to the figure in case the well produces both liquid and gas. This line was explained in section 127 with regard to the tanks of Figures 111 and 112. We say, then, that $I$ approaches $N$, and, if present, $Q$ approaches $N$, the latter continually remaining at a constant distance below $I$. At the end of the life of the reservoir, under the given conditions of production as shown by the position of the lines $K$, $J$, and $N$, either $I$ or $Q$ actually reaches $N$. The free surface of liquid descends from $a$ to $f$ in the absence of $Q$, or to some point $d$ above $f$ in the presence of $Q$. $N$ is a line which corresponds to the potential axis $X$ in pressure-time diagrams; here in place of time we have one lateral dimension of space.[3]

The line $I$ corresponds to the axis $X$ which we used in our study of the pressure gradients. It is clear that static and kinetic gradient curves and surfaces are not constant, but variable. For the static gradient curve we have the equation $y = P$, where $P$ varies with time, and for the kinetic gradient curve the equation

$$y = \frac{nk}{x^2} + l\left(P - \frac{f}{m} + \frac{f}{x}\right)$$

where not only $P$ varies but also $k$ and $f$, which depend, as we saw in section 88, upon $P$ and $R$. These constants assume an instantaneous succession of values—they in truth become variables. *It is possible to replace all variable quantities appearing in these equations by their equivalent expressions in terms of the square of time remaining.*[4]

All wells that produce from the same reservoir in the field, the orifices of which at the point $b$ in Figure 47 are at the same elevation, will continually possess identical values of the static pressure $S$.[5] These values remain identical regardless of different velocities of production at the various wells, such different velocities being possible by virtue of different values of the external friction and the constant back pressure at the wells. All these wells which have identical values of $S$, when provided with equal values of the constant back pressure $C$, will continually possess identical values of the potential pressure $P$. All will reach equilibrium at the same instant.[6] The total volumes

[3] See § 78, next to the last paragraph.

[4] In this way the succession of gradient curves and surfaces in this control may be appropriately expressed in terms of performance. As a consequence the gradients at any and every instant during the process of production become known specifically.

[5] We are here considering real, and not apparent values of $S$.

[6] They attain equilibrium at the instant when $I$ reaches $N$, and therefore at the instant when all primary function curves reach the $X$ axis. All these wells will have $I$, $N$, and $X$ in common throughout production. Where we deal with unequal apparent static pres-

which they individually produce will depend directly upon their individual velocities of production at any and every instant during life. Those wells provided with smaller values of external friction head will produce with greater velocities, and therefore they will produce greater volumes of fluid.[7] Any wells whose orifices may be at higher elevations will continually have smaller values of $S$, and any whose orifices may be at lower elevations will continually have greater values of $S$.[8] Any wells that are provided with smaller or greater values of $C$ will produce greater or smaller volumes, and will possess longer or shorter lives, respectively.[9]

The true static pressure of a natural reservoir can only be determined upon closing the orifice. This applies to the single well when it alone produces from the reservoir, and to all wells in a group when these produce from the reservoir. The group merely constitutes a multiple orifice. In the absence of $Q$ the value of the true static pressure can be taken as a direct indication of the height of $I$: that is, the height of the free surface of liquid in the formation. If $Q$ is present this pressure can only be taken as a direct indication of the location of $Q$.[10]

**141.** *Hydraulic versus Volumetric Control.*—It may be possible, of course, that the reservoir pictured diagrammatically in Figure 47 may be in any one of the three controls. We should not attempt to classify it on the basis of structural geology, nor yet on the basis of the lithological character of the productive formation. Classification is only possible on the basis of mechanics.[11] The control of any natural reservoir can be determined from the performance of any or all of its wells.

Section 58 contains a preliminary discussion concerning the determination of the control. As we see it in Figure 34, the proposed method appears to be dependent upon ideal performance, or at least upon a reasonably close approximation to such performance. The method is a practicable one, although

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sures in the presence of equal real static pressures, there are, for these apparent pressures, lines $I'_1$, $I'_2$, $I'_3$, and so on, which pertain to the individual wells. These lines approach $N$ common to all in harmonious percentage variation with each other and with the line $I$ itself. $I$ and all $I$-primes coincide with $N$ at the same instant.

[7] Life is the same for all, yet velocity is greater for some than others, and, in accordance with this control, $Vo = \frac{1}{2} VeL$.

[8] See footnote 7, § 131, page 351.

[9] This is in agreement with the relations shown by vertical cuts in Fig. 121.

[10] It seems not improbable that we may make practical use of these indications in fields of Hydraulic and Volumetric controls. It would thereafter be of interest to compare the elevation of $I$ or $Q$ with the elevation of the outcrop of the productive formation.

[11] I believe it is exceedingly unwise to judge the control by pure inference aside from mechanics. When we have determined the control with certainty, we undoubtedly shall find that structural and stratigraphic features are in agreement with the control, or at least that they are not in disagreement with it.

an interval of time is required for the purpose of observing successive values of the functions. The greatest disturbance to be encountered in this interval of time is due to alterations in the apparent static pressure which accompany the increase in the number of wells in the field, but this disturbance offers no serious obstacle.[12]

Without awaiting the interval of time we might perform a test at one of the wells. For this we would equip the well with a valve and a pressure gauge, or, perhaps better, a flow-meter with its gauges. Observations between pressure and velocity are to be made, and if these agree with the equation

$$Ve = K(PR)^{1/2}$$

as in Case 1 of the theoretic performance, sections 76 and 77, our reservoir is in either Hydraulic or Volumetric Control. But if these observations agree with the equation

$$Ve = K(PR)^{3/2}$$

we shall know that our reservoir is in Capillary Control. These equations permit us to make at once a most important distinction within the three controls, while a less important distinction between Hydraulic and Volumetric controls can, as it must, await an interval of time.[13] In order to be certain of the fact that a particular natural reservoir is in Hydraulic Control it is necessary to treat all wells in the field as a multiple orifice. In case the field is large, and the property is divided as to ownership, it will of course be impracticable to attempt to maintain a record of the true static pressure of the reservoir, for this would require the simultaneous closing of all the wells. It is simple enough, however, to maintain a record of the total rate of production from all the wells in the reservoir. This total rate should pertain to total fluid; for example, it should pertain to oil plus water in case both liquids are produced.[14]

In the discussion to follow, and in that of the succeeding section, we shall assume that the determination of the control has been made, and that the reservoir proves to be in Volumetric Control. Let us compare some of its characteristics with those of a physically identical container in Hydraulic

[12] Adjustments for any disturbance in values due to theoretic performance can be made. (See footnote 4, § 58, p. 108.) In reality Case 1 of theoretic performance causes the most serious disturbance, for $Ve$ alters its value while $P$ remains unaltered. Cases 2 and 3, in the absence of adjustments, merely cause irregularities in the curves of the two functions, but not such irregularities that permit crossing and recrossing of the curves.

[13] The first is most important on account of differences in the secondary functions of performance. In the second we deal with secondary functions of like nature.

[14] If the reservoir produces gas only, the total record pertains to gas only; if the reservoir produces oil with gas, or oil and water with gas, the total record pertains to oil plus water. Our system of mechanics is constructed in such a manner as to require separate treatment for liquids and gases in production from combination reservoirs.

Control. In this we shall risk a repetition of some of the facts which have been stated in connection with our first class of reservoirs. These remarks are under no circumstances to be attributed to reservoirs in Capillary Control.[15]

The decline-curve method of forecasting the performance of a well, or of a group of wells, is an appropriate one in Volumetric Control, but a decidedly inappropriate one in Hydraulic Control. In the latter, as we know, there is no analytical foundation for such a method, for the reason that there is no decline. There may be a diminishing of values as a result of alterations in theoretic performance during the history of production, but these alterations are clearly due to the acts of the operators. Nature alone is not responsible for them, as she is in the matter of mathematical decline. Forecasting in Hydraulic Control can only be done with the aid of a map of the field, and the results can thereafter be modified, if known alterations in theoretic performance are to be made in the course of production. The map method indeed possesses features that may well be used to advantage in supplementing forecasts by the decline curve in Volumetric Control.[16]

Figure 132 illustrates a typical contour-map of a producing field. While

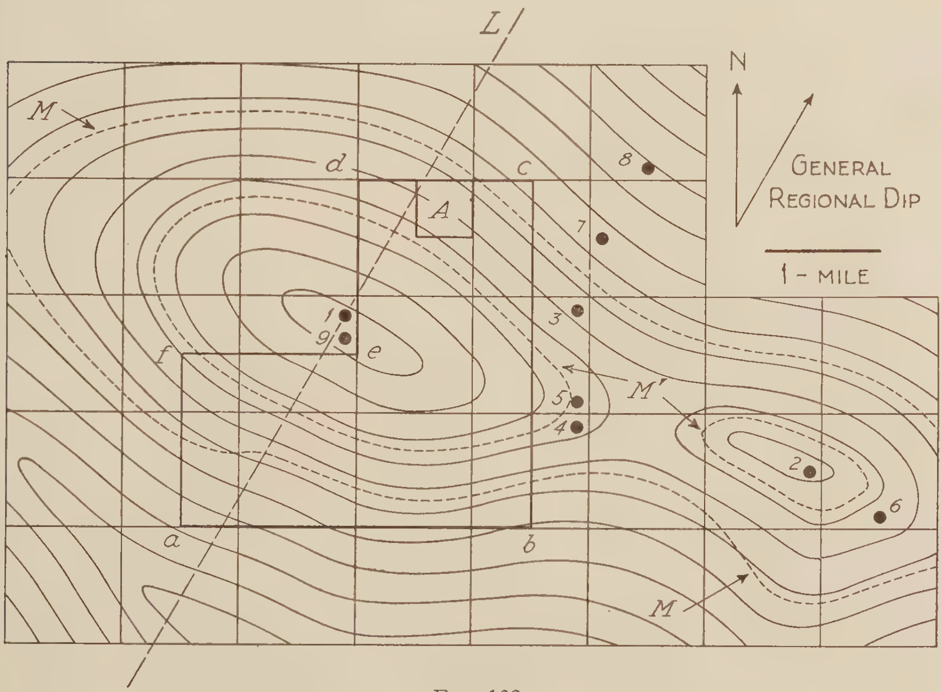



Fig. 132

[15] The differences between Hydraulic and Volumetric controls, on the one hand, and Capillary Control, on the other, can hardly be appreciated until we have studied Capillary Control.

[16] The procedure is to be explained in § 142.

the regional dip is east of north, the local area contains a structural feature which consists of a major and a minor dome that are connected by a "saddle" in the manner shown. The dotted line $M$ indicates the location of the edge-contact between water which occupies most of the porous and permeable formation and a pool of oil that exists in the immediate vicinity of the structural feature. This line is presumably known from the records of many wells which have been drilled in the field, only a few of these being shown in the figure.

We may believe the reservoir to be approximately ideal. The formation is uniform in thickness, and its texture is homogeneous throughout its lateral extent. The greatest deviation from the ideal consists in the fact that the structural feature is not symmetrical with respect to a centrally located point. A profile section on the line $L$, when extended sufficiently toward the southwest, appears as in Figure 47. Now if the available supply of water at the outcrop is equal to, or greater than, the total rate of production from all wells in the field, so that the free surface of liquid in the formation is maintained at a constant level, the reservoir is in Hydraulic Control; but if the total rate of production from the field exceeds the rate of replenishment of water at the outcrop, the field is in Volumetric Control.[17] In either case the events and conditions within the producing reservoir are the same, except in so far as these decrease in their intensity during decline in the latter control. The reservoir may be classified in accordance with the nature of its fluid, as in section 93, irrespective of the particular control, and those peculiar events which were discussed in the remaining sections of chapters xiii and xiv are to be reckoned with in either case.

If we say that the pool contains gas with the oil, we are to know from the records of performance that the static pressure of the reservoir is sufficiently great to overcome the resistant action of bubbles of gas throughout the mass of liquid; that is, to overcome what we term the Jamin action. When we speak of the reservoir here, we refer of course to the entire natural container. This extends over an area far greater than that shown within the figure; how far, we cannot say, unless we have accurately determined the limits of the formation with its present open texture and its contained fluid.

There should be no doubt concerning the source of the energy which causes this reservoir to produce. The static pressure at the wells is due solely to the weight of the column of liquid in the formation regardless of whether the reservoir is in Hydraulic or Volumetric Control. The wells can produce in the absence of gas. This we know from experience with water wells, where this liquid is not conducive to the existence of gas. *Oil is certainly conducive to the existence of gas; nevertheless its existence is not essential, but merely accidental, in the performance of this natural reservoir of the open type.*

[17] We are to find that Capillary Control cannot exist permanently in this reservoir, so long as there is a continuous inflow of any amount of water at the outcrop.

The potential reservoir is generally not co-extensive with the physical reservoir. In Figure 47 it is clear that the liquid below the point $f$ on the line $N$ plays no mechanical part in production, for its weight is offset by the weight of the column of liquid in the casing. If $N$ is caused to coincide with $J$, $f$ coincides with $j$, having passed through intermediate positions as $g$, $h$, and $i$, while $N$ is lowered. It is true that the actual liquid below $f$ is produced, for there is a replacement by the liquid above $f$. The potential volume registered by any or all wells in the field consists of that volume of liquid which is above $f$. *In Volumetric Control the decline-curve method of forecasting relates specifically to this volume.*

If the reservoir is in Hydraulic Control, we may be certain that Wells 1 and 2 will, given sufficient time, produce all the mobile oil within the pool. They cannot produce that which has become adsorbed upon the surface of the porous material; therefore the distinction between mobile and adsorbed oil is essential, as we have learned in earlier sections. If either of the wells were alone in the field, a smaller pool would remain trapped on the other dome. In general each minor closed structural feature must have at least one well upon it, in order that the pool may be completely drained.

During the course of production from the field the line $M$ encroaches upon the structural center or centers. Thus $M$ at some stage in the history of the field occupies the position $M'$.

Wells 3, 4, and 5 begin production with oil, but thereafter these successively turn to water. The same thing happens to Well 6. Wells 7 and 8 begin with and continue in water. Well 9 is sufficiently close to Well 1 to affect its production.

Every producing well in the field has its influence upon the movement of every point in the line of contact between the water and oil, both as to direction and as to lineal velocity. The influence of a single well upon various points varies inversely as the distance between the well and the points. The influence of many wells upon a single point is the "vectorial sum"[18] of the influences of the individual wells upon the point, combined with such influences as specific gravity, which might come into play because of the presence of more than one fluid in the vicinity of the structural feature, and the subordinate action of bubbles of gas in case oil, gas, and water are all present in the same vicinity.

*Every barrel of oil that is produced from one well in this pool means one less barrel of oil to be produced from the other wells, for all wells are drawing from the same container.* It is evident that the worth of an area $A$ diminishes in the course of production from wells on the same, or on higher contours.

[18] Two or more forces whose lines of action intersect, when combined by the well-known principle of the parallelogram of forces, give rise to a resultant force which is equivalent to the combined forces. This resultant is said to be the vectorial sum of the component forces. The same proposition holds with respect to lineal velocities.

By the time $M$ has passed to $M'$ its value as a producer of oil is reduced to zero. Even a larger area *abcdef* suffers from production at Wells 1 and 9. Fields wherein the reservoirs are of Hydraulic or Volumetric Control are not suitable places for oil reserves, unless the reserve covers the entire structural feature, or at least a satisfactory central portion of it.[19]

The actual productive formation may possess strata of various textures. If these are present, it is easily seen how fluid in the more compact strata is produced directly into those of a more open texture; only thereafter does it reach the well.[20] It is conceivable that these compact strata may produce in Capillary Control. If so, the fact cannot be known from the performance of the well, for the record will display only the control of the main open strata.[21]

If gas is produced with liquid, this gas will issue in accordance with the power and energy curves, in spite of the fact that the gas possesses no energy that is intrinsically its own. Capillary attraction aids in maintaining a more homogeneous mixture of gas and liquid from the bottom to the top of the formation. Durable foam will do likewise, and it will furthermore modify the proportional production of gas from the reservoir, in so far as it may interfere with the formation of gas pockets on any known or unknown domal structures within the vicinity of the field.

In Hydraulic Control any alterations in the proportional production of gas are forced upon the reservoir by the action of the operator or operators of the wells. A change in the velocity of oil production affects the amount of by-passing, and it might reasonably affect the action of foam, since in any case the duration of foam is finite and limited. At a sufficiently slow rate of production the duration of the foam may be partially or completely exceeded, thus allowing the segregation of oil and gas within the reservoir. In addition, the introduction of new wells in the reservoir, and the opening or closing of wells already in, particularly those situated on a lower contour of the structure, will force alterations in the proportional production of gas at the well. In Volumetric Control these forced alterations are likewise encountered in practice, but here we have an additional natural one that accompanies decline.

When gas is present with liquid in a reservoir of the open type, there are circumstances which allow the gas to act temporarily as a source of energy. If the entrance of fluid into the chamber at the bottom of the well is restricted,

[19] I do not pretend to say what would constitute a satisfactory central portion of the structure. We can be certain that neighboring wells down the dip of the structure are draining the area below the contour of that well among them which is highest on the structure. Our reserves in fields of Capillary Control are safe regardless of structure.

[20] We might say that the "orifice" for a compact stratum is the area of contact with the adjoining stratum of open texture.

[21] We are not warranted in assuming an internal production in Capillary Control within reservoirs of Hydraulic or Volumetric Control without evidence. This evidence can be obtained from the cores of test wells that are subsequently drilled for the purpose.

it is quite conceivable that the temporary closing of the well at the top will permit a pressure to accumulate within the chamber, and a sudden opening of the well will allow accumulated gas to expand. A volume of fluid which would otherwise require a longer time to issue from the well is produced in a comparatively short time. During this short time there is a small reservoir performing in Volumetric Control regardless of the control of the main reservoir behind it. With a temporary effect of this nature we should include the effects of inertia which are likewise temporary. Fluid at rest or in motion within the reservoir requires a brief time for an adjustment in accordance with alterations in theoretic performance.[22] When a valve at the well is opened or closed, we see that time, as measured in minutes or hours, must elapse before the fluid is steady, either in motion or at rest. This is particularly true of liquids, for their mass per unit volume is considerably greater than that of gas. *During the period of adjustment these effects are in Volumetric Control irrespective of whether the reservoir is in Hydraulic or Volumetric Control.*

142. *The forecast of performance.*—Is it not clear that the primary function curves give us information concerning the performance of the reservoir at large; that they make no distinction between two such liquids as oil and water? In Figure 132 we have, say, a pool of oil surrounded by water. The primary function curves, either for Hydraulic or Volumetric Control, tell us nothing of the instant at which a particular well will change in production from oil to water.

If, in virtue of the fact that the field is sufficiently drilled, the contact $M$ between the two liquids can be mapped at the beginning and end of a definite period of time in the history of production, say, as at $M$ and $M'$, with a fair degree of accuracy, the exhaustion of the pool is readily calculated, and consequently the change in production at any given well may become known with the same degree of accuracy. Let $A_1$ be the area inclosed by $M$, and let $A_2$ be the area inclosed by $M'$, then for the area $A_1$–$A_2$ we have produced a certain volume of oil $Vo'$, known from the record of the field. Now if $Vo''$ is the volume of oil within $M'$, yet to be produced, it is evident that

$$\frac{Vo''}{Vo'} = \frac{A_2}{A_1 - A_2}$$

$Vo''$ is thus known, for the areas $A$ are easily measured by means of a planimeter. A knowledge of the thickness and porosity of the formation is not required, but a uniform thickness and a homogeneous texture in lateral directions are assumed.

I do not say that this method has an appeal for accuracy, for I recognize

[22] Inertia is "the incapability of matter of altering the state into which it is put by any external cause, whether that state be rest or motion."—Young, *Mechanics,* page 117.

the difficulties that are involved. We are confronted with a situation. It is our duty to understand it, and do with it what we can. Perhaps a better method will some day be found.

It is possible that in some fields the approach of water at the well may be predicted in advance by a rise in the temperature of the oil. If the productive formation contains lime, and if this lime has ever been subjected to heat in place, a slight rise in the temperature of the oil, and perhaps a slight increase in the carbon dioxide contents of the gas, may be expected. The phenomenon is purely one of a chemical nature, for

$$CaCO_3 + \text{Heat} = CaO + CO_2$$

and

$$CaO + H_2O = Ca(OH)_2 + \text{Heat}$$

The carbon dioxide in the first reaction either is picked up by the liquids or it collects in pockets, depending upon the pressure to which it is subjected. Oil does not enter into such a reaction as the second one, for it appears to have no chemical affinity for the calcium oxide.[23]

A change in production from oil to water may be either gradual or sudden and complete. As previously stated, where both liquids are produced simultaneously, the rate of production curve should pertain to oil plus water. If desired, a second curve may be drawn upon the plat, one which separates oil and water. This sort of a record is rather satisfactory for the purposes of calculations.

When the reservoir is in Volumetric Control, which of two phenomena is to appear first? *Is the pool to be exhausted before the point of equilibrium indicated by the curves is reached, or is this point of equilibrium to be reached before the pool is exhausted?* If the latter holds, the decline-curve method of forecasting is complete in itself, but if the former holds, this method must be augmented by determinations to be made with the map.

Many of our fields are known to possess more than one reservoir. Various formations within the stratigraphic column are frequently capable of producing fluid when penetrated with the drill. While some of the strata may have oil, others of course may only contain water. That these formations constitute distinct reservoirs may be known by the fact that their values for the static pressure do not agree. Where porous formations are parted by thin shale beds of mere local extent, we do not have distinct reservoirs below and above these "partings."

The control need not be the same for all reservoirs in the same field. Whether they are or are not in the same control, their records, and the records of their wells, must be kept separately in order that these may have value in computations.

[23] The thermo-chemical reactions are well-known phenomena; nevertheless the notion of their applicability in the case of natural reservoirs is admittedly hypothetical.

It is useless to attempt to increase or "restore" the pressure within a natural reservoir of Hydraulic or Volumetric Control by pumping gas in at one or more wells. Indeed we may pump the gas, but the result is necessarily inappreciable; there is too great a space to be filled with our little pump.[24] It is conceivable that in fields where higher strata contain water under considerable head, such water can be let down into the productive formation beyond the confines of the pool of oil. With Nature's aid we may realize an appreciable result.

**143.** *Conversion of control.*—In section 100 we saw how easily reservoirs may be converted from Hydraulic to Volumetric Control. To simplify our study of Volumetric Control we immediately assumed that the rate of the replenishment of fluid is zero. Now let us say that this rate is not zero; it has a real value less than the rate of production from the reservoir. What happens?

In Figure 133 we have a velocity-time curve for production from a type solution tank. For the first five hours the reservoir is maintained in Hydraulic

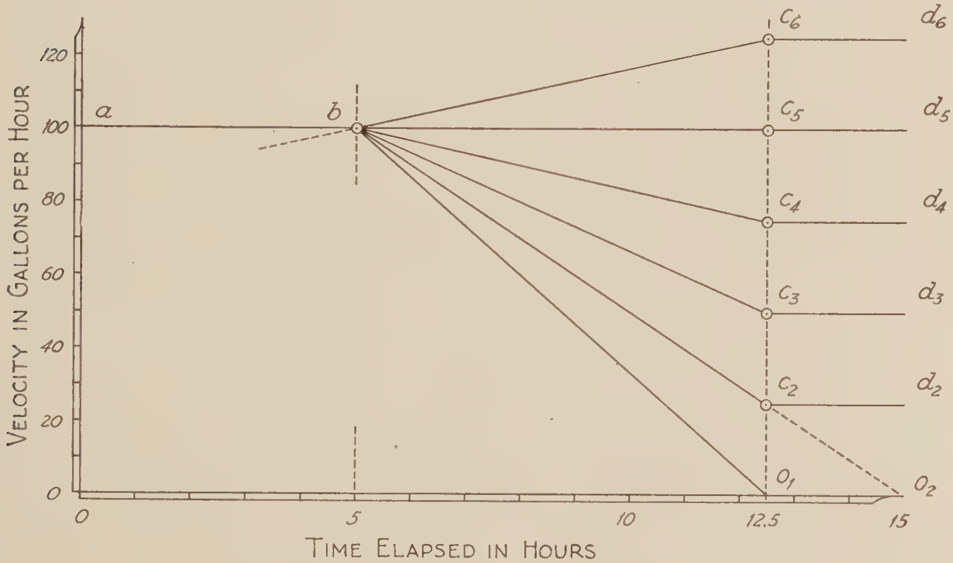



Fig. 133

Control, giving the production curve *ab*. If at the end of this period the inflow of liquid is completely shut off, the curve is, let us say, $bO_1$, the point $O_1$ denoting equilibrium in the simple Volumetric Control that we have heretofore studied. But if the rate of inflow, necessarily 100 gallons per hour for *ab,* is reduced to 25 gallons per hour, the production curve is thereafter

[24] Compare with Capillary Control, § 187.

$bc_2d_2$. The velocity declines from $b$ until it reaches the same value possessed by the rate of inflow, and the reservoir, after passing through a period of 7.5 hours in Volumetric Control, proceeds to produce once more in Hydraulic Control. To locate the point $c_2$ it is sufficient to draw a vertical line at the point $O_1$, and scale off the new rate of inflow. The path of $bc_2$ appears to approach a point of equilibrium at $O_2$, but clearly this point is not reached.

For a reduction from 100 gallons per hour to 50 and 75, respectively, we have the curves $bc_3d_3$ and $bc_4d_4$. There are evidently points $O_3$ and $O_4$ to correspond, such points lying beyond the limit of the figure. Where no change is made in the rate of inflow the curve is $bc_5d_5$; the point $O_5$ is at infinity to the right. If instead of a reduction we provide an increase in the rate of inflow, say to 125 gallons per hour, the curve is $bc_6d_6$. An imaginary point $O_6$ has come into finite space at the left, and rests at the intersection of the horizontal axis and $c_6b$ produced downward to the left.[25]

Corresponding to these velocity-time curves between $b$ and the points $c$ the pressure-time curves may be constructed. These are all true parabolas, as in Volumetric Control, with their vertices at $c$, except in the one point $c_5$. Below $c_5$ the axes of the parabolas extend upward in the usual manner, whereas above this point they extend downward, as in Figure 100 (p. 304). The functions travel on paths as though the points $c$ denoted an equilibrium.[26]

It is clear that at the points $c$, other than $c_5$, we have a conversion from Volumetric to Hydraulic Control. Now the paths $abcd$ are brought about by altering the rate of inflow of liquid; they might likewise be brought about by increasing the size of the orifice, the rate of inflow remaining unchanged. In this manner we would account for the fact that we not infrequently meet with such curves as $abc_2d_2$ in the production of oil from natural reservoirs. When a field is first discovered, a few wells, taken in a group as a multiple orifice, do not exceed the available rate of inflow of water at the surface, and consequently there appears to be no decline in performance. But as the field is provided with more wells, thus increasing the size of the multiple orifice, a point in time is reached when the total rate of production from the reservoir exceeds the available rate of inflow. Decline is established, as at $b$.

[25] For an increase of this sort we are assuming that the sides of the tank extend upward sufficiently to accommodate the required volume of liquid within it.

[26] The point approached as the time-location of equilibrium in Volumetric Control in reality proves to be the time-location of the conversion of control. This holds in conversions from Volumetric Control to Hydraulic Control and from Volumetric Control to Capillary Control. (See footnote 2, § 127, p. 332.) In Fig. 133 the point approached is determined graphically by means of the velocity-time curve. This method will not serve for conversions to Capillary Control. There is another method, however, which serves the two conversions equally well: namely, one based on pressure-time relations. The vertex of the parabola—a feature which the straight line lacks—is the point to be determined. Three observations are necessary and sufficient, as in the investigation of the final closed-in pressure of a reservoir, § 119.

Some time later the field is considered to be "drilled up"; the confines of the pool are located by outlying wells, and the owners of property overlying the pool are satisfied with the number of wells which they have drilled. In the meantime decline continues until such a time when the total rate of production from the reservoir just equals the available rate of inflow, and thereafter there appears to be no further decline. Some would say that under such circumstances they have "settled production."

If we reason logically in this, we should be able to note a correspondence between dry winters and a reduction in the total rate of production from the field, and between wet winters and an increase in the total rate. Presumably there would be a lag in time between the season and its reflectd influence. Just these circumstances have been actually observed in the field. From year to year we see a repeated conversion between Hydraulic and Volumetric controls, such that new values in the former are successively established.

So much for conversion between the first two controls; we must now take note of one of a different character between Volumetric and Capillary controls.

In section 140 we observed that within those reservoirs which contain gas and liquid in combination, in the presence of a porous medium, the pressure head may decline in Volumetric Control until a line $Q$ that was previously described meets and coincides with the line $N$. At any time this may happen there yet remains a pressure at the orifice, one which is represented by the distance between $Q$ and $I$. *The static pressure of the reservoir has reached a low value; it can no longer overcome the resistant action of the bubbles of gas throughout the mass of the liquid. As a result the liquid becomes securely locked in place throughout the porous medium, and cannot move except as pressure is relieved at the orifice. We shall say that the reservoir is now in Capillary Control.*[27] The laws of delivery differ from those for Volumetric Control.

Figure 134 (p. 394) illustrates this conversion. The line $ab$ is the velocity-time curve while $Q$ approaches $N$; at $b$ it has reached $N$, and the curve is thereafter a cubic parabola $bO_1$, $bO_2$, or some other $bO_n$, the point $O_n$ lying on the horizontal axis, to the right of the fifth hour. This point is not determinable from the data in Volumetric Control; it is only determinable by observing the performance of the given reservoir after conversion.

[27] This conversion from Volumetric Control to Capillary Control is a common occurrence in our artificial reservoirs which we use for investigating the behavior of natural reservoirs. The data obtained after conversion appear to be erratic or erroneous; we therefore are inclined to discredit them, and maintain our experiment in Hydraulic or Volumetric Control, giving preference to theoretic performance in the former one of the two. To maintain the experiment in these controls we simply provide a pressure sufficiently great to overpower the Jamin action. I confess that I cannot cite a particular instance of conversion from Volumetric to Capillary Control in natural reservoirs. I only say that it is possible.

The conversion to Capillary Control may be temporary or permanent, depending upon whether there is or is not an inflow of fluid.[28]

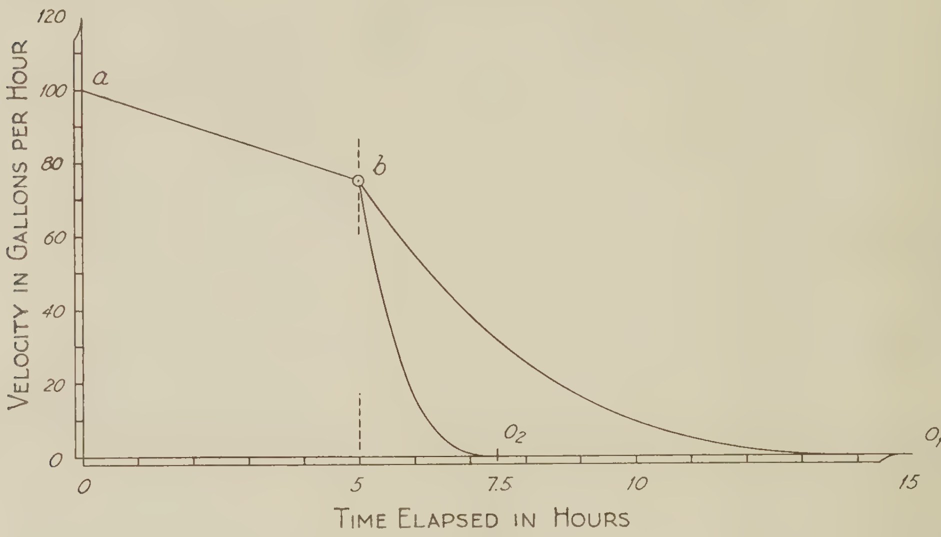



Fig. 134

While we observe the possibility of a conversion of control under proper circumstances, we are not to overlook the fact that reservoirs in any of the three controls can, and frequently do, begin to produce and continue to do so without conversion.

Let us now enter upon our investigation of Capillary Control.

[28] See footnote 17, § 141, page 386.

# Part IV. Reservoirs in Capillary Control

CHAPTER XXIV

# Field Data of Distinctive Habit

"We must not say, Let us begin by inventing principles whereby we may be able to explain everything; rather must we say, Let us make an exact analysis of the matter, and then we shall try to see, with much diffidence, if it fits with any principle."—VOLTAIRE

**144.** *Introduction.*—That we have natural reservoirs which produce oil, gas, and water—singly or in combination—in Hydraulic and Volumetric controls is certain. We may know from the behavior of the type reservoirs that a natural reservoir in a given field might belong to one or the other of these controls, depending upon the relation between the rate of replenishment of water and the rate of production of fluid from all its wells. It is not difficult to list those conditions which will most likely cause a group of wells to begin and maintain its production in Hydraulic Control. The formation, as we have found, must be in communication with the surface, and there must be a sufficiently heavy rainfall in the general region. Where property is owned in large blocks, and consequently where wells will tend to be few and far apart, excessively heavy rainfall is not necessary for the purpose. We can easily see how it is possible that wells in Mexico can deliver their fluid from month to month without decline in pressure and velocity.

Where either of these controls is possible in the fields of the United States, we must admit that the opportunity for Hydraulic Control exists once in their very early history, and again in their late history. Newly discovered fields can soon be drilled so thoroughly that their total rates of production exceed the available supplies of water by rainfall. As a result we have in them a period of decline in pressure and velocity. The fields are in Volumetric Control. We find these notably in the Pacific region, frequently in the Rocky Mountain and Gulf regions, and in scattered parts of the Mid-Continent and Eastern regions. As we observed in section 143, fields in this control can enter into Hydraulic Control again when the total rate of production has declined to the rate of replenishment by rainfall.

In the early stages of our present investigation, after defining our terms, we briefly reviewed those laws of physics which appear to have the greatest importance in the performance of reservoirs. From these laws, among them being included the fundamental principles of fluid mechanics, twice we proceeded to develop the laws for the behavior of the primary functions of per-

formance, and twice we continued onward with the study of the secondary functions. The first hold us at the mouth of the well, while the second transport us to the interior of the reservoir. Without doubt we have continually believed ourselves to be familiar with these two classes of reservoirs. They represent precisely, it seems, our conceptions of reservoirs in general. But is this proper? Is it not possible that reservoirs with different primary and secondary functions exist? Surely this must be true, for so many hundreds of our oil and gas wells, not to exclude water wells, notably in the Mid-Continent and Eastern regions, and less frequently in the Rocky Mountain and Gulf regions, appear to behave in a manner that is quite distinctive.

These hundreds of wells decline in pressure and velocity; that is, their reservoirs are in finite control. While they sometimes may begin their history according to the laws of Volumetric Control and thereafter change, most frequently they begin and maintain their production in what seems to be a new finite control.[1]

These reservoirs we shall now proceed to study. Where shall we begin?

I propose first to present evidence in support of the notion that we contend with a different control. In this we shall retrace the steps which led to its discovery. Those of us who are familiar with the records of oil and gas production in the regions mentioned above will at once recognize the facts that are now to be cited.

**145.** *New primary function relations.*—In our study of Volumetric Control we found the equation between velocity and time to be the following:

$$Ve = KT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (387)$$

where both velocity and time are expressed in potential units. The curve for this equation, when drawn upon the Cartesian plat, is a straight inclined line, the constant $K$ designating the slope of inclination with the horizontal axis. It is likewise a straight inclined line when drawn upon the logarithmic plat, as we learned in chapter xix. Here the slope is necessarily 1 to 1, for the exponent of $T$ is unity; $K$ merely fixes its location upon the plat.

What must we say if, in plotting the data on velocity and time for many hundreds of oil and gas wells, we find the curves to be straight inclined lines on the logarithmic plat with a consistent slope of 3 to 1? Perhaps they seldom, if ever, appear to have exactly this slope; nevertheless they clearly have a slope that is more nearly 3 to 1 than 1 to 1. They seem to approximate this slope so closely; sometimes they show a little less, and sometimes a little more, just as we might expect from the manner in which the observations are made. *Surely we must say that for these wells the equation between velocity and time is as follows:*

$$Ve = KT^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (388)$$

[1] See footnote 27, § 143, page 393.

*since the slope of the line on this plat clearly specifies the exponent of the function represented by the horizontal axis; that is,* T, *as in the case of Equation 387.*

In Figure 135, $B_1$ represents Equation 387, while $B_2$ represents Equation 388. These are declining lines on a parabolic plat to suit the nature of the

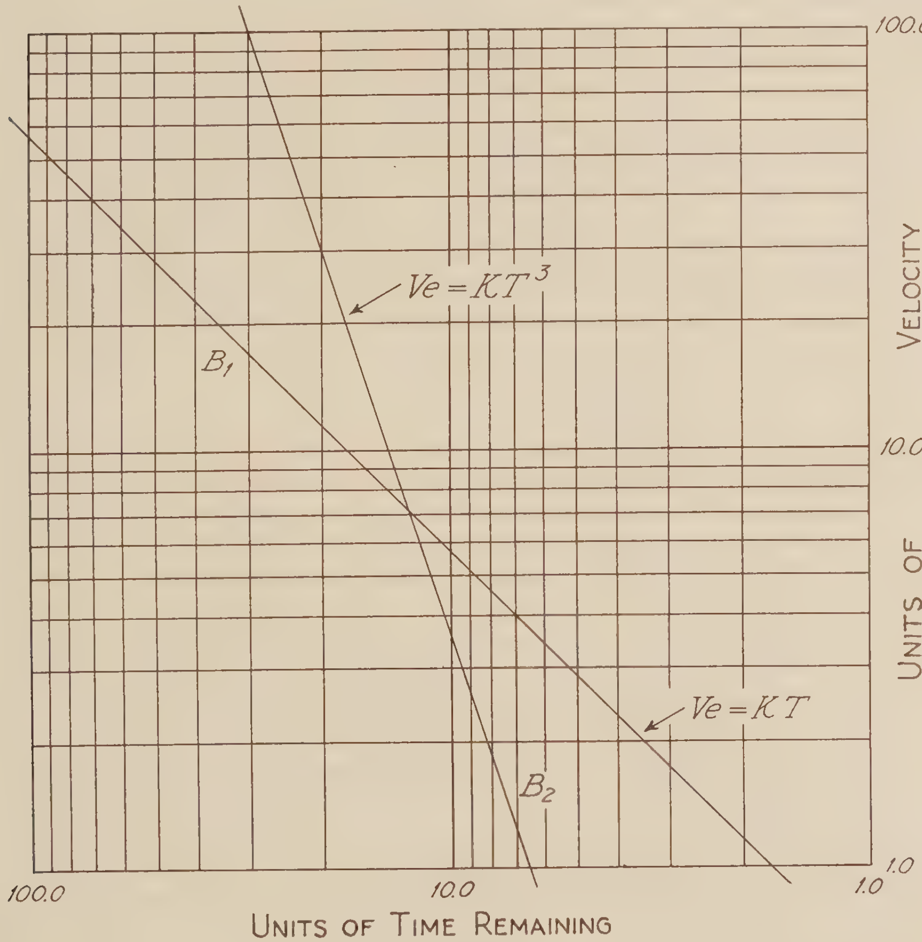



FIG. 135

equations. One is inclined at an angle of 45 degrees, and the other at 71 degrees, 34 minutes.[2] Perhaps the eye is not as competent to distinguish them on the basis of their angles as it is on the basis of ratios between altitudes and bases of triangles which we may picture them to form with the lines

[2] If the data for these lines were plotted on a hyperbolic plat—as of time elapsed in place of time remaining—and the resulting curve were straightened by shifting, such lines would appear to approximate those given in the figure. They still could be easily distinguished from each other by their respective slopes. (See footnote 3, § 120, p. 312.)

of the plat. In this light any confusion between these lines appears to be impossible.

Now if there is foundation to the truth of Equation 388 and its curve $B_2$, we should expect to find further evidence in support of it. Let us take the data on velocity and pressure for gas wells, inasmuch as data on pressure for oil wells are not commonly available today. In our study of Volumetric Control we found the following equation to hold between these functions:

$$Ve = KP^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (389)$$

where velocity and pressure are expressed in the usual potential units. The curve for this equation appears as $C_1$ in Figure 136. Its slope is 1 to 2, as

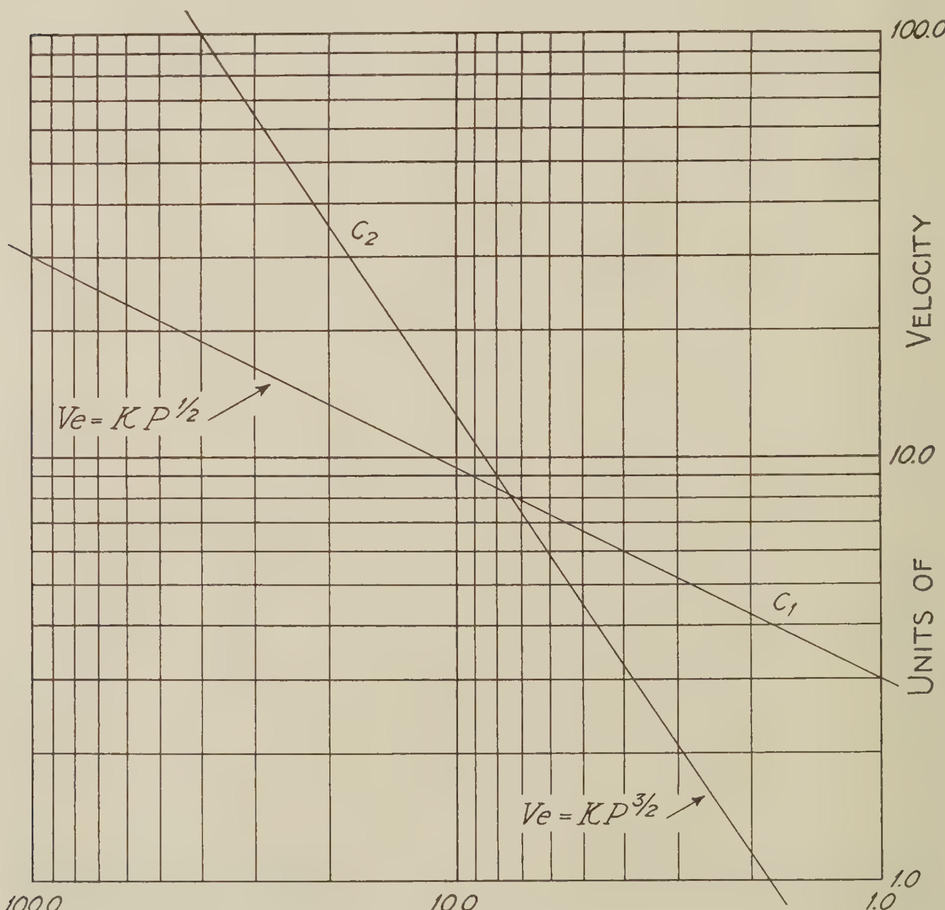



Fig. 136

specified by the exponent. Again what must we say if, in plotting the data on velocity and pressure for many hundreds of gas wells, we find the curves to be straight inclined lines with a consistent slope of 3 to 2 instead of 1 to 2?

*Surely we must say that for these wells the equation between velocity and pressure is as follows:*

$$Ve = KP^{3/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (390)$$

*in conformity with the slope, as is shown in the line* $C_2$ *of the same Figure 136.* As before, there appears to be little possibility of confusion between the two lines. One is inclined at an angle of 26 degrees, 34 minutes, and the other at 56 degrees, 19 minutes; thus one is less and the other is greater than 45 degrees. In their triangles of equal bases $C_2$ has an altitude three times that of $C_1$. This is certainly in agreement with the preceding figure, for in both the altitudes of triangles represent the same function.

Let us seek further evidence in support of the truth of these new equations and curves. We shall take the data on volume and pressure for gas wells. In Volumetric Control we found the following equation to hold between these functions:

$$Vo = KP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (391)$$

where once more these functions are expressed in potential units. For this equation we have the curve $D_1$ in Figure 137 (p. 402) with its slope of 1 to 1. Now in plotting the data for many hundreds of gas wells we find the curves to be consistent with a slope of 2 to 1 instead of 1 to 1. *For these wells the equation between volume and pressure must be as follows:*

$$Vo = KP^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (392)$$

*in conformity with the slope, as is shown in the line* $D_2$ *of this figure.* There is a less difference between the two lines than before. One is inclined at an angle of 45 degrees, and the other at 63 degrees, 26 minutes. In their triangles of equal bases $D_2$ has an altitude but two times that of $D_1$.

Possibly the difference between these lines is more easily observed when they are placed upon a Cartesian plat, as in Figure 138 (p. 403). Here data other than those of Figure 137 are used. In order to show the contrast between the present circumstances and those encountered in the study of Boyle's Law in potential phase, section 38, Figure 14, Equation 392 is changed to

$$P = KVo^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (393)$$

and the curve is given the same initial and final points as that for

$$P = KVo \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (394)$$

in the earlier figure. Thus we have $D_1$ and $D_2$ conveniently placed together in the present figure. We shall have occasion to compare these curves when we consider forecasting by pressure and volume.

If we take the data on pressure and time for gas wells, and plot them on either a Cartesian or logarithmic plat, we find no difference between the curves for wells that we know to be producing from reservoirs in Volumetric

Control and those for the many hundreds of wells we are now investigating. *So long as there is decline in pressure with respect to time, the equation appears to be as follows:*

$$P = KT^2 \quad \text{.......................} (395)$$

Thus $E_1$ and $E_2$ in Figure 139 (p. 404) are two parallel lines to conform to

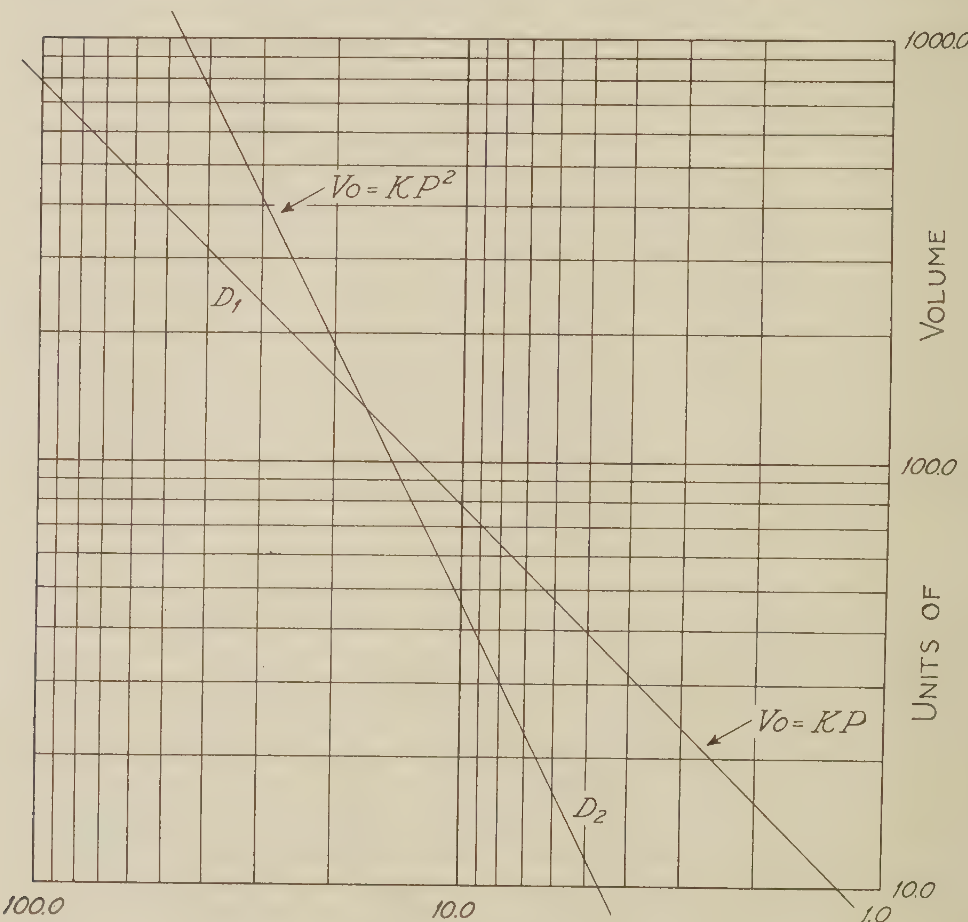



Fig. 137

their respective data of Figures 135, 136, and 137. They have a slope of 2 to 1, and an angle of 63 degrees 26 minutes.

The lines with slopes of 3 to 1 and 3 to 2, in accordance with Equations 388 and 390, respectively, are quite well known throughout the Mid-Continent and Eastern regions. The first one represents a cubic parabola. If corresponding data for Equations 388 and 392 are plotted on a Cartesian plat, such scales being chosen for velocity and pressure as to cause the

left-hand initial points to coincide, the velocity-time curve underlies the pressure-time curve, as $D$ underlies $B$ in Figure 34 (p. 108). This feature is particularly well known in the gas fields of the Eastern region. If these wells were producing from reservoirs in Volumetric Control, the velocity-time curve would overlie the pressure-time curve, as $C$ overlies $B$ in the same figure.

Such undeniable features as these, it seems to me, show conclusively that something different exists where they are noted. They are too consistent to be accounted for on the basis of errors in observation. Truly we may expect at least some errors to enter into our data, but I cannot believe that individuals, or groups of individuals, of various degrees of training in measuring with precision, can, while working in absolute independence of one another, furnish us with such consistent, supposedly erroneous data. They simply cannot be erroneous. They must be founded on truth.[3]

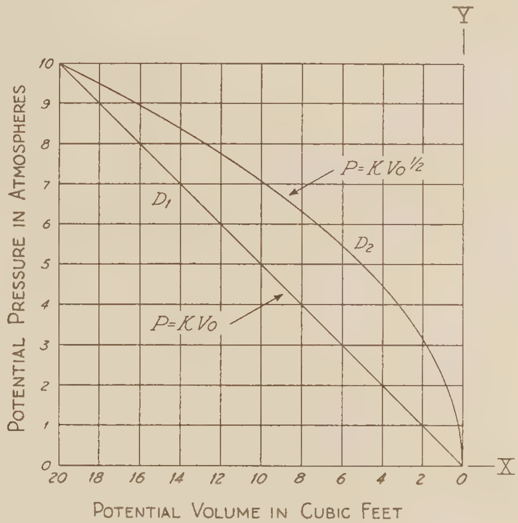



Fig. 138

Are these new equations perfectly consistent in themselves? This is certainly a question of first importance.

To repeat Equations 395 and 393 we have

$$P = KT^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\quad [395]$$

and

$$P = KVo^{1/2} \quad\ldots\ldots\ldots\ldots\ldots\ldots\quad [393]$$

The two right-hand members of these equations are evidently equal, for they are each equal to $P$. As a consequence we may write

$$Vo^{1/2} = KT^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\quad (396)$$

Now by squaring both terms we obtain[4]

$$Vo = KT^4 \quad\ldots\ldots\ldots\ldots\ldots\ldots\quad (397)$$

This is a new relation between potential volume and potential time, for in Volumetric Control we have

$$Vo = KT^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\quad (398)$$

[3] Such data have been recognized, yet accepted as unreasonable and erroneous for the past ten or twelve years.

[4] As usual, we are to give no heed to changes in the values of the $K$'s.

[ § 145

On the assumption that it is proper we may differentiate it with respect to $T$ in order to obtain the relation between velocity and time; thus,

$$Ve = KT^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [388]$$

the equation indicated by the line $B_2$ in Figure 135.

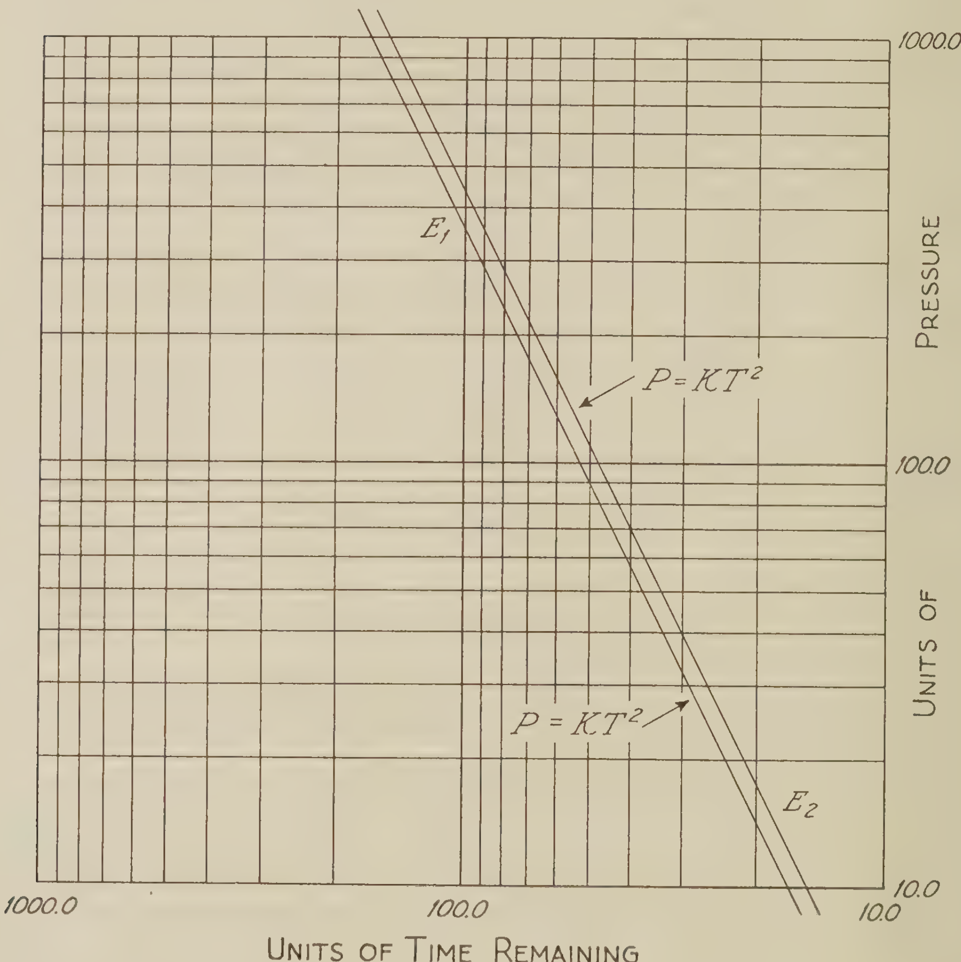



FIG. 139

To continue, we may raise Equation 395 to the three-halves power and obtain

$$P^{3/2} = KT^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (399)$$

Now it is evident that the left-hand members of Equations 388 and 399 are equal, since their right-hand members are identical; therefore we may write

$$Ve = KP^{3/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [390]$$

the equation indicated by the line $C_2$ in Figure 136.

We see that the new equations are perfectly consistent in themselves.

**146.** *New secondary functions.*—In observing the existence of new primary function relations in our field data we have taken, let us say, a first step toward the solution of an interesting problem concerning the performance of many natural reservoirs. We are not yet supposed to know the significance, nor the cause, of these equations. Let us provisionally assume them to be accurate and proper, and proceed to formulate a hypothesis that may account for them. In doing this we take the second step. We have no guide in this procedure except a purely logical reasoning founded upon our fundamental principles of fluid mechanics. While we deal in hypothesis we shall make any assumptions that appear to be of service, no matter how extravagant they may seem to us at the moment. But of course we must be reasonable in these assumptions; we cannot assume the properties of matter to be other than as we know them, and we cannot contravene the laws of physics that pertain to the general behavior of fluids.

We are to invent a partial explanation of one of our new equations. Let us choose the following:

$$P = KVo^{1/2}$$

or perhaps, better, its equivalent

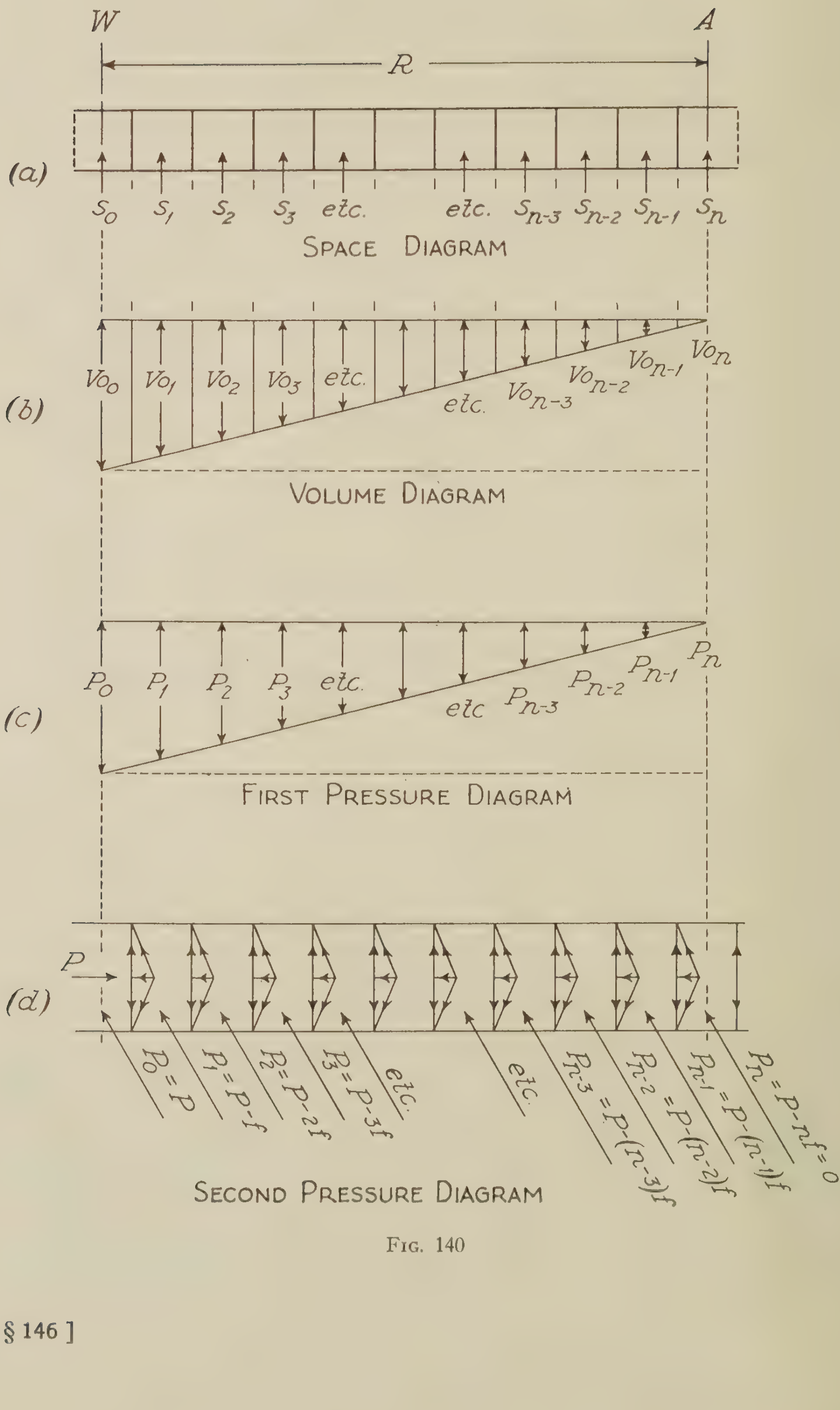
$$Vo = KP^2$$

What conditions can be arbitrarily assumed to exist within the reservoir to give this relation, such that when $P$ declines, say, from 100 units of pressure $Vo$ also declines from 100 units of volume in the following manner?

| $P$ | $Vo$ |
|---|---|
| 100 | 100 |
| 90 | 81 |
| 80 | 64 |
| 70 | 49 |
| 60 | 36 |
| 50 | 25 |
| 40 | 16 |
| 30 | 9 |
| 20 | 4 |
| 10 | 1 |
| 0 | 0 |

These values are in accord with Figure 138, provided the scales be changed to read from 0 to 100. It is of importance to note particularly that when $P$ possesses one-half of its initial value $Vo$ possesses one-quarter of its initial value, and not one-half as in Volumetric Control.

Let us be bold, and assume without reason that our reservoir interior, as measured outward from the well, is divided into chambers in the manner shown in Figure 140 ($a$) (p. 406). The well being at $W$ on the left, these chambers may be numbered toward the right; thus we have $S_0$ immediately at the well, then $S_1$, $S_2$, $S_3$, and so on, until at some distance $R$ from the well we



W
A
R
(a)
$S_0$ $S_1$ $S_2$ $S_3$ etc. etc. $S_{n-3}$ $S_{n-2}$ $S_{n-1}$ $S_n$
SPACE DIAGRAM
(b)
$Vo_0$ $Vo_1$ $Vo_2$ $Vo_3$ etc. etc. $Vo_{n-3}$ $Vo_{n-2}$ $Vo_{n-1}$ $Vo_n$
VOLUME DIAGRAM
(c)
$P_0$ $P_1$ $P_2$ $P_3$ etc. etc $P_{n-3}$ $P_{n-2}$ $P_{n-1}$ $P_n$
FIRST PRESSURE DIAGRAM
(d)
P
$P_0 = P$
$P_1 = P - f$
$P_2 = P - 2f$
$P_3 = P - 3f$
etc.
etc.
$P_{n-3} = P - (n-3)f$
$P_{n-2} = P - (n-2)f$
$P_{n-1} = P - (n-1)f$
$P_n = P - nf = 0$
SECOND PRESSURE DIAGRAM


FIG. 140

§ 146 ]

have $S_n$ with $S_{n-1}$, $S_{n-2}$, and so on, successively in front of it. These $n$ chambers are materially separated from each other in some unknown manner, and the hollow spaces within them are filled with, say, gas and oil. Now if it is possible that each successive chamber outward from $W$ delivers to the well a progressively diminishing portion of its fluid, presumably by virtue of the existence of the partitions between them, volume conditions can be represented in the manner shown in ($b$). The fluid to be delivered to the well by all chambers is shown in the area of the triangle with a base $R$, and that to be delivered by the separate chambers is shown in the partial areas which correspond to $S_0$, $S_1$, $S_2$, $S_3$, and so on, in ($a$). Thus we have $Vo_0$, $Vo_1$, $Vo_2$, $Vo_3$, and so on, which go to make up a total $Vo$.

Let us for a moment make a change in the manner of representing volume delivered from the chambers. Instead of areas we shall have simple lines whose lengths denote volume. These are shown in ($b$) with arrow points.[5] Now since we assume these individual chambers to be hollow, we may further assume them to be individual reservoirs in Volumetric Control. Regardless of their possible size, regardless of their being possibly of the open type, or possibly of the closed type, and therefore regardless of the possible source of energy possessed by their fluid, they appear to be able to perform in the manner of ordinary reservoirs in Volumetric Control. We can see no logical reason for supposing them to be in any other control.[6] If we are safe in this assumption, we may, according to pressure-volume relations in such control, that is, according to Boyle's Law in potential phase, represent pressures within the chambers by lines, as shown in ($c$). Thus we have $P_0$, $P_1$, $P_2$, $P_3$, and so on, corresponding to $Vo_0$, $Vo_1$, $Vo_2$, $Vo_3$, and so on, respectively. These two sets of functions clearly satisfy our definitions of potential pressure and potential volume.[7]

What dynamic conditions may we imagine to exist within the productive formation in order to account for a progressive diminution in potential pressure, as we proceed outward from $W$? One possibility readily suggests itself: namely, *that the partitions which are assumed to exist between the chambers exert a small, but real, force in a direction opposite to that of a pressure* P *at the well.* A pressure $P_0 = P$ at $W$ would then become $P - f$ just beyond the first partition, $P - 2f$ just beyond the second, $P - 3f$ just

[5] We know that $Vo$ can be represented either by areas subtended by the velocity-time curve or by lines, as ordinates of the volume-time curve. No novel feature is introduced here in these alternative methods of representing volume. The arrows are convenient for accentuating the comparative values of volume.

[6] In our past investigations we have placed no limits to the size which our reservoirs in Volumetric Control may possess.

[7] If, for any reason whatever, we choose to consider the small part of a large system as a reservoir complete in itself, we can do so. Under such circumstances this small part has its potential pressure and potential volume strictly in accordance with the original definitions of these functions.

beyond the third, and so on, where $f$ is the oppositely directed force that is exerted by each partition. At some distance, supposedly at $R$ distance from $W$, $P - nf = 0$; that is, a sufficient number of partitions, $n$, is able to offset completely the pressure $P$, so that $P_n$ is actually zero.[8]

An attempt to show the situation accurately is shown in ($d$). Whatever the nature of these partitions may be, they appear to act as stretched membranes. Before $W$ is drilled in, these membranes, we shall say, are plane; they are therefore represented properly by lines perpendicular to the lay of the formation. While $W$ is in the process of production we may imagine them to be distorted. It is as if we were to press a finger against the center of each membrane and cause it to assume a conical surface, as indicated by the inclined lines. In this distorted condition it offers a horizontal resistance in the direction shown.[9]

It would be fair to ask how conditions in ($d$) will provide the values for $P$ and $Vo$ given in the table above. Let the volume $Vo$ to be produced from $W$ be represented by the area of the triangle in ($b$). During the process of production both ($b$) and ($c$) are imagined to shift simultaneously toward the left, $W$ of course remaining in its place. When, for example, $P_{n/2}$ in ($c$) reaches $W$,[10] that is, when $P$ has declined to one-half of its original value, three-quarters of the initial area of the triangle in ($b$) has passed beyond the position of $W$. Three-quarters of the initial $Vo$ has been produced from $W$, and but one-quarter remains to be produced. The values in the table may be verified mathematically by dividing ($b$) into small areas and counting them off as they are caused to shift under the vertical edge of a sheet of paper held firmly in position over the surface of the drawing.

This fanciful conception of conditions and events within the productive formation would seem to be quite different from that which we have heretofore held. In our study of the secondary functions in Hydraulic and Volumetric controls we saw no evidence in support of any such ideas. Yet, if we reflect a moment upon our experiences in the field, perhaps these new inventions are not impossible. They might explain features which we have been unable to account for in the past. As an example, let us consider the significance of a distance $R$ as measured outward from a well. *We would infer*

[8] It is essential that we recognize in $P$ the effective pressure, the potential pressure, as measured at $W$. It is not, except at the initial instant of production, the pressure at points within the reservoir at any and all distances from $W$.

[9] Unless we understand clearly that the pressures $P_1$, $P_2$, $P_3$, and so on, represent successive potential pressures for the spaces, the figure in ($d$) may appear to be incorrect, in that the small forces $f$ may seem to point in the wrong direction. Their direction would certainly be wrong if we should say that $P$ represents continually the pressure of the reservoir at and beyond the distance $R$ from $W$.

[10] The subscripted $P$ exactly midway between $P_0$ and $P_n$ is obviously $P_{n/2}$, for whatever value $n$ may have.

*from Figure 140 that this distance denotes a limited radius of drainage of definite length.*[11] Now possibly this is a correct representation of the situation in many of our oil and gas fields. We shall suppose that we have four wells, such as $W_1$, $W_2$, $W_3$, and $W_4$, in Figure 141. These are regarded as being completely exhausted, for they appear to be in perfect equilibrium at zero potential pressure. On the assumption that they produced from one formation that is perfectly homogeneous in texture throughout all lateral directions, they each have circular drainage areas of radius $R$. Obviously a fifth well, $W_5$, subsequently drilled at a central location, should possess an initial pressure equal to that initially held by the original four, since the formation in the immediate vicinity remained untouched by them. It is possible that the volume of fluid to be delivered from the later well will be smaller than that received from each of the four early ones. This will depend solely upon the relation between the spacing of the wells, $WS$, and the length of $R$, as we might surmise from the appearance of the drawing. Now all this conforms exactly to experience. If the drainage radius extends to the physical limits of the reservoir, how could four wells reach equilibrium while an area between them continues to hold fluid under the original pressure? Clearly this fluid cannot so remain under pressure, unless there exists a limited radius $R$.

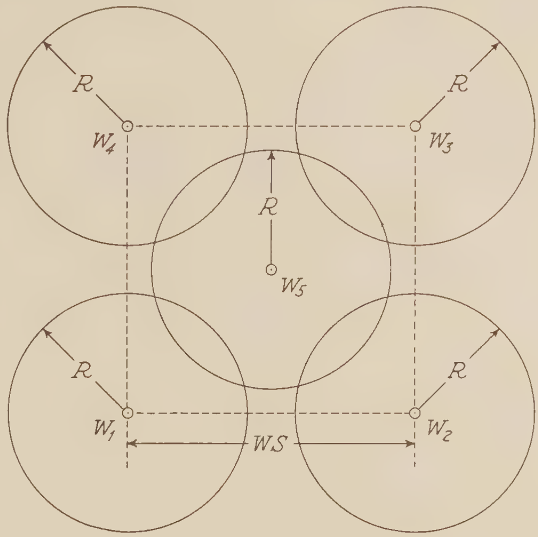


FIG. 141

This is but one of several good examples which might be cited in support of new secondary functions within many of our natural reservoirs. The others will come to light in due time. Meanwhile we must account for Figure 140.

[11] The inclined line of Fig. 140 (*c*) is not to be confused with a kinetic pressure gradient in Hydraulic and Volumetric controls, a gradient which we know to be asymptotic to a horizontal line. The forces $f$ are real (not zero, except when the globules are in equilibrium), even though small, and $n$ is finite, even though large. The product $nf$ is in any case a finite quantity; therefore the inclined line must intersect the horizontal line, and not remain asymptotic to it. This product $nf$, where $nf = P$, and consequently where $P - nf =$ zero, can be as small, or as large, as any known value of $P$. As an example, let us suppose that $n = 100{,}000$, and that $f = 1/100$ pounds per square inch, then $P = nf = 1{,}000$ pounds per square inch.

**147.** *The experiments of Jamin.*—We take a third step in attempting to fit these accepted primary function relations, and a highly imaginative basis for their accompanying secondary functions, with known principles. In brief our problem is this: Given the pressure diagram of Figure 140 ($d$), what known physical phenomenon can be offered to explain it satisfactorily?

I believe I have found the solution to the problem in the experiments of the French physicist, Jules Célestin Jamin, the records of which are obtained in *Comptes Rendus de l'Académie des Sciences,* Paris, and in *Leçons de Chimie et de Physique,* Société Chimique de Paris.[12]

Jamin's investigations concerned the function which plants fulfill in raising water through their tissues up to their leaves. As we know, plants lift their sap to heights that are greater than the equivalent head in feet due to the pressure of the atmosphere. This particular phase of Jamin's study is of no concern to us; we need only consider the records of his laboratory experiments with capillary tubes and porous materials. I shall quote from the first reference, and, in Figure 142, include drawings from the plate which accompanies the second.[13] The translation is somewhat free, yet accurate in detail. Paragraphs are here given numbers to facilitate subsequent reference to the text.

1. "If a capillary tube be taken, of one meter length more or less, and one of its ends be placed into communication with a vessel containing a vacuum, there will be a current of air passing through the tube from the atmosphere to the vessel. Then, if the finger with a wet cloth attached be placed at the free end of the tube, touching and held apart from such end alternately, many times and at equal intervals, it will be noted that globules of water, separated by bubbles of air, will traverse the tube with a speed which is at first very great, but which diminishes in proportion as the globules multiply, and which finally becomes nil. At this moment the operation ceases, but we have within the tube a chaplet[14] made up of alternating globules and bubbles. The apparatus prepared in this manner possesses very peculiar properties.

2. "When a pressure is exerted on one end of the tube, the first globule advances considerably, but the following ones move less and less, and the last globules remain immobile. If the pressure exerted is $P$, the movement trans-

[12] Following are more complete references:

1. "Mémoire sur l'équilibre et le mouvement des liquides dans les corps poreux," *Comptes Rendus,* Tome Cinquantième, Janvier–Juin 1860.

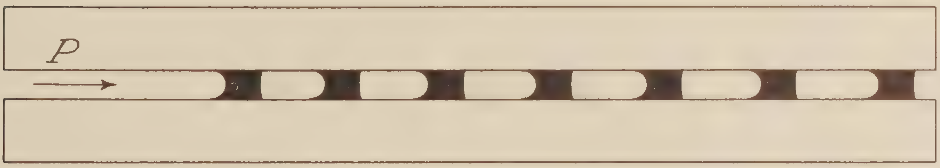
2. "Leçons sur les lois de l'équilibre et du mouvement des liquides dans les corps poreux," professées les Fevrier et Mars, 1861, devant la Société Chimique de Paris.

The first paper appears in three parts, pages 172, 311, and 385. Of the three the first one is important. The others are interesting, but not essential to our problem.

[13] Jamin's first paper contains no drawings. The drawings given here are again considered in § 148.

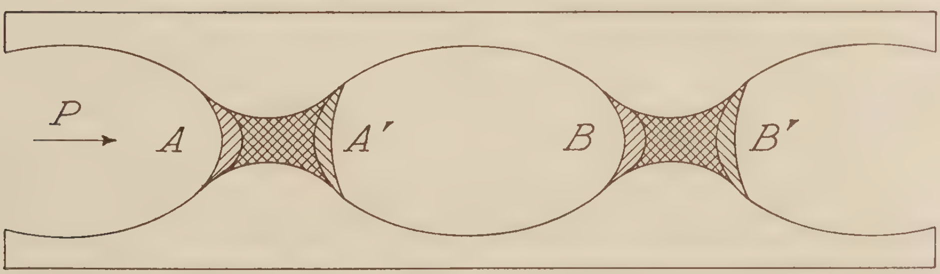
[14] I find the word "chapelet" sometimes rendered as "chain" in English.—S. C. H.

(a) EQUILIBRIUM THROUGHOUT




(b) PRESSURE APPLIED AT THE LEFT


(c) RADII OF CURVATURE ALTER




(d) CONTRACTIONS AND EXPANSIONS

FIG. 142

mitted to the globules affects a definite number $n$; if such pressure is $2P$, the perceptible movement extends over a number $2n$.[15] In general, the effect of the pressure will be noted over a number proportional to such pressure. Consequently, the opposite end of the tube will not be affected until such a time as the difference of pressures existing on the two ends of the tube reaches a point proportional to the number of globules in the chaplet, and this limit extends indefinitely if the number itself increases indefinitely. I have thus been able, at one of the extremities of a very fine tube containing a great number of globules, to exert a pressure of three atmospheres, and to maintain the same during fifteen days without detecting the slightest displacement of the globules of liquid.

3. "Inversely, if a partial vacuum is applied at one end of the tube, the first bubble of air expands considerably, the next one less, and so on, the last ones remaining in repose as long as the diminution of the pressure has not reached the limit proportional to the number of globules. For the purpose of the experiment a long tube containing a great number of globules and bubbles was attached to the upper end of a mercury column, and it was noted that the mercury maintained its level to the same absolute degree as though the upper extremity were perfectly sealed.

4. "These experiments show that the pressure exerted at one end of the tube diminishes by successive steps, each step a constant quantity for each bubble of air. This is easily explained.

5. "It is probable, to be exact, that the first action of the pressure $P$ is to hollow out the anterior meniscus of the first globule and increase the radius of curvature of the posterior meniscus. The difference increases little by little to a maximum point; consequently the globule transmits, to the bubble immediately following, a pressure of $P - f$. That which happens to the first globule is then repeated on the second, and successively on all the others up to the last, which transmits a pressure of $P - nf$. If this pressure is equal to that of the atmosphere, $A$, a state of equilibrium exists.

6. "In developing this idea it is shown that the chaplet can assume an infinite number of positions of equilibrium, of which the conditions for each one can be calculated, and the experiment verifies the results of such calculations. . . . .

7. "I have made experiments for measuring the value of the resistance limit $f$ which one globule alone can sustain against the pressure. I have found that it is independent of the length of the globule, but that . . . . it increases according to a very rapid progression as the diameter becomes smaller and smaller. (I examined three tubes in which the water was raised by the action of capillary attraction to 35, 145, and 199 millimeters, and in these I placed chaplets whose globules possessed a constant length equal to approximately

[15] These, and other symbols to follow, are changed to agree with our symbols with which we are already familiar.—S. C. H.

a millimeter. I found that the limit of resistance was equal to 6, 35, and 54 millimeters. We can easily see that these numbers increase more rapidly than the first ones.)[16] Mercury produces effects of much greater intensity; alcohol and oil do not offer any resistance to the pressure.[17]

8. "When the capillary channel, in place of being cylindrical, is a succession of contractions and expansions, it possesses properties even more curious. After once having been wet, some of the water adheres, forming a film on the walls, and such water soon collects at the contractions, producing interrupted globules. A chaplet is thus formed spontaneously, and because of the form of the channel, it is easy to see that the properties which we have just studied in connection with the cylindrical channel are exaggerated in a surprising manner. A tube which had eight contractions, in truth exceedingly narrow, sufficed to hermetically seal the upper end of a barometric column, and even offset a pressure of two atmospheres.

9. "If such a tube be completely filled with water, and pressure be applied at one of its extremities, the water percolates without difficulty. But if this pressure is exerted by compressed air, this air replaces the water in chamber after chamber, leaving globules at each contraction, and these, offering a resistance which increases with their number, will finally offset the pressure.

10. "Inversely, when the chambers are filled with air, and a column of water is driven in by pressure, it refills the chambers successively, destroying the globules and annulling their resistance, finally filling the entire tube and allowing percolation.

11. "The results may be applied to porous materials, in which the existence of channels alternating in contractions and expansions can be acknowledged. When one fills with water a porous cell of an electric battery, a porous clay water jar, a cup made of plaster, or any other hollow vessel of porous material, a pressure exerted upon this water causes it to filter to the exterior. But the interior can be exhausted to a complete vacuum without causing the air of the atmosphere to pass through the moist walls.

12. "When the porous vessel, filled with water, is itself submerged in a bath of water, and a pressure is exerted upon the interior by means of compressed air, this air commences to drive the water to the exterior. But when the water has disappeared, the air will not percolate, and the pressure can be

[16] For the data given in Jamin's first paper, the first part of which I am quoting here, I substitute more detailed information from his second one. We find his ratios to be, respectively,

$$1 : 4.14 : 5.69$$

and

$$1 : 5.84 : 9.00$$

From these we can say that Jamin found the approximate limit of resistance for one globule to vary directly as the five-halves power of the height of capillary attraction; that is, inversely as the five-halves power of the diameter of the tube.—S. C. H.

[17] Jamin's observations on the behavior of the tubes with alcohol and oil are incorrect.—S. C. H.

increased to two, three, and in some cases four atmospheres without the smallest bubble of air passing through the walls. This pressure can be maintained indefinitely, absolutely as if the walls were not pierced by capillary channels."

**148.** *The experiments repeated.*—Is it not clear that in porous productive formations we contend with minute channels which radiate in all lateral directions from the wells, that these channels possess contractions and expansions in virtue of the existing pores with their communicating canals, and that the chaplets exist wherever gas is present with liquid? *What are the chambers in Figure 140 but pores with their bubbles of gas, and what are the partitions but globules of liquid which cling tenaciously to the walls of the communicating canals?*

Because of their importance in our investigation these experiments have been performed perhaps somewhat more elaborately in the laboratories at Stanford University. Jamin's observations were found to be correct, except the one with regard to alcohol and oil, as recorded in Paragraph 7. These act in a like manner, but with less intensity than water.

The tests with tubes of smooth bore were made with ordinary capillary tubing of heavy wall. The pieces selected measured approximately a meter in length, the outside diameter measured 6 millimeters, and the diameter of the bore measured 0.5 millimeter. The following liquids were used: ordinary water, not distilled; alcohol, kerosene, and gasoline; crude oil of 25, 30, 35, and 40 degrees Baumé; and mercury. When the liquids wet the glass, the globules appear as shown in black, and the bubbles as shown in white, in Figure 142 (*a*) and (*b*) (p. 411).[18] With mercury the situation is reversed; the globules appear as in white and the bubbles as in black. The bubbles were invariably of air, although natural gas might have been used with like results.

As explained by Jamin a pressure applied at one end of the tube with its alternating globules and bubbles causes a compression of the air in the bubbles, and consequently the globules slide in the manner indicated in (*b*). If a vacuum is applied at the same end of the tube, the air expands, and the globules slide in the opposite direction; that is, to the left instead of to the right in the figure. The number of globules affected is proportional to the intensity of the pressure or vacuum.[19] If these globules and bubbles are of

[18] To represent conditions more accurately I should show a thin film of the wetting liquid lining the bore of the tube throughout its length. Within this lining the globules either slide or remain at rest. Physically this film provides a smoother movement of the globules, while analytically its presence may be ignored in view of the fact that the result is the same whether we consider it present or absent. We shall treat this film as if its liquid were in the adsorbed state. (See § 26.)

[19] The intensity of pressure refers to amounts above or below atmospheric pressure, this pressure obviously being the basis of comparison. Atmospheric pressure represents the state for zero movement of the globules in one direction or the other. To double the pressure or vacuum, for example, means to double the difference between the atmospheric pressure and the higher or lower pressure, as the case may be.

constant length throughout the tube, that is, if the tube is homogeneously filled with them, it is easily seen that *R*, the radius of action of *P*, is proportional to the intensity of *P*, and that this holds regardless of whether *P* is greater or less than the pressure within the tube when this is in the state of equilibrium shown in (*a*).

Jamin seems not to have been definitely certain in regard to the radii of curvature of the menisci separating the globules and bubbles, although he gives the drawing as in (*c*), to which I have added the radii.[20] With a magnifying glass he might have observed equal radii for the globule *abcd* in equilibrium, and the alteration caused by the application of pressure at the left, as shown by the same globule in *a′b′c′d′*. When the globule is in the latter position, it is clearly in a "distorted" condition. How can we question its ability to offer a resistance *f* to the pressure which deforms it?[21]

We see that the distortion of the globule *a′b′c′d′*, as here drawn, may be due either to a pressure on the left or to a vacuum on the right. There is in fact no analytical difference between the two cases, for in both it is simply a matter of the existence of a greater pressure on the left than on the right.

In regard to the value of the resistance limit *f* which one globule alone can sustain we can say that this information is of little worth to us in the present investigation. We are, of course, interested in relative values only, in so far as these show us the comparative effects of a given pressure upon a given tube that is equipped with various liquids in successive tests. A series of tests of this nature reveals the fact that the resistance *f* depends upon three factors, as follows:

*a*) The diameter of the capillary bore.

*b*) The surface tension of the liquid in contact with air, or natural gas, as the case may be.

*c*) The force of adhesion between the liquid and the glass, or other material forming the wall of the tube.

The limit of *f* does not depend upon *P*.[22]

Tubes of extremely small bore were prepared by stretching the ordinary tubing. Lengths of approximately 25 centimeters were heated in the middle

[20] If, in § 147, Paragraph 5, I should have translated Jamin's "Il est probable" to mean "It is provable," rather than "It is probable," I must withdraw the remark.

[21] Obviously we cannot question the fact in the light of experimental results. The globules certainly behave as a stretched membrane.

[22] Actual experiment with the tubes will show that the resistant force offered by a globule, on the careful application of pressure, begins at zero and increases gradually to the limit *f*. To be exact, then, our figures should include at the end of *R* opposed to *W* a region—of one or more globules—wherein the resistance is shown to change gradually from zero to *f*, or from *f* to zero. As in the case of the film lining the bore of the tube, this region is ignored in view of the fact that the result is the same whether we consider it present or absent.

over a fish-tail Bunsen burner, thus melting about 10 centimeters of the tube. The two ends were then pulled apart to 3 meters or more. By actual measurement in one case it was found that, for a selected portion of the middle, 0.95 centimeter of the original tube had stretched to 171.0 centimeters. This length represented the part having fairly uniform bore;[23] thus the tapered ends were ignored in the measurement. A section of the tube drawn in this manner is shown in Figure 143 ($a$), where we see that the outside diameter

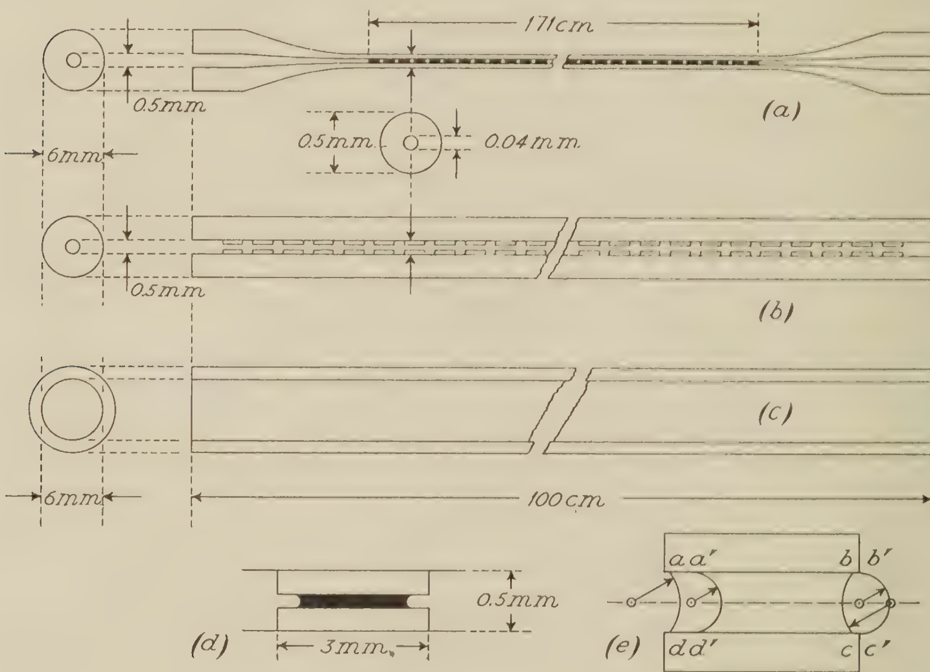



FIG. 143

is reduced from 6 to 0.5 millimeters, and that the bore is reduced from 0.5 to 0.04 millimeters. The tests were performed with one of the heavy ends attached, in order to facilitate connections with the pressure and vacuum pumps.

Jamin prepared his tube with contractions and expansions by carefully heating the tube at successive points with a blowpipe. Figure 142 ($d$) is a reproduction of his drawing. While the contractions in the drawing appear to be perfectly alike, to make them so in the laboratory seemed a too difficult task. A tube as shown in Figure 143 ($b$) proved to be successful. Pieces of 3 millimeter length were broken off the middle section of the tube in ($a$), such portions being selected by virtue of their having an outside diameter very

[23] Its uniformity was tested by measurements made on the length of a bead of mercury that was given successive positions along the tube.

slightly in excess of the inside diameter of the ordinary capillary tube. The latter was warmed over a heated plate, causing it to expand, and the cold pieces were then slipped into place by means of a fine wire. In order to slide the piece rapidly into its position the wire was bent at right angles for each successive set. This permitted a quick run of the wire up to a hilt without requiring fine adjustments as the piece approached its place. It is possible to construct a tube with contractions and expansions of quite homogeneous "texture" in this manner. Globules of wetting liquid, when in equilibrium, appear as in Figure 143 (*d*). Mercury forms globules in the chambers between the canals. Such a tube shows effects which greatly exceed those of Jamin's tube. As in (*e*), a globule *abcd* shifts to *a′b′c′d′*, and the meniscus at the end farther from the greater pressure assumes a negative radius of curvature. Thus the globules overhang the edge of the canals, and as a consequence the limit *f* is greater than it otherwise would be. To the above factors upon which *f* is said to depend must be added the following:

*d*) The texture of the capillary channel within the radius of action of the pressure.

Since sliding is impossible, the globules can be seen to "break and make," in order that air or gas may pass their localities.

This tube shows very effectively the results of Jamin's tests, as recorded in Paragraphs 9 and 10. Water percolates easily, so long as the pressure is applied by means of water, but as soon as the pressure is applied by means of compressed air or gas, successive globules are set up in the manner, and with the result, he describes.

Successful tests were also made with hollow tubes filled with sand. A glass tube so used is shown in Figure 143 (*c*). A similar tube of brass, one inch in diameter and eight feet long, was likewise used in these experiments. The sand must be allowed to pack firmly into place under the influence of the applied pressure; therefore one end should be securely closed with a stopper or plate having the necessary small orifice.

Jamin's experiments with porous materials were also repeated. No attempt was made to elaborate upon his work. His results, as recorded in Paragraphs 11 and 12, were fully verified.

I have proposed to call this resisting action due to a series of alternating globules and bubbles "Jamin action" in recognition of the work done by its discoverer.[24] We find this action present in every reservoir system that contains a porous medium and liquid and gas in combination. Where the pressure *P* is sufficiently great to overpower this action, that is, where *R*, the radius of action of *P*, is greater than the radius of the reservoir so conditioned, the system performs either in Hydraulic or in Volumetric Control, as we have heretofore noted. *But where the pressure* P *is not sufficiently great to overpower this action, that is, where* R, *the radius of action of* P, *is less than the radius*

[24] See § 29.

*of the reservoir so conditioned, the system performs in what we shall know as Capillary Control.*

We must now proceed to show that these tubes and porous materials possess secondary functions that are in accord with the observed primary functions illustrated in Figures 135, 136, 137, 138, and 139. Indeed the latter have been verified in the laboratory.

CHAPTER XXV

# Ideal Performance and Its Primary Functions

"A new result is of value, if at all, when in unifying elements long known, but hitherto separate and seeming strangers one to another, it suddenly introduces order where apparently disorder reigned. It then permits us to see at a glance each of these elements and its place in the assemblage. This new fact is not merely precious by itself, but it alone gives value to all the old facts it combines."—HENRI POINCARÉ

**149.** *The type reservoirs.*—The convenience and advantage of selecting certain artificial reservoirs to serve as types became evident in our study of Hydraulic and Volumetric controls. Now for the same reason with respect to Capillary Control I suggest that we adopt two varieties of Jamin's tubes as types. Thus we shall have the tube of smooth bore, as shown in Figure 142 (*a*) and (*b*), and that with contractions and expansions, as shown in Figure 143 (*b*). The latter we substitute in place of Figure 142 (*d*).

Both tubes possess a high degree of homogeneity throughout their comparatively short lengths. This is essential, for we are to form our opinions concerning the performance of natural reservoirs, where the radius of action of the pressure may reach and exceed half a mile, upon the performance of tubes that will likely not exceed a few feet in length. Furthermore, these tubes are perfectly transparent; we can observe their internal behavior without difficulty, and we may be certain of the fact that during the time of our experiments they are in Capillary Control. For these reasons the tubes are more suitable as type reservoirs than a porous-filled tank, either as in Figure 111 or as in Figure 112,[1] and more suitable than a porous-filled tube as in Figure 143 (*c*).

If we understand the performance of these tubes we cannot err greatly

[1] Inasmuch as well-known tanks have been selected as type reservoirs in the preceding controls, it might appear to be appropriate from the viewpoint of uniformity to select a tank for the present control. We can well afford to sacrifice uniformity for the sake of utility. That tanks filled with sand can perform in Capillary Control is certain. (See § 127.) The mechanical treatment of sands and slimes in cyanide plants for gold and silver ores is largely influenced by Jamin action. This action is encountered in the "sand tanks" and in the vacuum or pressure filters for slime. The liquid is water containing KCN or NaCN in solution, either of which liberate free bubbles of HCN in contact with the crushed ore.

in our opinions regarding the performance of any porous-filled reservoir, whether this be artificial or natural.

The tubes are selected, then, for their evident analytical properties. Both varieties are alike in these, although it can readily be seen that they possess slightly different physical properties. *In one the globules slide during the process of production from the tube, while in the other they break and make.* We may safely assume both to represent faithfully the actual conditions to be found in our various natural reservoirs. Porous formations, as we know, possess their expansions and contractions in pores and minute communicating canals. We shall later decide that in natural reservoirs which produce gas alone the globules break and make, whereas in those which produce oil and gas in combination they break, make, and slide simultaneously throughout the entire mass of porous material that is giving up its fluid.

We are to refer to both types of tubes; sometimes to one and sometimes to the other. They have their individual characteristics, and of these we must take advantage in order to facilitate our analysis. We may be assured that whatever analytical conditions are found to exist in one will likewise be found in the other.

The fundamental principles of the mechanics of Jamin's capillary tubes are difficult. I do not believe they can be thoroughly grasped without recourse to experimentation in the laboratory.

**150.** *From secondary to primary functions.*—To account for new primary function relations we found it necessary to consider the basic principles of the secondary functions. We must turn about, and wander more slowly over paths along which we skipped in arriving at our present location.

Figure 144 ($a$) represents an ideal homogeneous tube with its contractions and expansions. While in fact only ten globules are distorted within a radius $R$ measured from $W$, we should imagine the tube to contain many more in such condition. It is as if we were to part the drawing in the middle, separate the ends, and insert therein a great number of globules properly encased in the same sort of tubing. Thus in general we may say that there are $n$ partitions and $n$ chambers within the distance $R$, as indicated by the symbols $S_0$, $S_1$, and so on, up to $S_n$. It is for convenience only that we confine our discussion to the ten "couplets" appearing in the figure.[2] *We must realize that, in the process of breaking and making, some globules are open at the instant others are closed, and that in fact, among those which are closed, all values of resistance between zero and the limit* f *are simultaneously in*

[2] The particular advantage to be gained by using a small number of couplets in our drawings lies in the fact that the triangles for pressure and volume, as these have already appeared in Fig. 140, are complete. If sections in the middle were missing, the inclined lines would not be continuous. They would be portions of lines that are parallel upon extension; consequently the mechanics to be illustrated by the drawings might be rendered somewhat obscure.

*action.* We should carry in mind a number $n$ in the order of perhaps several thousands, so that we may say with a very high degree of accuracy that homogeneous conditions exist continuously throughout the distance $R$. This

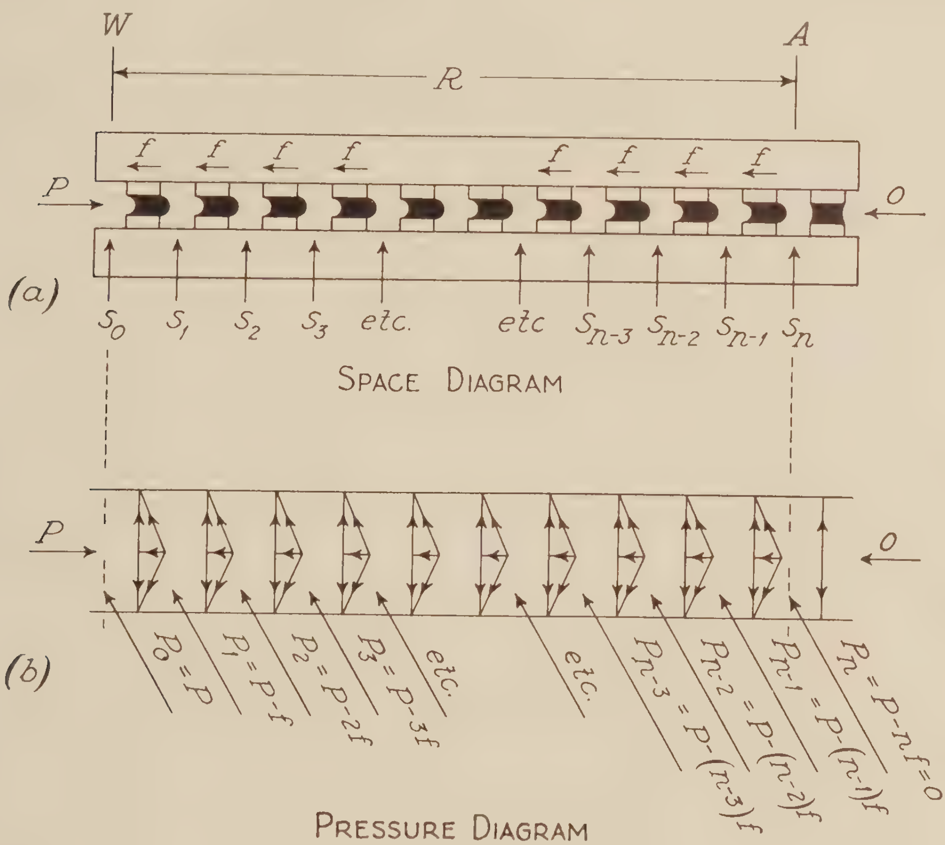



FIG. 144

statement would be based on the assumption that average conditions for all unit portions of $R$, say for all lengths of 10 meters each, are identical at each and every instant.

In accordance with Jamin's observations

$$R = KP \qquad (400)$$

where $P$ is the pressure which he applied at the orifice of the tube, as shown on the left in the figure. This equation simply states a fact that may easily be verified: namely, that the radius of action of the pressure depends upon the intensity of the pressure. Of course the equation refers to any specific tube. If we were to consider all possible tubes in a group, we would say that the radius of action depends upon the following factors:

*a*) The intensity of the pressure.

*b*) The value of the limit of resistance $f$, which in turn is dependent upon four factors listed in section 148.

*c*) The number of $f$'s per unit length of tube.

$R$ is finite so long as $P$ is finite, and $R$ does not extend from the orifice to the other end of the tube unless $P$ is sufficiently great in its intensity.

The pressure $P$, according to our system of mechanics, is a potential pressure. It complies with the definition given to this so-called pressure, namely, that

$$P = S - C \quad (401)$$

wherein $S$ is the static pressure of the tube, as measured at the orifice, and $C$ is the constant back pressure exterior to the orifice of the tube. Under the conditions of the experiment, and in accordance with the figure, $C$ is equal to $A$, the pressure of the atmosphere.

$S_0$, $S_1$, $S_2$, and so on, designate the succession of chambers in the tube. We shall say that these symbols also represent the values of the static pressure in these chambers. Each is smaller than its predecessor by an amount $f$, the value of the resistance due to the globule in its distorted condition. The first chamber is in direct communication with the orifice; consequently

$$S_0 = S \quad (402)$$

and its corresponding

$$P_0 = P \quad (403)$$

Now the situation with respect to potential pressures is shown in (*b*) of the same figure. These diminish progressively from left to right, until finally, to satisfy the condition that

$$S_n = A \quad (404)$$

it follows that

$$P_n = P - nf = 0 \quad (405)$$

as explained in connection with the identical diagram in Figure 140 (*d*).[3]

Before $P$ is applied at $W$, it is evident that a potential pressure of value zero, or a static pressure of value $A$, exists in all chambers. Now the tube, instead of being equipped for equilibrium at this pressure, might well be filled under conditions which provide an equilibrium at a higher pressure. When the orifice of this tube is opened to the atmosphere, an adjustment takes place throughout $n$ chambers in accordance with a constant back pressure

$$C = A \quad (406)$$

[3] In § 146 we assumed that the partitions act as stretched membranes, and therefore we had inclined lines in Fig. 140 (*d*). Now according to the known menisci of the globules these lines should be replaced by curved lines. This circumstance is ignored, however, since the analytical result is the same whether we regard the surfaces as conical or as pseudo-hemispherical.

We say that the tube, as a reservoir, produces fluid until a new equilibrium is established. This situation is shown in Figure 145. Production has ceased; for $S_0$ we have

$$P_0 = 0 \quad \text{.......................} \quad (407)$$

and for $S_n$,

$$P_n = P \quad \text{.......................} \quad (408)$$

the original potential pressure at the orifice $W$. Beyond the distance $R$ from $W$ the tube remains intact; production at the orifice has not disturbed the fluid in this section of the tube to the slightest extent.

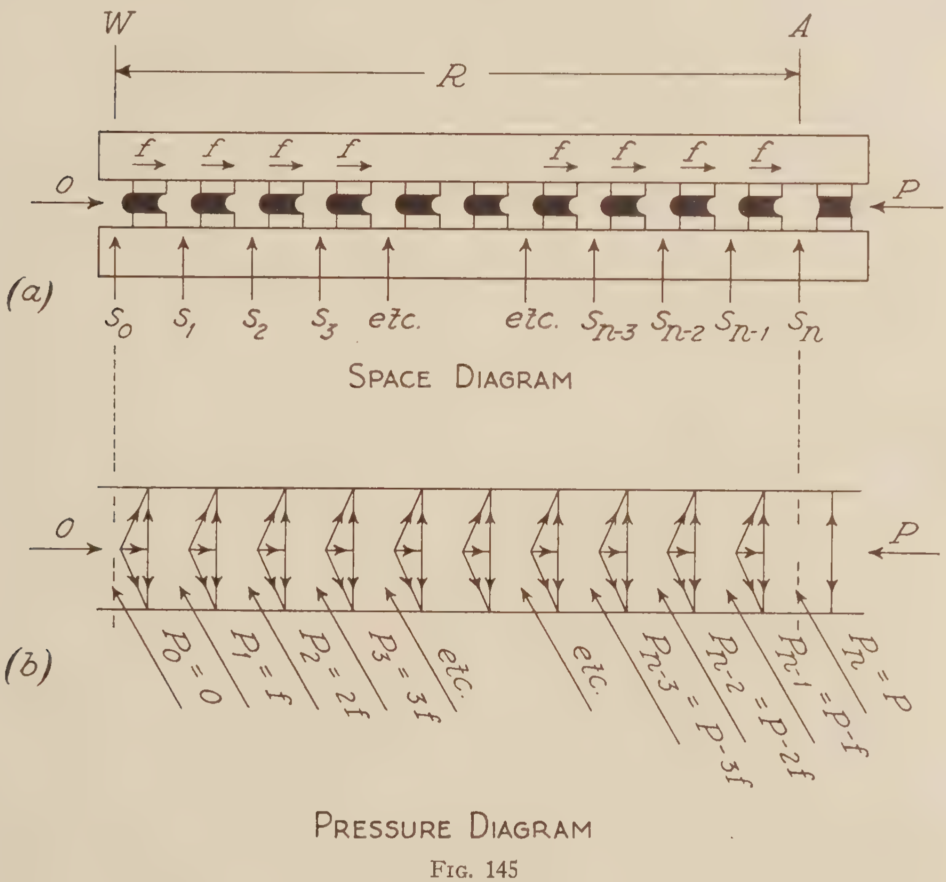



FIG. 145

Suppose, however, that our tube were equipped for equilibrium at a pressure $A$, as in the first instance, and a vacuum were applied at $W$. Production takes place in the manner noted in the second instance. Figure 145 serves in both instances. Now $P$ is given a value equal to $A$ if the vacuum is perfect, and less than $A$ if only partial. The radius $R$, as always, depends upon this value of $P$.[4]

[4] See footnote 19, § 148, page 414.

The mechanics of Figures 144 and 145 are identical. *One simply represents a reservoir formerly in equilibrium at a uniform low pressure in all its chambers and now "filled" at a higher pressure, while the other represents one formerly in equilibrium at a uniform high pressure in all its chambers and now "emptied" at a low pressure.* The reasoning accompanying the description of Figure 140 appeared to me to be more easily followed if carried out on the basis of filling rather than on that of emptying.[5] The analytical result is the same in either case. Now that we have found the principles of that figure to fit with those of Jamin's tubes we should preferably continue on the basis of Figure 145 when we deal with production.

**151.** *Pressure-volume relations.*—The method of analysis we pursued in our study of Volumetric Control is particularly suitable in the present control. What we did at that time will now be repeated. In so far as we are to begin with conditions which we know to be different, we may expect our derivations to be different.

Let us determine analytically the relations between pressure and volume during the process of production from the tube of Figure 145 ($a$). In this I suggest frequent comparison with our procedure in section 103, where we dealt with an analogous situation in Volumetric Control. We should first depict a series of drawings composed of Figures 145 ($a$), 140 ($b$), and 140 ($c$), in succession from the top of the plat to the bottom, in order that we may start off as we did with Figure 80. These two figures would now represent a complete analogy.[6] Having done this we are prepared to proceed.

We have already admitted that the individual chambers comply with the description of reservoirs in Volumetric Control, and that therefore we may safely assume them to perform individually in this control. We say, then, that for each separate chamber

$$P = KVo \qquad \qquad (409)$$

But we see that the globules between the chambers hold back a progressively increasing amount of fluid, as measured outward from $W$, and that consequently a progressively decreasing amount of fluid is delivered from these same chambers. Let us have once more the triangular area representing the volume delivered from the successive chambers, as in Figure 146 ($a$). In the process of production from the tube this triangle moves toward the left. Ten of its positions are here indicated. If we count off the areas as they pass under a vertical line at $W$, that is, if we count off volume as it is produced at $W$, we have data for the curve $abc$ of Figure 146 ($b$). It is observed that Figure 146 is completely analogous to Figure 81.

[5] In view of this we see clearly that the $f$'s in Fig. 140 ($d$) point in the proper direction. (See footnote 9, § 146, p. 408.)

[6] The three parts of such a proposed drawing would, part by part, correspond to those of Fig. 80. I deem it unnecessary to present the drawing.

If we were to write in place of Equation 409

$$Vo = KP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (410)$$

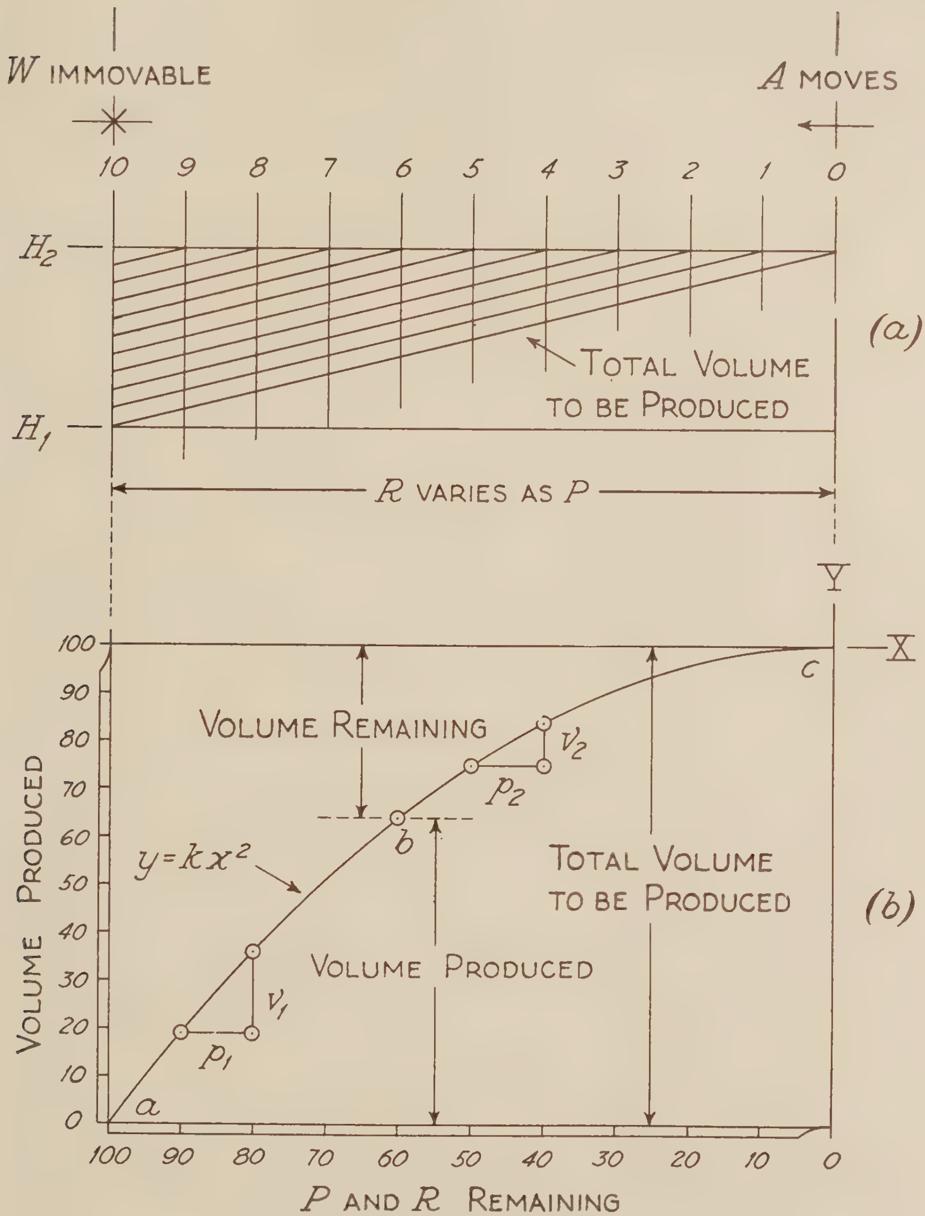



FIG. 146

for these chambers in Volumetric Control, and integrate the expression with respect to the pressure $P$, we would obtain

$$Vo = KP^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (411)$$

In fact this is just the operation that is performed in reasoning from (*a*) to (*b*) in Figure 146. The effect of the globules is cumulative, as we might have known from our earlier discussions concerning them. Wherever effects are cumulative, we perform an analogous mathematical operation which we call integration.[7] The same situation arose in section 103, where we integrated Equation 174,

$$Vo = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (412)$$

and obtained Equation 175,

$$Vo = KP \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (413)$$

Now as illustrated in Figure 147 (*a*) we integrate the straight inclined line

$$y = kx$$

in which $y$ is the volume delivered from individual chambers for any $x$ or $R$, and consequently for any $P$ obtained by measurement at $W$, with respect to $x$. The result is

$$y = kx^2$$

a true parabola with its vertex at the origin of co-ordinates.

In Figure 147 (*b*) we have the relative curve between pressure and volume in potential phase. Figure 146 (*b*) is simply modified so as to show the values of the functions during production in their percentage ratios. The relative constant has the value 1/100; therefore the relative equation is

$$Vo = \frac{1}{100}P^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (414)$$

Titles and scales in the figure are placed on all sides in order to emphasize the relation between them. As usual, there are eight possible positions for this plat. Figure 147 is completely analogous to Figure 82.[8]

It is to be noted in Figure 146 (*b*) that for equal intervals $p$ in the potential pressure $P$ we have unequal volumes $v$ produced or withdrawn from a potential volume $Vo$, and the magnitude of the inequality depends upon the horizontal locations of the $p$'s with respect to $P$, that is, with respect to the value of $P$ possessed by a given reservoir at any instant during the process of production. Thus in the figure $v_1 \neq v_2$, whenever $p_1 = p_2$. This is true wherever both $p_1$ and $p_2$ might be located between $a$ and $c$ on the curve.

[7] The cumulative effects of velocity are expressed as volume, and the cumulative effects of power are expressed as energy. Even in these we integrate to obtain the analytical expression of the effects. See §§ 87 and 88, where the cumulative effects of internal friction were obtained by integration. In § 113 we discussed areas subtended by certain function-pressure curves. Of course areas mean integration or cumulative effects. We integrate either with respect to time or with respect to pressure, depending upon the nature of the investigation.

[8] Incidentally we see examples of the reversal of scales in (*b*) of Figs. 82 and 147. (See footnote 14, § 137, p. 374.)

The relations between the potential functions of pressure and volume upon production from capillary tubes are verified by actual tests in the laboratory.

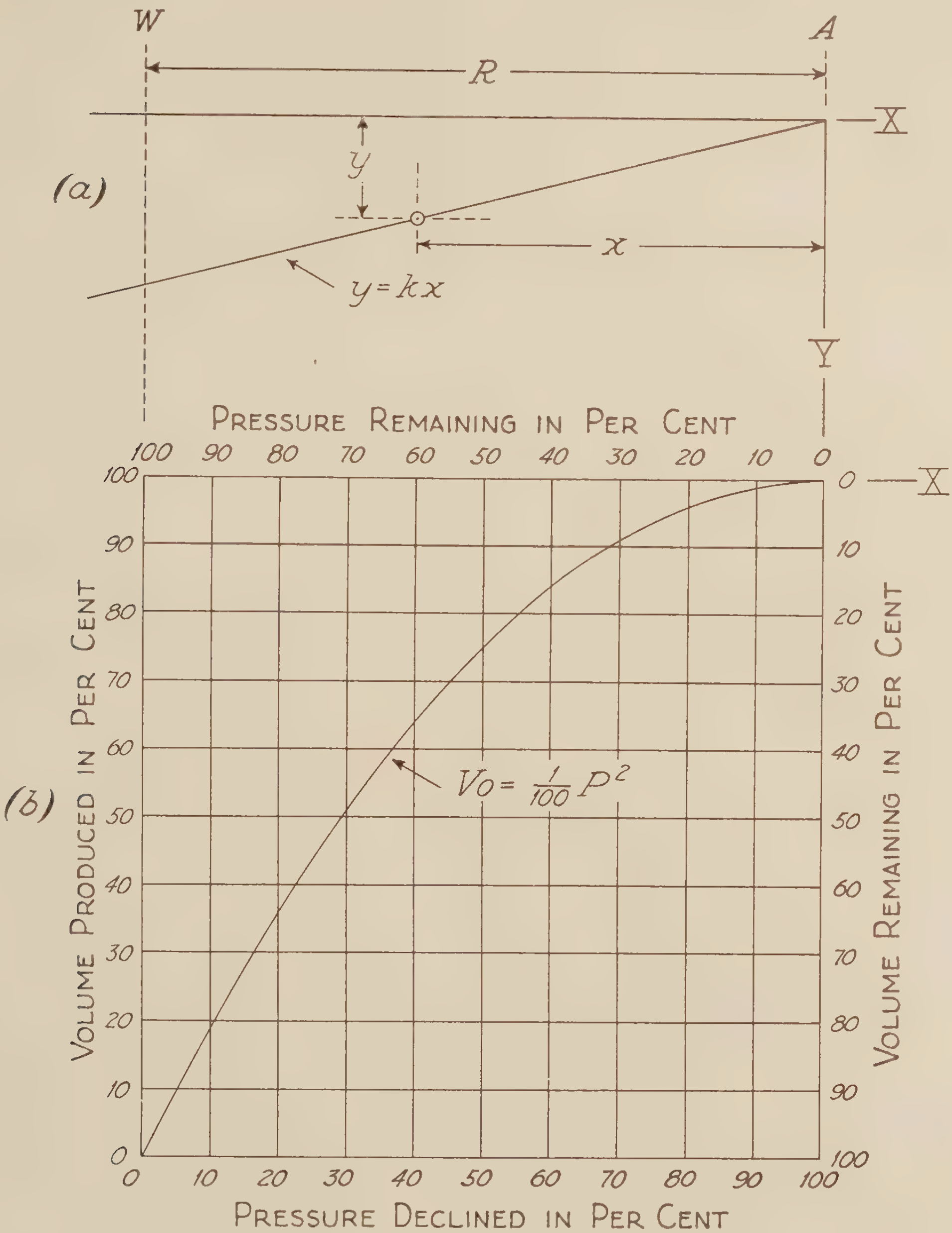



Fig. 147

For this purpose we should use preferably a tube of smooth bore, equipped with globules and bubbles in the usual manner. Accessory apparatus consists

of a tank of compressed air having an orifice-pipe furnished with a valve, heavy rubber tubing, a glass or metal tee to insert in the line, and a mercury manometer connected therewith. A portion of the tube is shown in Figure 148 (*a*). With the globules in equilibrium at atmospheric pressure the

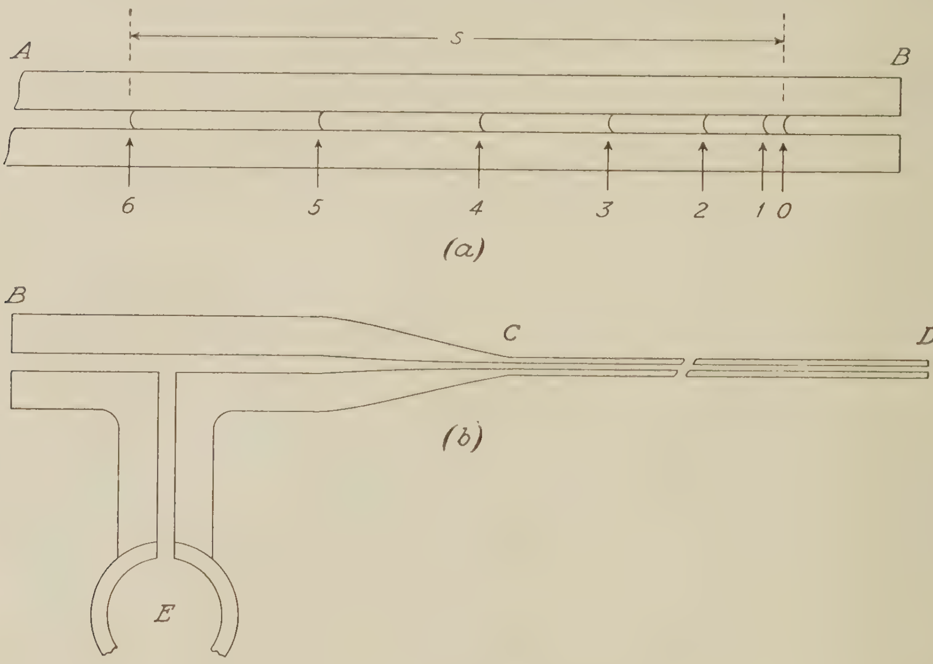



FIG. 148

anterior meniscus of the first globule from the operating end *B* denotes a potential zero for both pressure and volume. Air is admitted carefully until the manometer shows, let us say, 2 centimeters of pressure above that of the atmosphere. The meniscus now stands at Position 1 in the figure. For this potential pressure there is a corresponding potential volume the value of which is to be determined. This volume of air, under compression, occupies the space denoted by the distance between Positions 0 and 1—a space that may be expressed either in cubic units or, if the tube is of accurate uniform bore, in linear units of direct measurement. Now by Boyle's Law in absolute phase this amount of space can be converted into its equivalent at atmospheric pressure.[9] We have from

$$pv = k \qquad (415)$$

the more convenient expression

$$p_x v_x = p_c v_c \qquad (416)$$

[9] We can measure spaces and treat the data as if they pertained directly to volume. In the analogous experiment of § 103 the conversion by Boyle's Law was rendered unnecessary by providing the space *C* in Fig. 80 (*a*).

where the subscripts $s$ and $c$ denote standard atmospheric conditions and conditions under compression, respectively. The quantity $v_s$ is the required potential volume.

A series of observations and computations follow. The data are idealized. As observed, the successive increases in potential pressure are equal at 2 centimeters of mercury. A barometric pressure of 76.0 centimeters and a constant temperature (of unspecified degree) are assumed.

TABLE OF DATA

| Number of Observation | Absolute Pressure | Space Measured | Potential Pressure | Potential Volume |
|---|---|---|---|---|
| 0 | 76.00 | 0.000 | 0.00 | 0.00 |
| 1 | 78.00 | 0.975 | 2.00 | 1.00 |
| 2 | 80.00 | 3.800 | 4.00 | 4.00 |
| 3 | 82.00 | 8.340 | 6.00 | 9.00 |
| 4 | 84.00 | 14.480 | 8.00 | 16.00 |
| 5 | 86.00 | 22.100 | 10.00 | 25.00 |
| 6 | 88.00 | 31.100 | 12.00 | 36.00 |
| . | ..... | ...... | ..... | ..... |

The positions of the meniscus for the six observations are shown in the figure.[10]

To illustrate the use of Equation 416 let us consider Observation No. 2. Here we have

$$p_s = 76.00$$
$$v_s = \quad ?$$
$$p_c = 80.00$$
$$v_c = 3.800$$

Then in accordance with this equation

$$76.00\, v_s = 80.00 \times 3.800$$

therefore

$$v_s = 4.00$$

That is to say, a potential volume of 4.00 units corresponds to a potential pressure of 4.00 centimeters of mercury.[11]

[10] In the actual experiment the zero point as read is unreliable, for the globule is not exerting a force $f$ in this position; consequently the curvature of the meniscus is not the same as it is for the other positions. The zero point should be calculated from two other observations, the required equations being as follows:

$$\frac{Vo_1}{Vo_2} = \left(\frac{P_1}{P_2}\right)^2$$

and

$$Vo_1 - Vo_2 = v$$

This is a special case in the general problem to be treated in the next section.

[11] In this experiment the appropriate applications of pressure-volume relations in absolute and potential phases are evident.

By inspection of the table we see that the potential equation between pressure and volume in this experiment is as follows:

$$V_o = \frac{1}{4}P^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (417)$$

in accordance with the units in which volume is expressed. To obtain two times the pressure, the volume of air at atmospheric pressure to the left of Position 0 in the figure must be made four times as great, and to obtain three times the pressure this volume must be made nine times as great, and so on, in accordance with parabolic variation. When these values of potential functions are plotted on a Cartesian plat, their curve appears as in (*b*) of Figures 146 and 147. These agree with the curves $D_2$ of Figures 137 and 138.

*The volume to be produced by a reservoir in Capillary Control varies as the square of the potential pressure measured at the orifice.* I suggest that we call this Jamin's Law for pressure and volume.[12] The law can also be stated as follows: The potential pressure measured at the orifice of a reservoir in Capillary Control varies as the square root of the volume to be produced.

152. *Pressure-time relations.*—An actual test upon the relations between the potential functions of pressure and time may be carried out with our apparatus set up in a modified way. For this we need only change the accessories. In combination with our tube of Figure 148 (*a*) another tube as shown in (*b*) is used. This is made of the same tubing as in the first experiment. There is a tee between *B* and *C*, and to this is attached, by fusion, a capillary stopcock *E*. The portion *CD* is drawn in the usual manner, and its end at *D* is sealed. The desired diameter of bore depends upon the length of the drawn portion. Through this the production of air, regulated and maintained at a very slow rate by means of its friction, is to take place.[13]

The tube thus prepared is joined to the first one by means of a short piece of heavy rubber tubing. Now air is carefully admitted from the tank through *E*, until the first meniscus stands back from *B*, as for example, at Position 6 in (*a*). *E* is then closed, and the tip at *D* is broken off. Readings are to be taken between volume displacement and time elapsed. The former is thereafter successively converted into potential volume and potential pressure in accordance with the preceding experiment upon the same tube (*a*), and the latter is changed to time remaining.[14] An idealized set of data and

[12] This clearly corresponds to the following law: *The volume to be produced from a reservoir in Volumetric Control varies directly as the potential pressure measured at the orifice.* We now have a new principle which parallels one that has been known for some three hundred years.

[13] The tube can be drawn to diameters less than 0.02 millimeter, if desired.

[14] Knowing the pressure-volume relations for the tube, and given the data for volume and time, we are able to compute the data for pressure and time. By this method we are not concerned with expansion of air in the part of the apparatus to the right of Position 0. The units of time adopted here are unspecified.

computations follows. These values are based upon the same assumptions as before.

TABLE OF DATA

| Number of Observation | Absolute Pressure | Space Measured | Potential Volume | Potential Pressure | Potential Time |
|---|---|---|---|---|---|
| 0 | 88.00 | 31.10 | 36.00 | 12.00 | 6.00 |
| 1 | 84.33 | 15.65 | 17.36 | 8.33 | 5.00 |
| 2 | 81.33 | 6.65 | 7.11 | 5.33 | 4.00 |
| 3 | 79.00 | 2.16 | 2.25 | 3.00 | 3.00 |
| 4 | 77.33 | 0.43 | 0.44 | 1.33 | 2.00 |
| 5 | 76.33 | 0.02 | 0.03 | 0.33 | 1.00 |
| 6 | 76.00 | 0.00 | 0.00 | 0.00 | 0.00[15] |
| . | ..... | .... | .... | .... | .... |

It is evident that the potential equation between pressure and time in this experiment is

$$P = \frac{1}{3} T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (418)$$

The potential equation between pressure and volume is, of course, the same as in the first experiment: namely,

$$Vo = \frac{1}{4} P^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (419)$$

If the above values of potential pressure and potential time are plotted on a logarithmic plat, their curve appears as $E_2$ in Figure 139. $E_2$ is parallel to $E_1$—pressure-time curves in Volumetric and Capillary controls are alike.

In both of these experiments upon the performance of Jamin capillary tubes it is to be particularly noted that the potential reservoir system which we are testing lies entirely to the left of Position 0 in Figure 148 ($a$). How far to the left does it extend? The answer is that it extends over the distance $R$, the radius of action of the pressure $P$ at Position 0. But must we not admit that there is a space to the right of Position 0 which performs simultaneously with that designated as the potential system? That this is true is obvious in both experiments, and particularly so in the second one. Not only is there a space to the right of our zero in the tube ($a$) itself, but also there is space in any accessory apparatus which might be attached to this tube. All such space taken together performs as a reservoir in either Hydraulic or Volumetric Control, depending upon the conditions of the experiment.[16] Our experimental procedure calls for the performance of a comprehensive system which we deliberately separate into two minor systems, one of which is potential and the other accessory. Our observations are taken in a manner

[15] In the actual experiment the zero point of time remaining is difficult, if not impossible, to detect. It may be calculated by means of equations to be presented in § 153.

[16] In the first test this space performs in Hydraulic Control, theoretic performance, Case 2; and in the second it performs in Volumetric Control, ideal performance.

that provides data for the performance of one to the exclusion of the performance of the other. It is thus that we learn of the performance of our type reservoirs in Capillary Control.

A laboratory system of this sort necessarily provides an accessory system of dimensions comparable to those of the potential system. Fortunately—in virtue of the transparency of our tubes—we can analyze the performance of that system alone in which we are interested. In natural systems, for which the tubes are to serve as types, the accessory system is exceedingly small in comparison with the size of the potential system. Here the expansion chamber at the bottom of the well and the casing or tubing of the well constitute the accessory system, while the potential system lies within the porous productive formation. In this case the performance of the accessory system can be completely ignored in practice.

With slight modifications in our methods we can perform the same experiments upon tubes which possess contractions and expansions. The analytical results are identical, for expressions like Equations 418 and 419 are fulfilled. As previously noted, the physical performances of the two kinds of tubes are somewhat different. There is no sliding of the first globule through a variable distance $s$, Figure 148 ($a$), accompanied by the sliding of following globules through progressively shorter distances; but instead the globules break and make in order to allow the usual compression of the gas in the bubbles—now chambers—to take place.

We already know that $R$, the radius of action of the pressure $P$, varies with $P$ in a given Jamin capillary tube. This is stated mathematically in Equation 400 (p. 421): namely,

$$R = KP \quad \text{(420)}$$

Now in our second experiment with the tubes we find that in general

$$P = KT^2 \quad \text{(421)}$$

of which Equation 418 is a specific example. In accordance with these two equations we may write the following:

$$R = KT^2 \quad \text{(422)}$$

an expression which states that during the process of production from a Jamin tube the radius $R$ varies as the square of time remaining.

In our first experiment we found that in general

$$Vo = KP^2 \quad \text{(423)}$$

of which Equation 419 is a specific example. We may also write this in the form

$$P = KVo^{1/2} \quad \text{(424)}$$

for the purpose of combining it conveniently with Equation 420, above. This results in

$$R = KVo^{1/2} \quad \text{(425)}$$

an expression which states that the radius $R$ varies as the square root of the potential volume of fluid in the system.[17]

Equations 422 and 425 indicate the fact that the bubbles, or chambers, attain successively a state of equilibrium. The establishment of this equilibrium is progressive during the process of production; the farthest bubble or chamber attains it first, the next one secondly, and so on along the tube until the last one—that in communication with the orifice—attains it. When this last one is in equilibrium, the entire system is at rest. This progression is easily noted in our experiments. According to Figure 145 the pressure under which equilibrium exists progressively increases by an amount $f$ for each globule, as distances increase from the end of the tube nearer the orifice.

The relative curve, introduced in section 102, serves in the same capacity in Capillary Control as in Volumetric Control. Since the relations between pressure and time are identical in both controls, the same relative curve for these functions is applicable to both. Figure 78 will therefore serve in the present control without the slightest modification. We found the relative equation to be

$$P = \frac{1}{100} T^2 \qquad (426)$$

and the points for the curve were given in the section to which we are here referred. This curve is subject to stretching and compressing in the same manner, and with the same significance, as before. It continually repeats itself in this operation.

**153.** *Volume-time relations.*—A Jamin capillary tube of smooth bore, when properly equipped with globules and bubbles at equilibrium under a static pressure $S$ greater than a constant back pressure $C$, can produce liquid and gas in combination. Obviously the produced liquid is derived from the globules which slide along the tube and escape at the end, while the produced gas is derived from the bubbles. In order that the globules may escape at the end of the tube, their "Positions 0" must not lie within the tube in the manner shown in Figure 148 ($a$), but they must lie at points exterior to the tube. If the situation is as illustrated in the figure, the tube can produce only gas.

Again, a Jamin capillary tube having contractions and expansions, when properly equipped with globules and bubbles under the conditions just noted, can produce liquid and gas in combination. In order that it may do so, the tube must contain liquid in excess of the amount required to fill the contractions. If the quantity of liquid is equal to, or less than, this amount, the tube can produce only gas.

*Neither tube can produce liquid alone.* This we shall account for when we investigate the energy possessed by the fluids within the tube. In this fact

[17] Our present system is clearly one in which there is tubular flow. We cannot expect these equations with $R$ to be correct in a natural reservoir, where there is radial flow. (See § 172.)

we observe a difference between possibilities in Capillary Control, on the one hand, and Hydraulic and Volumetric controls, on the other. The difference exists among artificial "type reservoirs" and natural reservoirs alike.

In our studies of the first two controls we found it necessary to differentiate between the primary function curves for the production of gas when this fluid alone is produced, and when it is produced in combination with liquid. We must do likewise in the present control. The equations which we are now to derive pertain to gas, if only gas is produced from our tubes, and to liquid, if both gas and liquid are produced. We leave the equations that pertain to gas which accompanies liquid for subsequent discussion concerning energy and power.

If by experimentation we are satisfied with the two equations,

$$P = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (427)$$

and

$$P = KVo^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (428)$$

we may proceed at once to determine all other fundamental and derived primary function equations for this control. To do this we merely duplicate the analytical computations made in our investigation of Volumetric Control. We shall expect results which do not agree with our former determinations, for Equation 428 is not the one previously demanded by the control. It is a very easy matter to verify our analytical computations by means of the data which we obtained in the second experiment.

To repeat steps taken in section 145, we equate the two right-hand members of the given equations. Thus

$$Vo^{1/2} = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (429)$$

and this when squared becomes

$$Vo = KT^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (430)$$

*The potential volume of fluid to be delivered from a reservoir in Capillary Control, under given conditions of production, that is, in ideal performance, varies as the fourth power of time remaining.* The curve for this equation is a parabola of the fourth power, just as in Volumetric Control we found the curve for energy-time relations to be.[18] When the present curve is inverted, it becomes the cumulative production curve for this control. Thus we have Figure 149 comparable with Figure 84 for the preceding control. In consideration of the description given the latter figure in section 104 further comment upon the present figure is unnecessary.

The relative curve between volume and time is shown in Figure 150 (p. 436). Its equation is

$$Vo = \frac{1}{1{,}000{,}000} T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots (431)$$

the relative constant, as usual, being $K = 10^{2-2n}$, as explained in section 109.

[18] An assortment of curves of the fourth power can be found in Fig. 88.

The value of $n$ is now 4. This curve is subject to identically the same manipulation as its correspondent for Volumetric Control in Figure 83. As always, a reservoir makes a full sweep through its relative curves from each and every instant during the process of production.[19] The points for the present curve are as follows:

| $T$ in Per Cent | $Vo$ in Per Cent |
|---|---|
| 0 | 0.00 |
| 10 | 0.01 |
| 20 | 0.16 |
| 30 | 0.81 |
| 40 | 2.56 |
| 50 | 6.25 |
| 60 | 12.96 |
| 70 | 24.01 |
| 80 | 40.96 |
| 90 | 65.61 |
| 100 | 100.00 |

It is interesting to note that when the potential pressure has declined to 50 per cent, that is, when time remaining has been reduced to 70.71 per cent, the potential volume in the present control has declined to 25.00 per cent, whereas in Volumetric Control, under like circumstances, it has declined to 50.00 per cent. Seventy-five per cent of the volume has been withdrawn from the reservoir instead of 50 per cent. This observation agrees with the curves of Figure 138.

Let us now investigate volume-time relations with respect to our experiments with the Jamin tube of Figure 148. For this particular system we found that the two following equations hold:

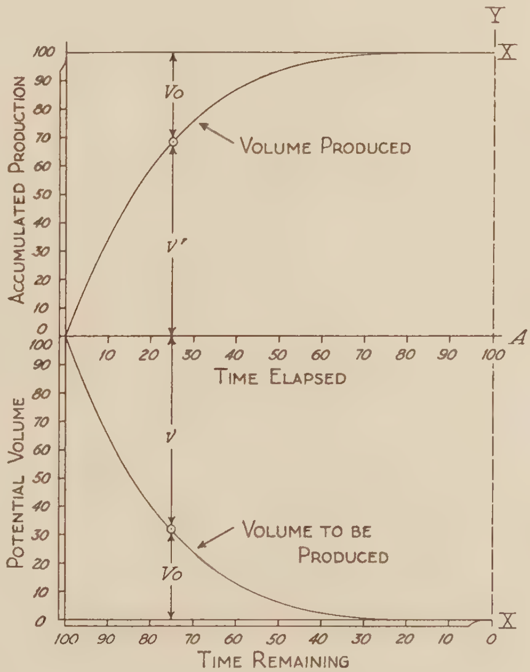



Fig. 149

$$P = \frac{1}{3} T^2 \qquad (432)$$

and

$$Vo = \frac{1}{4} P^2 \qquad (433)$$

[19] See § 102.

By combining these we learn that

$$Vo = \frac{1}{36} T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (434)$$

Our procedure in determining pressure-time relations involved potential volumes at successive instants. In fact we may say that in the first experiment we merely calibrated the tube in regard to pressure and volume, for the purpose of reading the latter in place of the former in the second experiment. Thus our table of data in section 152 contains observations upon volumes, for each one of which there is a corresponding observation on time. If Equation 434 is correct, the values for volume and time conform to it; and this they do. By substituting any of the values of $T$ into the equation we obtain the recorded volume.

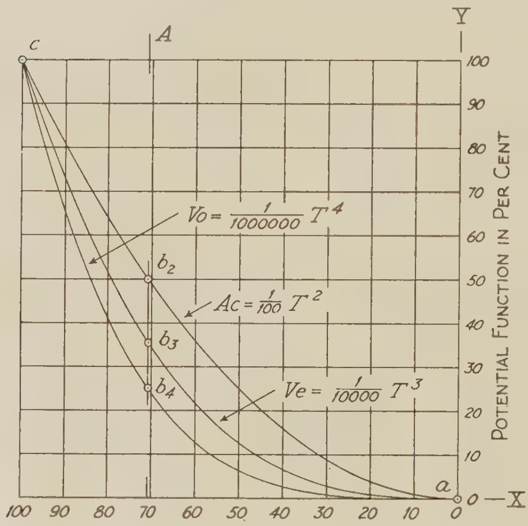



FIG. 150

When we may be assured that our potential system is so large that any accessory system joined thereto can be entirely ignored, as explained in the preceding section, we are able to make computations analogous to those of section 104. There we imagined circumstances wherein the reservoir is inaccessible at all points other than at its orifice. Corresponding to Equations 181 and 182 (p. 255) we now have

$$Vo_1 = KT_1^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (435)$$

and

$$Vo_2 = KT_2^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (436)$$

On dividing the first equation by the second we obtain

$$\frac{Vo_1}{Vo_2} = \left(\frac{T_1}{T_2}\right)^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (437)$$

an equation that apparently contains four unknown quantities. If we measure the volume produced between the instants $T_1$ and $T_2$ we have

$$Vo_1 - Vo_2 = v \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (438)$$

where $v$ is this volume. $T_1$ and $T_2$ become known by observing the corresponding values of $P_1$ and $P_2$; thus we finally have two equations with but

two unknown quantities. They may be solved simultaneously in the usual manner.

To illustrate our computations let us say that we find

$$P_1 = 12.00$$

and

$$P_2 = 3.00$$

These are values of the pressure at instants such that

$$T_1 - T_2 = 3.00$$

According to the relative curve between pressure and time we see that when the pressure has declined to 25 per cent, time remaining has declined to 50 per cent; therefore the 3.00 units of time between $T_1$ and $T_2$ represent the 50 per cent of time remaining which has elapsed. Consequently we know that

$$T_1 = 6.00$$

and

$$T_2 = 3.00$$

Let us say that according to measurement we find

$$Vo_1 - Vo_2 = v = 33.75$$

Now we have all that is required to solve Equations 437 and 438 simultaneously. More conveniently we may refer directly to the relative curve between volume and time. There we see that when time remaining has declined to 50 per cent, the potential volume has declined to 6.25 per cent; therefore the 33.75 units of volume represent 93.75 per cent of the potential volume withdrawn from the reservoir. If

$$93.75\% = 33.75$$

then

$$100\% = 36.00 = Vo_1$$

and

$$Vo_2 = 2.25$$

These values correspond to Observations 0 and 3 in the second experiment.

As a matter of fact we need not consider $T_1$ and $T_2$, as the quantities of time required to produce $Vo_1$ and $Vo_2$, at all. It is sufficient to read the pressures and measure the volume produced between the readings. Our two equations are then

$$\frac{Vo_1}{Vo_2} = \left(\frac{P_1}{P_2}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (439)$$

and Equation 438 as given. Now we may refer to the relative curve between pressure and volume, or to the relative curves for pressure and time and volume and time. In the latter case we cut the pressure-time curve with a vertical line and note where this line in turn cuts the volume-time curve.

We have agreed that $R$, the radius of action of the potential pressure $P$, is to be measured outward to the left from Position 0 in Figure 148 ($a$). There is within this distance $R$, irrespective of the length that it may measure at any instant, a certain amount of fluid that is either in motion or capable of being set in motion toward Position 0, but of this fluid—which may be conveniently designated "the mobile fluid"—only one-half can reach and pass this position by the time $R$ becomes equal to zero.[20] In other words, if $Vo$ is the potential volume of the system at any instant, then there is a mobile volume of fluid $Vo'$ within the same radius $R$ such that[21]

$$Vo' = 2Vo$$

This can be observed as the case in Figure 140 ($b$). $S_0$ contains a potential volume

$$Vo_0 = 100\%$$

and a mobile volume

$$Vo'_0 = 100\%$$

while $S_n$ at a distance $R$ from $S_0$ contains a potential volume

$$Vo_n = 0\%$$

and a mobile volume

$$Vo'_n = 100\%$$

Between $S_0$ and $S_n$, as of the entire system included within the radius $R$, there is a potential volume

$$Vo = \tfrac{1}{2}\, Vo_0 \times R$$

and a mobile volume

$$Vo' = Vo_0 \times R$$

Like the potential volume, the mobile volume is dependent upon the values of $S$, the static pressure, and $C$, the constant back pressure. More specifically, it is dependent upon $S - C$, the value of the potential pressure $P$ of the given system.

[20] The potential reservoir, as defined in § 19, possesses a physical dimension $R$ in these tubes, but this does not mean that all fluid within $R$ is potential volume. As we know, we learn of the size of the potential reservoir by measuring the area subtended by the velocity-time curve for the gas, if gas alone is produced, and for the liquid, if liquid and gas are produced. Mobile volume is always of the same nature as potential volume.

[21] The expression following holds only for tubular systems. For radial systems we must write $Vo' = 3Vo$. This will be explained later.

CHAPTER XXVI

# Ideal Performance and Its Primary Functions
## *(Continued)*

"It appears to me . . . . that the conviction is constantly gaining ground, that in the present more advanced state of science those only can experimentalize profitably who have a clear-sighted knowledge of theory, and know how to propound and pursue the right questions; and, on the other hand, only those can theorize with advantage who have great practice in experiments."—HERMANN VON HELMHOLTZ

**154.** *Static and kinetic effects of Jamin action.*—As I have said before, I do not believe that the mechanics of Jamin's capillary tubes can be thoroughly grasped without recourse to experimentation in the laboratory. Familiarity with the action of these tubes places us in a position to understand the behavior of all natural reservoirs which contain liquid and gas intimately distributed throughout their porous mediums, regardless of the control in which they may be.

In experimenting upon these tubes we soon learn that if we apply too great a pressure at one end, the action of the globules and bubbles is overcome. $P$, we say, is greater than the sum of all the $f$'s. Under these conditions they are not reservoir systems in Capillary Control, but they are merely conduits, serving as accessories to a greater system wherein the pressure $P$ originates. As such they are parts of a major system which is either in Hydraulic Control or in Volumetric Control.

But if the pressure is not greater than the sum of all the $f$'s, the globules and bubbles dictate the performance of the tubes. These tubes are now independent of that greater system wherein $P$ originates. Communication between the source of $P$ and the tubes can be completely closed without influencing their performance in any manner. They are complete reservoir systems in themselves; and they are in Capillary Control.

When we connect a tube with its contractions and expansions to a tank of compressed air or gas, wherein a pressure $P$, greater than the sum of all the $f$'s, can be maintained, flow takes place through the tube. If we measure its rate, we find that it is exceedingly small in comparison with the rate realized in the absence of globules of liquid in the tube, $P$ being the same in either case. *A series of alternating globules and bubbles produces an effect equivalent to that of a very high viscosity.* Granted that gases appreciably reduce the viscosity of liquid with which they come in contact under pressure—in virtue of the fact that they are dissolved in the liquid—any beneficial effects due thereto are

offset and greatly exceeded by the impedient effects of bubbles of free gas. We can with complete safety ignore the former in the presence of the latter.

I suggest that we speak of these high viscosity effects as the "kinetic effects of Jamin action."[1] They are encountered in natural reservoirs of Hydraulic and Volumetric controls, provided, as previously noted, gas is present with the liquid in the reservoir. They are also encountered in natural reservoirs of Capillary Control, which, as we now know, only exist in case gas is present with the liquid in the reservoir.

Having performed this experiment with one tube, we may readily imagine what can happen in case two tubes are connected with the tank of compressed air. Let us suppose that the two are physically identical when they are without globules. One is equipped with globules of water and the other with globules of oil. Furthermore, we might suppose the tube with water to be less perfectly filled with globules; there are, we shall say, only one-tenth as many globules in action at any instant within this tube. The surface tension of the water can be considered to be three times that of oil. Now the situation is simply that the sum of all the $f$'s in the tube containing globules of oil is three and one-third times the sum of all the $f$'s in that containing globules of water. We can easily see that the lineal velocity of flow through the latter tube is greater than that in the former one. We are indeed carried back to the discussion given in section 99 concerning reservoirs of oil and water, with gas. Obviously it is immaterial whether the tank of compressed air is performing in Hydraulic or Volumetric Control, so long as $P$ is greater than the sum of all the $f$'s in the tube containing globules of oil.

It is conceivable that $P$ can have a value greater than the sum of all the $f$'s in the tube containing water, and yet less than the sum of all the $f$'s in the tube containing oil. The first tube continues to produce in the control dictated by the tank of compressed air, while the second is converted into a system in Capillary Control. It is conceivable that just such a situation might arise in natural reservoirs of Volumetric Control upon a sufficient decline in $P$.

To vary the conditions slightly we might imagine the two tubes "texturally different," yet containing globules of the same liquid. One of the tubes has more minute contractions and greater numbers of them per unit of length. Again we see that the sum of all the $f$'s is greater in the one than in the other. This situation received brief mention in section 141, where we admitted that compact strata may, in Capillary control, produce directly into other strata of a more open texture, the latter carrying the fluid thence to the well.

The effects of Jamin action are subordinate in Hydraulic and Volumetric controls. They are limited, as we see, to those effects we describe as kinetic. The rate of production is lowered by virtue of the presence of this action,

[1] I mean "kinetic" in the sense in which we have used the word before. These effects are non-existent when the fluid is at rest in the system. Compare with the use of the word in the expression "kinetic pressure gradient."

but its curve with time complies with the same mathematical equation as though the action were absent.

In Capillary Control the effects of Jamin action are predominant. Not only are the subordinate kinetic effects present, but also greater ones which we may well call "static effects."[2] Let us connect again one capillary tube, as before, with the tank of compressed air. We shall now allow the complete system to produce as long as it will in Volumetric Control. At some instant the tube becomes independent of the tank, being converted, as we say, to Capillary Control. A short time later flow ceases completely; yet there remains a pressure $P$ greater than $C = A$, the pressure of the atmosphere, within the tank. Now the entire apparatus will remain in this state indefinitely, as observed by Jamin himself. $P$ simply declined in value until it became equal to the sum of all the $f$'s in the tube.[3]

These static effects are the ones we have been studying since we began chapter xxiv. While their analytical importance at once became obvious, I believe their economic importance cannot come to light forcibly unless we grasp the significance of our argument concerning volume-time relations, as presented in section 153.

Aside from corroborating evidence in the field, such as that cited in sections 145 and 146, it is, I know, difficult to accept freely the principles which are founded on the static effects of Jamin action. When we hear of it for the first time we immediately think of the nature of pressures that can exist within masses of fluids, and it seems to us that there is somewhere a contradiction to well-known facts. Most of us feel that only so-called hydrostatic pressures can be transmitted by fluids; that is, only pressures of equal intensities in all possible directions about a point can be transmitted to the walls of the container.[4] Some of us might say that the mechanics of the tubes as we describe them, specifically with regard to the static resistance $f$ which a globule can offer in a direction opposed to that of movement dictated by the pressure $P$, contravenes the second law of thermodynamics.[5] Both of these attitudes are incorrect.

What facts concerning pressures must we believe to exist with respect to the fluids within the Jamin tube? It seems to me that we are obliged to admit the following:

*a*) That we deal with hydrostatic pressures. These, as just noted, are ex-

[2] I mean "static" in the sense in which we have used the word before. These effects can be said to exist when the fluid is either at rest or in motion. Compare with the use of the word in the expression "static pressure gradient." The terms static and kinetic have been borrowed from the physicist. He uses them in describing two kinds of friction. There is a perfect analogy in the situation with respect to fluids and solids.

[3] Here we see clearly that the sum of all the $f$'s is identical with $j$ associated with $Q$ in §§ 127 and 140.

[4] This belief is obviously founded on Pascal's Principle.

[5] See § 51.

erted equally in all directions about a point within the mass of fluid, whether this be liquid or gas.

*b*) That we furthermore deal with pressures whose exertion is directed in single lines. These, we have said, constitute the $f$'s, as opposed to $P$, in a direction denoted by $R$.

*c*) "To every action there is always an equal and contrary reaction." This, Newton's third law of motion, is applicable to pressures of (*a*) and (*b*) in their combined effects.

We clearly observe the effects of the pressures of (*b*) in our experiments with the tubes. Shall we say that what we see is not true? Let me quote H. H. Dixon on this very type of pressures within masses of fluid:[6]

"Water under suitable conditions can transmit a tension just like a rigid solid. . . . . In fact, in tension experiments the liquid becomes capable of sustaining and transmitting tensile stresses only when it is adhering completely to a rigid envelope which confers on the liquid a pseudo-rigidity. The state of tension then persists because the stretching forces act solely against the cohesive properties of the liquid (i.e., in an endeavor to separate the water molecules from one another—a separation which a liquid is able to withstand as well as a solid)."

Liquids, then, have the very property we have ascribed to them, while they exist in globular form, adhering completely to a rigid envelope furnished by the inner wall of the capillary tube. I am not bringing to light properties of matter hitherto unknown in attempting to account for Jamin action.

**155.** *Velocity-time relations.*—The potential volume of fluid to be delivered from a reservoir in this control, in so far as it depends upon Jamin action, depends solely upon the static effects of this action. Changes in its value during the process of production denote potential velocity. The mode of change in volume, that is, the path described by the velocity-time curve, likewise depends solely upon the static effects of this action. The numerical values of the velocity at each individual instant during the process of production depend upon both static and kinetic effects of this action, if the reservoir is in Capillary Control.

When the volume of fluid within a reservoir at any instant may be defined as a function of time, the first derivative of the expression, with respect to time, is a statement of the relation between velocity and time. Thus from

$$Vo = KT^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (440)$$

we obtain by differentiation with respect to $T$ the relation

$$Ve = KT^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (441)$$

[6] H. H. Dixon, *Transpiration and Ascent of Sap in Plants*, pages 84–88, Macmillan. The quoted statement is based upon a simple laboratory experiment entitled, "The Cohesion of Water and Glass." I believe this experiment to be of interest to anyone concerned with the mechanics of production.

*The potential velocity of fluid issuing from a reservoir in Capillary Control, under given conditions of production, varies as the cube of time remaining.* The curve for this equation is a cubic parabola, just as in Volumetric Control we found the curve for power-time relations to be. If then, in Figure 90 (p. 266), we change ordinates to read "Units of Potential Velocity," we have a variety of curves which suit this control. The curve $B_2$ of Figure 135 conforms to Equation 441. If we should add to this figure the curves corresponding to those of Figure 90, we would have a series of lines parallel to $B_2$. All would differ only in possessing individual intercepts on the sides of the frame, since they are distinguished only by their respective values of the constants $K$.

The relative curve between velocity and time is shown in Figure 150. Its equation is

$$Ve = \frac{1}{10{,}000}T^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (442)$$

where the relative constant is determined in the usual manner. The points for this curve are as follows:

| $T$ in Per Cent | $Ve$ in Per Cent |
|---|---|
| 0 | 0.00 |
| 10 | 0.10 |
| 20 | 0.80 |
| 30 | 2.70 |
| 40 | 6.40 |
| 50 | 12.50 |
| 60 | 21.60 |
| 70 | 34.30 |
| 80 | 51.20 |
| 90 | 72.90 |
| 100 | 100.00 |

It is interesting to note that when the potential pressure has declined to 50 per cent, that is, when time remaining has been reduced to 70.71 per cent, the potential velocity in the present control has declined to 35.35 per cent, whereas in Volumetric Control, under like circumstances, it has declined to 70.71 per cent. Two reservoirs, one in each control, may be imagined to begin production at the same rate and under equal values of potential pressure, and thereafter, when both have declined to 50 per cent of their original values of pressure, one is producing at two times the rate of the other.

We have had occasion before to refer to the fact that if the velocity and pressure curves are drawn with their vertical scales such as to cause both to start from the same point in the upper left-hand corner of the plat, in this control the former underlies the latter. Regardless of the fact whether this starting-point represents the initial instant or any subsequent instant in the life of the reservoir, the relative position of the two curves on the plat remains invariable.[7]

[7] See footnote 2, § 58, page 107, and footnote 12, § 141, page 384.

The area subtended by any potential velocity-time curve represents potential volume.[8] The geometrical relation between velocity and volume, as functions of time in the present control, is illustrated in Figure 151. Here, in a manner corresponding to Figure 86, both curves start from a point $q$ as a matter of convenience, and not of necessity. The following equation may be written:

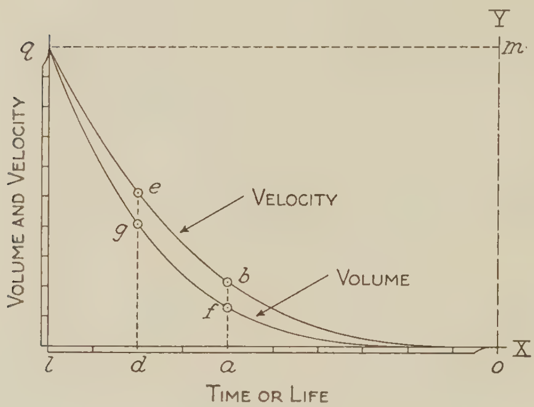



Fig. 151

$$\frac{\text{Area } Oab}{\text{Area } Ode} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

The areas subtended by portions of the velocity-time curve are directly proportional to corresponding ordinates of the volume-time curve.[9]

In Figure 152 we have the velocity-time curve once more. It is to be noted that the area, subtended by any portion of it that is greater than, equal to, or less than $Oq$, is one-fourth the area of the rectangle inclosing such portion. Thus the area subtended by $Ob$ is one-fourth the area $Oabc$, the area subtended by $Oe$ is one-fourth the area of $Odef$, and so on. To calculate any subtended area we would integrate the particular equation of the curve, say,

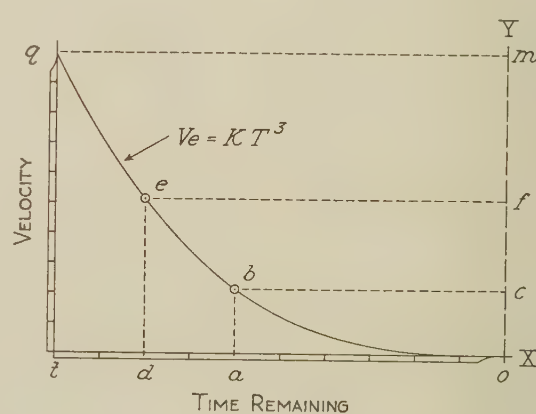



Fig. 152

$$Ve = K_1T^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (443)$$

and obtain thereby the expression

$$Vo = K_2T^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (444)$$

where $K_2$ is one-fourth of $K_1$.[10] We see that the algebraic and geometrical

[8] If the velocity-time curve is ideal, the area is obtained by integrating its equation; but if it is not ideal, the area is conveniently measured with a planimeter. Here, as usual, we assume that it is ideal for the purpose of developing principles.

[9] See footnote 7, § 105, page 259.

[10] As in Hydraulic and Volumetric controls, the constant of integration is here zero.

relations between velocity and volume agree. We may say that *the potential volume of a reservoir in this control, at any instant during its life, is equal to one-fourth the potential velocity at the instant, multiplied by the time in life remaining.*[11] Obviously the units for velocity, volume, and time must correspond.

From our experimental data given in section 152 we can easily compute velocities of flow during the test. This is done by subtracting each of the successive values of the volume from its immediately preceding value. Thus we are able to set up the following:[12]

TABLE OF DATA

| A<br>Potential<br>Time | B<br>Potential<br>Volume | C′<br>Average<br>Velocity |
|---|---|---|
| 6 | 36.00 | |
| | | 18.63 8/9 |
| 5 | 17.36 1/9 | |
| | | 10.25 |
| 4 | 7.11 1/9 | |
| | | 4.86 1/9 |
| 3 | 2.25 | |
| | | 1.80 5/9 |
| 2 | 0.44 4/9 | |
| | | 0.41 6/9 |
| 1 | 0.02 7/9 | |
| | | 0.02 7/9 |
| 0 | 0.00 | |

The values in Column C′ are, as designated, average velocities. Each represents the average rate of flow during an entire unit of time; for example, between the instants 5 and 4 the volume changed from 17.36 1/9 to 7.11 1/9; consequently the average rate between these instants was 10.25, but clearly the velocity was greater at the instant 5 than at the instant 4. It was not constant at 10.25 during this time, but had it been constant at this rate it would have provided for the same change in volume as was actually provided by the variable rate.

These average velocities satisfy a cubic equation. (Since the equation is not a potential one, we shall not bother with it, but we shall be content to

[11] We have, then, the following constants to be applied to the product of rate of production and time for the purpose of computing the volume quantitatively:

| | |
|---|---|
| Hydraulic Control | 1 |
| Volumetric Control | ½ |
| Capillary Control | ¼ |

(See §§ 63 and 105.)

[12] In the earlier table the repeating decimals were dropped after the second place. We now need complete numbers; therefore the fractions are included.

show that it is cubic without the necessity of determining it. This we leave for the next section.) The subtraction of successive values of volume in this way is approximate differentiation of volume with respect to time.[13]

Now we have the specific equation between volume and time in this experiment. It is, as we recall,

$$Vo = \frac{1}{36} T^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (445)$$

By differentiation this becomes

$$Ve = \frac{1}{9} T^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (446)$$

With this equation we can calculate the instantaneous values of the rate of flow for any instant whatever. Thus we can set up the following:

TABLE OF DATA

| A<br>Potential<br>Time | C<br>Potential<br>Velócity |
|---|---|
| 6 | 24.00 |
| 5 | 13.88$\frac{8}{9}$ |
| 4 | 7.11$\frac{1}{9}$ |
| 3 | 3.00 |
| 2 | 0.88$\frac{8}{9}$ |
| 1 | 0.11$\frac{1}{9}$ |
| 0 | 0.00 |

The values in Column C represent the exact rate of flow precisely at the recorded instants. Of course the velocity is different at every instant between 6 and 0. Of an infinite number of different values during this period of time but seven are tabulated. It would indeed be difficult, if not impossible, to measure these instantaneous values of velocities in the course of our experiment. The best we might be able to do is to reckon time in smaller units, and measure the volume produced in these shorter periods. Thus if we should divide the units above into tenths, we might measure the volume produced in the interval between 6.0 and 5.9, or perhaps even better, between 6.05 and 5.95, to obtain a value more closely approximating the exact value at 6.

**156.** *Acceleration-time relations.*—Changes in the value of the potential velocity during the process of production denote potential acceleration. The mode of change in velocity, that is, the path described by the acceleration-time curve, depends solely upon the static effects of Jamin action, in so far as it depends at all upon this action. The numerical values of the acceleration at each individual instant during the process of production depend upon both static and kinetic effects of this action, if the reservoir is in Capillary Control.

[13] Whenever we construct a curve with ordinates representing the changes in the value of a function, we have an approximate differential curve of the function.

When the velocity, or rate of flow from a reservoir, at any instant may be defined as a function of time, the first derivative of the expression, with respect to time, is a statement of the relation between acceleration and time. Thus from

$$Vc = KT^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (447)$$

we obtain by differentiation with respect to $T$ the relation

$$Ac = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (448)$$

*The potential acceleration of fluid issuing from a reservoir in this control, under given conditions of production, varies as the square of time remaining.* The curve for this equation is a true parabola. Whereas acceleration is zero in Hydraulic Control, and constant at 100 per cent in Volumetric Control, it now declines parabolically from 100 per cent to zero.

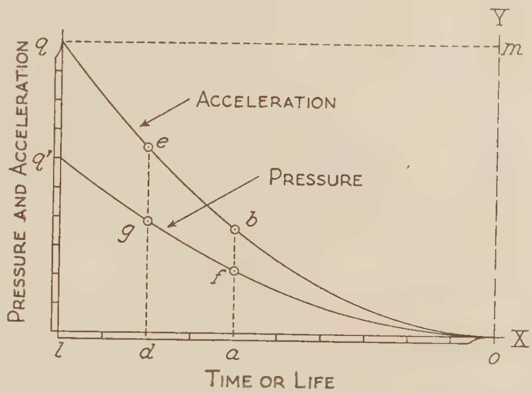



Fig. 153

Pressure and acceleration vary identically with respect to time remaining. Their equations and curves, when these pertain to one and the same reservoir in this control, are alike, except that ordinarily we may expect them to possess different values of their constants $K$. The situation is shown in Figure 153. In virtue of the relation between the curves we may write

$$\frac{\text{Ordinate } ab}{\text{Ordinate } de} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

The values of acceleration are directly proportional to corresponding values of pressure, and vice versa. Again we may write

$$\frac{\text{Area } Oab}{\text{Area } Ode} = \frac{\text{Area } Oaf}{\text{Area } Odg}$$

The areas subtended by portions of the acceleration-time curve are directly proportional to corresponding areas subtended by the pressure-time curve, and vice versa.

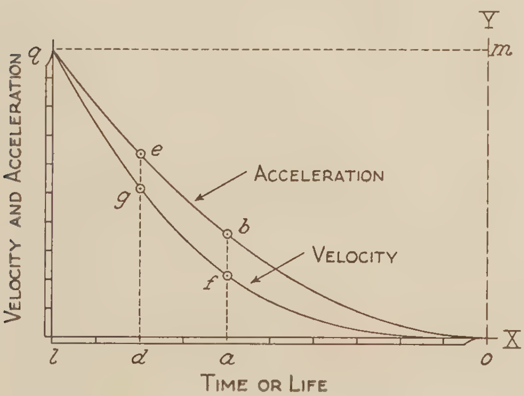



Fig. 154

Figure 154 shows the relation between the curves for acceleration and

velocity. The areas subtended by the former represent units of the latter. Now we may write

$$\frac{\text{Area } Oab}{\text{Area } Ode} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

The areas subtended by portions of the acceleration-time curve are directly proportional to corresponding ordinates of the velocity-time curve. Because of the relations between acceleration and pressure we may say that *areas subtended by portions of the pressure-time curve are directly proportional to corresponding ordinates of the velocity-time curve.* The situation with regard to the curves for pressure and velocity are now the reverse of those found in Volumetric Control.[14]

The relative curve between acceleration and time is shown in Figure 150. Its equation is

$$Ac = \frac{1}{100}T^2 \quad \ldots\ldots\ldots\ldots (449)$$

This curve coincides with the one between pressure and time.

In section 155 we learned that the potential volume of a reservoir in this control, at any instant during its life, is equal to one-fourth the potential velocity at the instant, multiplied by the time in life remaining. To express this mathematically we say that

$$Vo = \frac{1}{4} VeT \quad \ldots\ldots\ldots\ldots (450)$$

Now from the fact that the acceleration-time curve is a parabola, and that the area which it subtends represents velocity, we may say that the potential velocity of a reservoir in this control, at any instant during its life, is equal to one-third the potential acceleration at the instant, multiplied by the time in life remaining. And to express this mathematically in turn we say that

$$Ve = \frac{1}{3} AcT \quad \ldots\ldots\ldots\ldots (451)$$

By substituting this value of velocity into Equation 450 we have

$$Vo = \frac{1}{12}AcT^2 \quad \ldots\ldots\ldots\ldots (452)$$

*The potential volume of a reservoir in this control, at any instant during its life, is equal to one-twelfth the potential acceleration at the instant, multiplied by the square of time in life remaining.*[15]

[14] See footnote 7, § 105, page 259. In Volumetric Control the areas subtended by portions of the velocity-time curve are directly proportional to corresponding ordinates of the pressure-time curve.

[15] We recall that acceleration is the change in the rate of production. For "acceleration" in the above expression we may substitute "pressure," drop the fraction, and change the equation to a variation; thus, $Vo$ varies as $PT^2$. That is to say, the volume to be produced from a reservoir in this control varies as the product of potential pressure and the square of time remaining.

To continue with our calculations concerning the experimental data given in section 152 we can easily compute accelerations during the test. This is done by subtracting each of the successive values of velocity from its immediately preceding value. Thus we have the following:

TABLE OF DATA

| A Potential Time | C′ Average Velocity | D′ Average Acceleration |
|---|---|---|
| 6 | | |
| | $18.63\frac{8}{9}$ | |
| 5 | | $8.38\frac{8}{9}$ |
| | 10.25 | |
| 4 | | $5.38\frac{8}{9}$ |
| | $4.86\frac{1}{9}$ | |
| 3 | | $3.05\frac{5}{9}$ |
| | $1.80\frac{5}{9}$ | |
| 2 | | $1.38\frac{8}{9}$ |
| | $0.41\frac{6}{9}$ | |
| 1 | | $0.38\frac{8}{9}$ |
| | $0.02\frac{7}{9}$ | |
| 0 | | |

The values in Column D′ are, as designated, average accelerations. Each represents the average change in the rate of flow during an entire unit of time; for example, when the average velocity changed from 10.25 to $4.86\frac{1}{9}$, clearly such a change amounted to $5.38\frac{8}{9}$. Acceleration was greater at the time of the former reading than at the time of the latter, but had it been constant at this calculated amount it would have provided for the same change in velocity as was actually provided for by the variable change in the rate.

These average accelerations satisfy the equation of a true parabola. The subtraction of successive values of velocity in this way is approximate differentiation of velocity with respect to time.

Inasmuch as the specific equation between velocity and time in this experiment is

$$Ve = \frac{1}{9}T^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (453)$$

we can obtain the equation between acceleration and time by differentiating it. Thus

$$Ac = \frac{1}{3}T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (454)$$

In accordance with this equation we may set up the following:

TABLE OF DATA

| A<br>Potential Time | D<br>Potential Acceleration |
|---|---|
| 6 | 12.00 |
| 5 | 8.33⅓ |
| 4 | 5.33⅓ |
| 3 | 3.00 |
| 2 | 1.33⅓ |
| 1 | 0.33⅓ |
| 0 | 0.00 |

The values in Column D represent the exact change in the rate of flow precisely at the recorded instants.

In order to show that the subtraction of succeeding values of a function is approximate differentiation, let us perform two mathematical operations. First we shall continue subtracting the experimental data, beginning with Column D′. Thus we have the following:

TABLE OF DATA

| D′<br>Average Acceleration | E′<br>Change in D′ | F′<br>Change in E′ |
|---|---|---|
| 8.38 8/9 | | |
| | 3.00 | |
| 5.38 8/9 | | 0.66⅔ |
| | 2.33⅓ | |
| 3.05 5/9 | | 0.66⅔ |
| | 1.66⅔ | |
| 1.38 8/9 | | 0.66⅔ |
| | 1.00 | |
| 0.38 8/9 | | |

The values in Column F′ are invariable. If the values in Column D′ satisfy the equation of a parabola, those in E′ and F′ must satisfy the equations of a straight inclined line and a straight horizontal line, respectively. It is easily seen that they do so. But can we be assured that these successive subtractions, carried through Columns C′, D′, E′, and F′, have a definite relation to the performance of this particular reservoir? In other words, do the values in these columns refer back properly to Column B (p. 445), where we started with exact values of the potential volume? To determine this we merely differentiate Equation 454 twice. Thus by the first differentiation we have

$$\frac{d(Ac)}{dT} = \frac{2}{3}T \qquad (455)$$

and by the second,

$$\frac{d^2(Ac)}{dT^2} = \frac{2}{3} \qquad (456)$$

This, our second mathematical operation, leads us to the same result as the first one. The values in Columns C′, D′, E′, and F′ refer back properly to Column B. We can say that $T$ possesses exponents that successively differ by unity in these columns, beginning with four and ending with zero.[16]

**157.** *Energy-time relations.*—In our second experiment above, fluid is produced from that portion of the apparatus which we have called the potential system because of energy previously stored within the system. This energy, after having been stored, is possessed by the gas in the bubbles or chambers of the tube. The gas is in a compressed state, and it is capable of performing work upon expansion. As we know, this work can be performed only in case a constant back pressure, less in amount than the pressure within the system, is provided at an open orifice. The work which the system performs is done against this constant back pressure. The result of the work performed is the production of fluid at the orifice.

*Our potential system constitutes a reservoir of the closed type. It is closed by the chaplet of globules and bubbles beyond* R, *the radius of action of* P. As in all reservoirs of the closed type, compressed gas is the source of the energy. Furthermore, reservoirs of the closed type cannot produce liquid alone, for gas must be present in order that any fluid can be produced,[17] and if liquid is produced, gas must accompany it in virtue of the fact that it is soluble in the liquid.

We are then able to summarize the situation with regard to reservoirs in Capillary Control in the following terms: ($a$) they are of the closed type; ($b$) their lateral dimensions are defined by the radius $R$; ($c$) their energy is due solely to gas in the compressed state; ($d$) they can produce either gas alone or liquid and gas in combination, but they cannot produce liquid alone. These are the facts as we find them in our artificial systems composed of Jamin capillary tubes, and they must be acknowledged as such in any of our natural systems that are found to perform in the same manner as the tubes. *These natural systems must be conceded to be bundles of Jamin capillary tubes radiating in all directions about any well that might be drilled into them.*

Now if the system in this control possesses a certain amount of potential energy $E$, how may this amount vary with time in the process of production?

[16] As a matter of fact we lose track of the zero point in time by approximate differentiation. When we finally arrive at Equation 456, the zero point no longer serves as a vertex of the curve, nor as a point of intersection of the curve with the horizontal axis. In other words, the origin of co-ordinates does not define the position of the curve, and, irrespective of the position of this origin, the straight horizontal line is at the correct distance above the horizontal axis.

[17] We already know that a reservoir of the closed type in Volumetric Control cannot produce liquid alone. A reservoir of the open type in that control produces liquid alone in virtue of the fact that air enters from the top and replaces the liquid produced.

As we have done before, we may multiply together the equations for pressure and volume. Thus

$$P = K_1 T^2 \qquad (457)$$

and

$$Vo = K_2 T^4 \qquad (458)$$

by multiplication give us

$$E = K_1 K_2 T^6 \qquad (459)$$

This expression can be reduced to

$$E = KT^6 \qquad (460)$$

*The potential energy possessed by a reservoir in this control, under given conditions of production, varies as the sixth power of time remaining.* The curve defined by this relation is a parabola of the sixth power. While the value of the constant $K$ appears to be equal to the product of the $K$'s in the original equations, as a matter of fact we shall find in the next section that to compute the amount of potential energy in a reservoir at any instant during life, the sixth power of time remaining must be multiplied by two-thirds the product of the $K$'s in the pressure and volume equations.

The statement was made in section 107 that in Volumetric Control the potential energy possessed by a unit volume of fluid varies as the square of time remaining, and that the number of unit volumes[18] within the reservoir of that control simultaneously varies as the square of time remaining, with the consequence that the amount of potential energy within the reservoir varies as the fourth power of time remaining. This situation was considered further with reference to Bernoulli's Theorem. Now for our system in Capillary Control the situation is a parallel one. The potential energy possessed by a unit volume of fluid varies as the square of time remaining, and the number of unit volumes within the reservoir of this control simultaneously varies as the fourth power of time remaining; consequently the amount of energy within the system varies as the sixth power of time remaining. The fact that the number of unit volumes within the reservoir system varies in the manner stated is evident from the potential equation between volume and time. It will be observed that in both statements the first variations agree, and it is to be particularly noted that in the present case this variation agrees with the fact that we have recognized the individual bubbles and chambers of the Jamin tubes to be in Volumetric Control. Under the present circumstance we are concerned with Bernoulli's Theorem in the same manner as before. We take it again in the form:

$$E = W\left(z + \frac{p}{w}\right) \qquad (461)$$

[18] "Unit volumes" here pertain to portions of the potential volume of the reservoir, and not to portions of the volume which happens to exist within the reservoir.

The symbols have the meanings attached to them in section 43. Now so long as we confine our attention to $W =$ one unit of volume of fluid,

$$E = \left( z + \frac{p}{w} \right) = K_1'T^2 \ldots\ldots\ldots\ldots\ldots (462)$$

but where we admit that

$$W = Vo = K_2'T^4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (463)$$

then from Equation 461

$$E = K_1'K_2'T^6 \ldots\ldots\ldots\ldots\ldots\ldots (464)$$

The primes are once more introduced for the purpose of avoiding a conflict with the discussion immediately following Equations 459 and 460.

The relative curve between energy and time is shown in Figure 155. The relative constant, in accordance with the rule,

$$K = 10^{2-2n}$$

is equal to one, divided by ten billion, inasmuch as $n$ is now equal to six. Numbers which contain so many ciphers are more conveniently expressed as powers of ten; therefore we should prefer to write the relative equation in the following manner:[19]

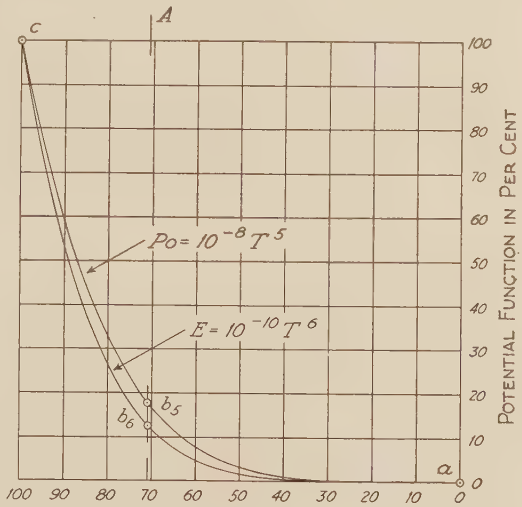



Fig. 155

$$E = 10^{-10}T^6 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (465)$$

The specific equation for energy and time in connection with our experiment will be considered in the next section.

**158.** *Power-time relations.*—When the energy within a reservoir at any instant may be defined as a function of time, the first derivative of the

[19] The exponential method of expressing large and small numbers is in common use among physicists. It is a simple matter to translate such expressions into the more familiar form by writing down 1 and following it with as many ciphers as the number in the exponent. In the present case

$$10^{-10} = \frac{1}{10^{10}}$$

and for the denominator we write 1 followed by ten ciphers.

expression, with respect to time, is a statement of the relation between power and time. Or, power is the product of pressure and velocity.

To repeat our derivation of energy-time relations we accept the two following equations:

$$P = K_1T^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (466)$$

and

$$Vo = K_2T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (467)$$

as proper, and multiply them to obtain

$$E = K_1K_2T^6 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (468)$$

as in Equation 459. This by differentiation becomes

$$Po = 6K_1K_2T^5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (469)$$

But if power is the product of pressure and velocity, we might differentiate Equation 467 for

$$Ve = 4K_2T^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (470)$$

and multiply this by Equation 466. Thus

$$Po = 4K_1K_2T^5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (471)$$

Obviously Equations 469 and 471 disagree. Before we decide between these let us analyze power-time relations in general.

The two equations reduce to

$$Po = KT^5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (472)$$

where the value of $K$ is as yet undecided. *Potential power, or the rate of displacement of potential energy, varies as the fifth power of time remaining.* The curve defined by this relation is a parabola of the fifth power.

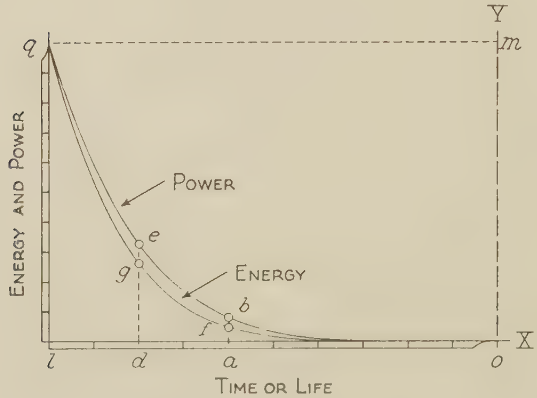



Fig. 156

Corresponding to Figure 151 for volume and velocity we may construct Figure 156 for energy and power. The following relation holds in this figure:

$$\frac{\text{Area } Oab}{\text{Area } Ode} = \frac{\text{Ordinate } af}{\text{Ordinate } dg}$$

The areas subtended by portions of the power-time curve are directly proportional to corresponding ordinates of the energy-time curve.[20]

[20] See footnote 7, § 105, page 259.

In Figure 157 we have the power-time curve once more. It is to be noted that the area subtended by any portion of it that is greater than, equal to, or less than $Oq$ is one-sixth the area of the rectangle inclosing such portion. Thus the area subtended by $Ob$ is one-sixth the area $Oabc$, the area subtended by $Oe$ is one-sixth the area $Odef$, and so on. To calculate any subtended area we would integrate the particular equation of the curve, say,

$$Po = K_1T^5 \ldots (473)$$

and obtain thereby the expression

$$E = K_2T^6 \ldots (474)$$

where $K_2$ is one-sixth of $K_1$. we may say that *the potential energy of a reservoir in this control, at any instant during its life, is equal to one-sixth the potential power at the instant, multiplied by the time in life remaining.* As in the earlier case with velocity and volume, so now the units for power, energy, and time must obviously correspond.

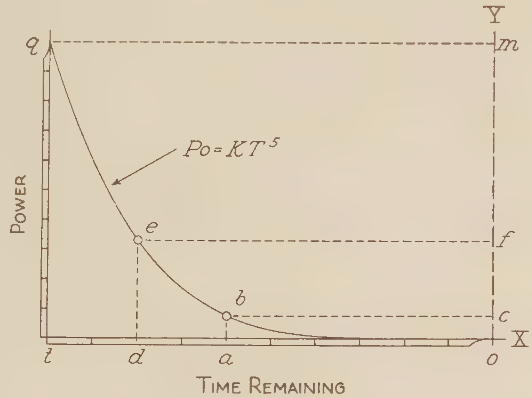



FIG. 157

Let us now refer to our experiment. We have the following data for the instant at which production starts:

$$P = 12.00$$

$$T = 6$$

$$Vo = 36.00$$

and

$$Ve = 24.00$$

$P$ appears in centimeters of mercury, and the units for the other functions are unspecified. We have also the following specific equations for the system:

$$P = \frac{1}{3}T^2$$

$$Vo = \frac{1}{36}T^4$$

and

$$Ve = \frac{1}{9}T^3$$

According to the values of $P$ and $Ve$ at the instant it is clear that

$$Po = 288.00 \text{ units}$$

The total potential energy possessed by the system at the instant is, according to the above rule,

$$E = \frac{1}{6} \times 288.00 \times 6 = 288.00 \text{ units}^{21}$$

On the other hand, if we multiply the values of $P$ and $Vo$ at the instant, we obtain $E = 432.00$ units, just one and one-half times the amount. We already know from our experience in Volumetric Control that to obtain the proper quantitative relation between energy and the product of pressure and volume we should first obtain the quantitative relation between power and the product of pressure and velocity. Upon integration we have the proper expression.[22] In the present instance the amount 288.00 units is proper. The product $K_1K_2$ in Equation 468 must be multiplied by 2/3 before it is converted into Equation 469. *The potential energy possessed by a reservoir in Capillary Control at any instant during its life is equal to 2/3 the product of potential pressure and potential volume.* This is substantiated by the fact that the area subtended by the curve between volume and pressure is 2/3 of the area of the rectangle inclosing it.

We will recall that in Hydraulic Control, sections 65 and 66, we found

$$E = K_1K_2PVo$$

and that in Volumetric Control, sections 107 and 108, we found

$$E = \tfrac{1}{2} K_1K_2PVo$$

Now we find that for Capillary Control

$$E = \tfrac{2}{3} K_1K_2PVo$$

If we imagine three systems, one in each of the controls, wherein the values of $K_1$, $K_2$, $P$, and $Vo$ are the same throughout, their respective potential energies are in the ratio

$$1 : \tfrac{1}{2} : \tfrac{2}{3}$$

The area subtended by a curve between potential pressure and potential volume represents units of potential energy. Thus for Hydraulic Control, in Figure 46 (p. 129), the area representing potential energy occupies the en-

[21] These units, as usual, can be converted into foot-pounds. (See footnote 9, § 15, p. 23.)

[22] In the science of the mechanics of solid particles, and also in the science of thermodynamics, we find invariably that energy is defined first, and power thereafter. I have preferred to follow these precedents, although it is evident that for the mechanics of fluids power should be defined first and energy thereafter. Perhaps we have a better conception of energy, and a better conception of the relation between energy and the controls, by being led into an unfamiliar situation and forced to wend our way out of it.

tire rectangle defined by any particular value of volume. Again, for Volumetric Control, in Figure 82 (*b*) (p. 251), the area representing potential energy occupies one-half the entire rectangle defined by 100 per cent values of pressure and volume, whatever amounts these values represent. Once more, for Capillary Control, in Figure 147 (*b*) (p. 427), the area representing potential energy occupies two-thirds the entire rectangle defined by 100 per cent values of pressure and volume, whatever amounts these values represent.[23] These areas agree with the foregoing ratio.

The striking difference between reservoir systems in Volumetric and Capillary controls is easily observed in Figure 138 (p. 403). If we imagine two systems, one in each of these controls, wherein 100 per cent values of pressure and volume are the same for both, it may appear to us in connection with this figure, where the potential energy in one is shown to be

$$\tfrac{2}{3} - \tfrac{1}{2} = \tfrac{1}{6}$$

in excess of the potential energy in the other,[24] that there is some principle involved which contravenes the second law of thermodynamics. But there is no such principle involved in the situation. *The two systems are fundamentally different; in actual practice they cannot be compared on the basis of equal potential energies for equal potential pressures and potential volumes.*[25] The situation is the same as that between two systems, one in Hydraulic Control and the other in Volumetric Control. For these we have unequal potential energies for equal potential pressures and potential volumes. The potential energy in one is seen to be

$$1 - \tfrac{1}{2} = \tfrac{1}{2}$$

in excess of the potential energy in the other.

[23] For this control we must specify the area subtended by the pressure-volume curve, and not that subtended by the volume-pressure curve. By revolving the plat in its own plane we observe the subtended areas change from 2/3 to 1/3.

[24] The fraction is 1/6 on the basis of the entire rectangle inclosing the curves. On the basis of the area subtended by the curve for Volumetric Control the fraction is 1/3. Then in comparing two reservoirs in the finite controls we can say that the one in Capillary Control possesses one-third more energy than the one in Volumetric Control. Otherwise expressed, the ratio between the energies possessed by the two is

$$1\tfrac{1}{3} : 1$$

[25] Only in an analytical investigation such as ours can they be compared. We might argue that in the case of the system in Hydraulic Control we continually add energy while energy is being displaced by production. This appears to be a reasonable argument. But then we might also argue that in the beginning—at whatever time the system might have been created as such—more energy was introduced into the system in Capillary Control than into that in Volumetric Control. The greater amount of energy is necessary in view of the fact that the resistant forces offered by the globules at their menisci must be overcome in the performance of work which results in production.

To return to our experiment we have yet to determine the specific equations for power and energy. By multiplying the equations

$$P = \frac{1}{3} T^2 \quad \ldots\ldots (475)$$

and

$$Ve = \frac{1}{9} T^3 \quad \ldots\ldots (476)$$

we have

$$Po = \frac{1}{27} T^5 \quad \ldots\ldots (477)$$

By integration this becomes

$$E = \frac{1}{162} T^6 \quad \ldots\ldots (478)$$

The values of power and energy at the recorded instants in the experiment can be calculated by means of these equations.

The relative curve between power and time is shown in Figure 155. Its equation is

$$Po = 10^{-8} T^5 \quad \ldots\ldots (479)$$

The relative constant is now one, divided by one hundred million. The points for the two curves of this figure are as follows:

| $T$ in Per Cent | $E$ in Per Cent | $T$ in Per Cent | $Po$ in Per Cent |
|---|---|---|---|
| 0 | 0.00 | 0 | 0.00 |
| 10 | 0.00+ | 10 | 0.00+ |
| 20 | 0.01 | 20 | 0.03 |
| 30 | 0.07 | 30 | 0.24 |
| 40 | 0.41 | 40 | 1.02 |
| 50 | 1.56 | 50 | 3.12 |
| 60 | 4.67 | 60 | 7.78 |
| 70 | 11.76 | 70 | 16.81 |
| 80 | 26.21 | 80 | 32.77 |
| 90 | 53.14 | 90 | 59.05 |
| 100 | 100.00 | 100 | 100.00 |

To facilitate accurate construction of the curves, intermediate points, as 45, 55, 65, and so on, should be added to the tables.

These curves are subject to identically the same manipulation as their correspondents for Volumetric Control in Figure 89 (p. 265).

CHAPTER XXVII

# Ideal Performance and Its Primary Functions *(Concluded)*

"Aristotle says that an iron ball of one hundred pounds falling from a height of one hundred cubits reaches the ground before a one-pound ball has fallen a single cubit. I say that they arrive at the same time. You find, on making the experiment, that the larger outstrips the smaller by two finger-breadths; now you would not hide behind these two fingers the ninety-nine cubits of Aristotle, nor would you mention my small error and at the same time pass over in silence a very large one."—GALILEO

**159.** *Summary of the fundamental relations.*—We have analyzed the six fundamental primary function curves for Capillary Control. At whatever instant we begin our reckoning time, whether it be the initial instant of production or any subsequent convenient instant, life, or time remaining, is mathematically finite. And if we begin our reckoning at a subsequent convenient instant, the paths traveled by the functions of performance in their decline are right-hand portions of their complete paths, as these are customarily traced on co-ordinate plats. Furthermore, reckoned from any instant whatever in the life of the reservoir, all functions make a complete sweep through their respective relative curves.

Our curves belong to the parabolic family, of the general equation $y = kx^n$, wherein the exponent $n$ is a positive integral number, greater than one and less than seven, and the constant $k$ is a positive number, either integral or fractional. We place the origin of the curves at the right, and reckon time in terms of time remaining.

The following table gives the six relations as we have found them in the preceding sections:[1]

CAPILLARY CONTROL

FUNDAMENTAL PRIMARY FUNCTION RELATIONS

| | |
|---|---|
| Pressure-Time | $P = KT^2$ |
| Volume-Time | $Vo = KT^4$ |
| Velocity-Time | $Ve = KT^3$ |
| Acceleration-Time | $Ac = KT^2$ |
| Energy-Time | $E = KT^6$ |
| Power-Time | $Po = KT^5$ |

[1] A table of these relations in the three controls is given in Appendix A.

Although in our discussion no $X$ axes other than the potential ones, to which these equations refer, have been mentioned, we must admit that, as in Hydraulic and Volumetric controls, in every case we also have the absolute and atmospheric axes. As we learned in the beginning, the potential axes do not coincide with the absolute axes unless the reservoir produces into a perfect vacuum, nor with the atmospheric axes unless the reservoir produces into the atmosphere. In our experiment with the Jamin tube, production was permitted to take place into the atmosphere, and here, then, the potential axes coincide with the atmospheric axes.

When the six primary functions of performance are arranged according to the exponents of time with which they are associated, they appear as follows:

| Function | Exponent |
|---|---|
| Pressure and Acceleration | 2 |
| Velocity | 3 |
| Volume | 4 |
| Power | 5 |
| Energy | 6 |

Already we have observed that the areas subtended by the velocity and power curves represent units of volume and energy, respectively. Now we may say that, in virtue of the fact that the exponents differ successively by one in the list, the areas subtended by the curve for any of the functions represent units of the function succeeding in the list. Furthermore, from what we have observed in the preceding chapter, we may say that, in virtue of the same fact, the curve representing the change in the value of any of the functions can be used as the curve of the function preceding in the list.

These fundamental curves can be plotted as straight lines on a Cartesian plat. To do this we would plot the root of the function indicated by the exponent of $T$ as ordinates, and time as abscissas, or we would plot the function as ordinates, and time raised to the power indicated by the exponent of $T$ as abscissas. Again, we might plot as ordinates the velocity divided by pressure, the velocity divided by acceleration, the volume divided by velocity, the power divided by volume, and the energy divided by power, with time as abscissas. If we use either of these methods for the logarithmic plat, the lines have a slope of 1 to 1.

Such principles as the foregoing ones are, of course, purely geometrical. They apply alike to Volumetric and Capillary controls, though the arrangement of the functions in the list differs for the two classes of reservoirs.[2]

A further method of particular value in Capillary Control is afforded by differential calculus. It is based upon volume-pressure relations, and it

[2] See §§ 109 and 125.

therefore possesses advantages due to these relations. From section 151 we have

$$Vo = KP^2$$

and this by differentiation with respect to $P$ becomes

$$\frac{dVo}{dP} = KP$$

(As usual, we pay no attention to changes in the $K$'s.) The quantities on the left have the following significance: $dVo =$ the volume produced from the reservoir during any interval of time between, say, the instants $t_1$ and $t_2$; $dP =$ the difference between the pressures at $t_1$ and $t_2$. It is therefore evident that the entire fraction on the left represents the volume produced per pound decline in pressure, as of the interval of time duly specified. Such use of values obtained by observations reduces the operation to approximate differentiation, as explained in section 156.

The quantity on the right is recorded pressure. Its value for the interval may be taken as the arithmetical mean of the pressures at $t_1$ and $t_2$.

For a series of intervals, for which we have corresponding sets of data, the values in accordance with the differential equation may be plotted, the quantities on the left as ordinates and those on the right as abscissas. The result is an inclined straight line. While the zero point is lost by approximate differentiation on the left, it is found again by the arithmetical mean on the right; consequently the result is exact. Data for $Vo$ are invariably expressed in potential units, but those for pressure are most usually not. In so far as we are now concerned, pressure may be expressed in accordance with any of its three possible zeros. When expressed in potential units the inclined line passes through the origin of co-ordinates; otherwise it passes either to the right or to the left of the origin. The lateral shift required to make it pass through this origin indicates the correction to be made in changing to potential units. For example, if the data are in gauge pressures, although there is a registered constant back pressure against production, the amount of shifting indicates the value of this registered constant back pressure.

This is the method of "the calculus of observations."[3] It is remarkable for its accuracy. The data concerning performance may not be complete, lacking, say, the description of the constant back pressure against which production takes place, yet the method is none the less exact. In the present problem we have the combined advantages of volume-pressure relations, the straight line, and accuracy in spite of deficient data.

It will be advantageous to have all relative curves for Capillary Control on one plat; therefore these are grouped in Figure 158 (p. 462), corresponding to Figure 92 for Volumetric Control.

That a reservoir of this control in its performance may fulfill these relative curves, each one of which depends upon $T$ as a function of performance,

[3] For the method of the calculus of observations see the textbook of Whittaker and Robinson, as listed in Appendix D.

requires performance itself to be ideal, and this in turn requires the reservoir to be ideal. Now ideal reservoirs are unknown to us, as we have already admitted on many occasions; consequently we cannot expect these curves to

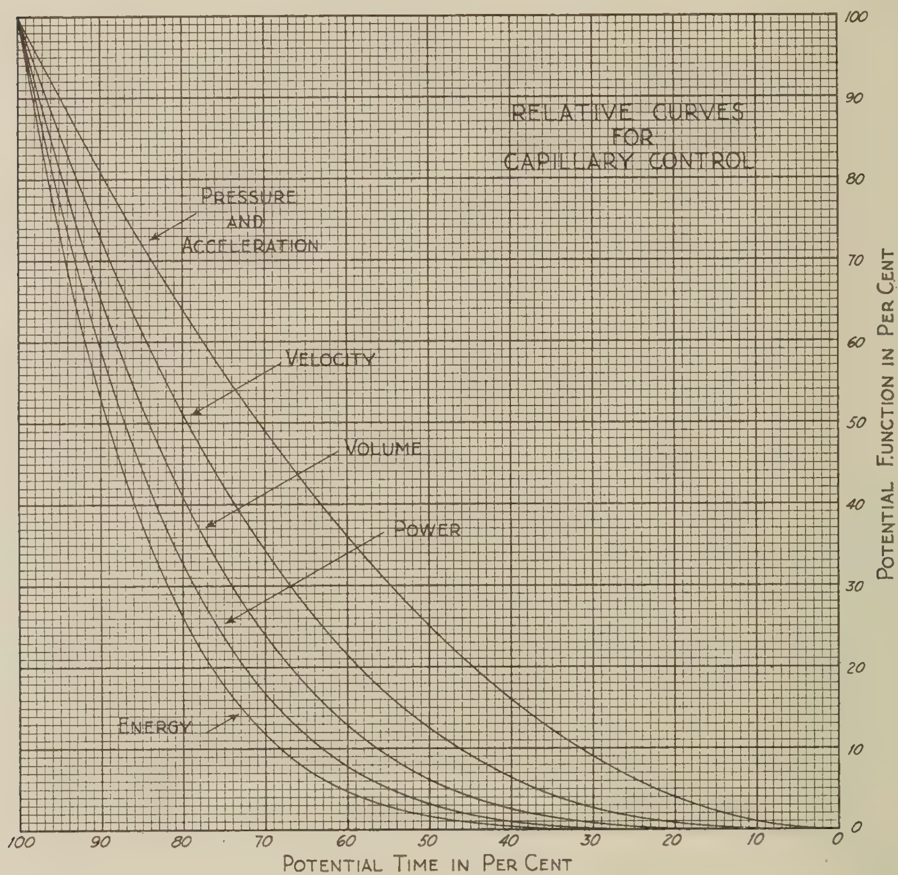



FIG. 158

be exactly fulfilled in practice. These curves do nevertheless define the laws of delivery from the type reservoirs, and the laws of delivery from all other reservoirs that approximate them in their physical state. These laws serve not only as a basis for reckoning performance with respect to time, but also as a basis of determining the derived primary function relations, all of which are independent of $T$ as a function of performance, and some of which are independent of time as a matter of definition, as explained in section 41. Furthermore, these laws serve as a basis of theoretic performance in Capillary Control. The laws for Volumetric Control are not applicable to reservoirs in the present control.

**160.** *Derived primary function relations.*—It is once more clear that $T$ may be eliminated between any two of the six fundamental equations, with

the result that relations which are not dependent upon $T$ in any manner are obtained. There are, as we know from our studies in the other controls, fifteen of these "derived primary function equations." The list for this control follows:

DERIVED PRIMARY FUNCTION RELATIONS

| | |
|---|---|
| Pressure-Volume | $P = KVo^{1/2}$ |
| Velocity-Pressure | $Ve = KP^{3/2}$ |
| Acceleration-Pressure | $Ac = KP$ |
| Pressure-Energy | $P = KE^{1/3}$ |
| Pressure-Power | $P = KPo^{2/5}$ |
| Velocity-Volume | $Ve = KVo^{3/4}$ |
| Acceleration-Volume | $Ac = KVo^{1/2}$ |
| Volume-Energy | $Vo = KE^{2/3}$ |
| Power-Volume | $Po = KVo^{5/4}$ |
| Acceleration-Velocity | $Ac = KVe^{2/3}$ |
| Velocity-Energy | $Ve = KE^{1/2}$ |
| Velocity-Power | $Ve = KPo^{3/5}$ |
| Acceleration-Energy | $Ac = KE^{1/3}$ |
| Acceleration-Power | $Ac = KPo^{2/5}$ |
| Power-Energy | $Po = KE^{5/6}$ |

These equations may be compared with their correspondents in sections 69 and 110.[4]

Any of the derived relations may be expressed in the order reversed to that given above. If this is done, the exponent on the right becomes inverted. The equation for any couplet can be written down by the rule presented in section 110; therefore we need only remember the exponents of $T$ in the six fundamental equations.

Relative curves can be constructed for all possible couplets. The value of the relative constant is found in the usual manner; namely, by the equation $K = 10^{2-2n}$, where $n$ is the value of the exponent on the right-hand side of the equation. The logarithmic plat of Figure 107 can be used in this control, if the new lines that are now called for are added.

As explained in section 125, the curves for the derived relations can be converted into straight lines on the Cartesian plat by plotting the required root or power of one or both functions. It is evident that one of the couplets already represents a straight-line relation: namely, that expressing the relation between acceleration and pressure. Thus if we plot the change in the rate of production as ordinates against pressure as abscissas, the curve is a straight line on the Cartesian plat.

[4] The three sets of equations are repeated in Appendix B. It is of interest to note that no two correspondents are identical. Had we included in our list of primary functions the acceleration of energy, or the change in power, we would have had among the derived relations in the finite controls the equation $Ae = KVo$, wherein $Ae$ signifies the acceleration of energy. The equation is correct in either of the two controls; therefore we might say that in the finite controls *the change in the rate of displacement of energy is always proportional to the volume yet to be produced from the reservoir.*

Next in importance to the relations between pressure and volume are those between velocity and pressure. We now have

$$Ve = KP^{3/2} \quad \text{.......................} \quad (480)$$

in place of

$$Ve = KP^{1/2} \quad \text{.......................} \quad (481)$$

as we found in Volumetric Control. The present equation is not equivalent to Torricelli's Theorem. If the effect of the globules and bubbles within the reservoir system is a cumulative one, as it has been said to be, then while Equation 481 holds true for the individual bubbles Equation 480 must hold for the system at large; that is, the integration of Equation 481 with respect to pressure must agree with Equation 480. This we observe to be the case.

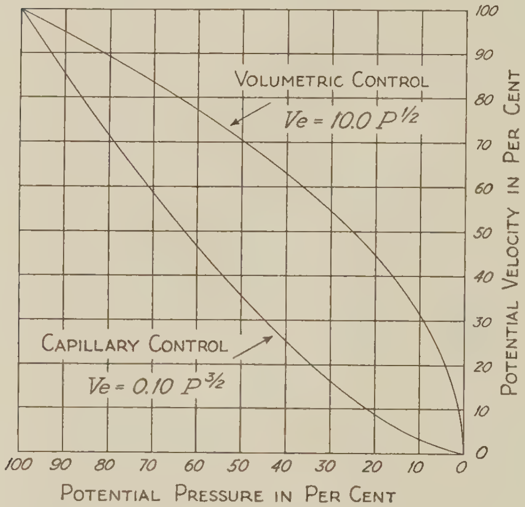



Fig. 159

The contrast between velocity-pressure relations in the two finite controls is readily seen in their relative curves, as these appear in Figure 159. At the point 100-100 one starts with a slope of 1 to 2, while the other starts with a slope of 3 to 2, in conformity with their respective exponents. The two approach 0-0 on opposite sides of the straight diagonal line from corner to corner. When the pressure has declined to 50 per cent of its value, velocity in the two systems possesses the values previously noted in section 155. These curves are represented by the lines $C_1$ and $C_2$ in Figure 136, the latter being, however, specific instead of relative curves.

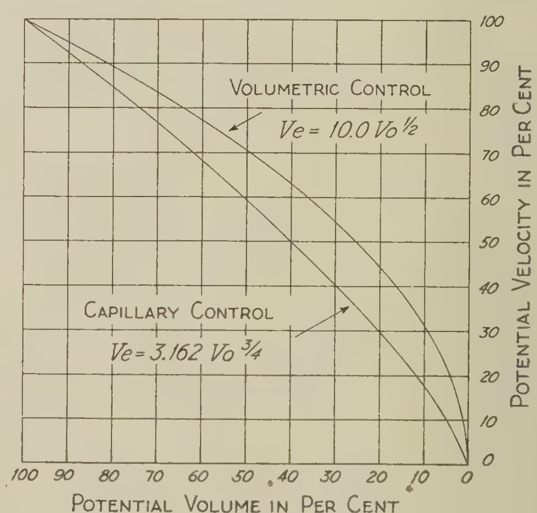



Fig. 160

Again, the contrast between velocity-volume relations is readily seen in Figure 160. The dif-

ference is not so evident as it is in the preceding figure, for both curves remain on the same side of the straight diagonal line.

**161.** *Forecasting by pressure and volume.*—In our earlier studies we found that three of the couplets among the derived primary function relations are absolutely independent of time. These are pressure and volume, pressure and energy, and volume and energy. The function $T$ is eliminated in these couplets, and the expressed functions do not involve time in their definitions. Such relations should, as stated in section 111, be given preference in forecasting the future performance of a reservoir, for, regardless of alterations in the conditions of production, they are accurately fulfilled. Where a reservoir produces either liquid alone or gas alone, predictions should be based upon pressure-volume relations. Where a reservoir produces both liquid and gas, predictions upon the liquid should be based upon pressure-volume relations, and predictions upon the gas, as well in this control as in the preceding one, should be based upon either pressure-energy or volume-energy relations. By means of the latter relations we compare the volume of the liquid with the volume of the gas.

While the so-called "graphic solution by Boyle's Law" is based upon pressure-volume relations in potential phase, we see in Figure 138 that in applying this method we cannot now rely upon the projection of a straight line as we could in Volumetric Control. A parabola, we admit, is not conveniently projected. We can easily avoid the necessity of doing this, however, by converting the parabola into a straight line. This we know to be done by plotting either the pressure against the square root of the volume or the square of the pressure against the volume, if the Cartesian plat is held in preference; otherwise the line is already straight if the values are plotted on the logarithmic plat. In any case the graphic solution is rendered serviceable in Capillary Control.

The graphic solution by Boyle's Law, in either finite control, is equivalent to the method that has been prescribed for the use of the relative curve. The former should perhaps be given preference when the data are numerous and at the same time subject to errors in observation.[5] It is clear, of course, that we cannot ignore the control even in such a graphical method. If we attempt to pass a straight line through points which properly lie on a parabola, we overestimate the volume to be produced from the reservoir under the given conditions of production. All straight lines that may be made to pass through points on the parabola in Figure 138 strike the horizontal axis somewhere between 0 and 20 to the right of this zero; thus in the position of 20 to the left of the zero we would necessarily place a number greater than 20, yet less than 40.

[5] It is easy to see that six or twelve accurate observations offer better data for computations than 365 poor ones.

The relation between pressure and volume in Capillary Control does not permit us to say that equal amounts of fluid are produced for equal amounts of decline in the pressure. The law frequently spoken of as that of "equal production per pound decline," while being perfectly appropriate in Volumetric Control, is not at all applicable in Capillary Control. Let us consider three successive readings on pressure and their relation to the corresponding volumes of fluid remaining within the reservoir.[6]

$$P_1^2 = KVo_1 \quad \text{(482)}$$

$$P_2^2 = KVo_2 \quad \text{(483)}$$

and

$$P_3^2 = KVo_3 \quad \text{(484)}$$

Now by subtraction we obtain

$$P_1^2 - P_2^2 = K(Vo_1 - Vo_2) \quad \text{(485)}$$

and

$$P_2^2 - P_3^2 = K(Vo_2 - Vo_3) \quad \text{(486)}$$

The first of these may be divided by the second:

$$\frac{P_1^2 - P_2^2}{P_2^2 - P_3^2} = \frac{Vo_1 - Vo_2}{Vo_2 - Vo_3} \quad \text{(487)}$$

Here the numerator and denominator on the left represent differences of squares of pressure, while those on the right represent differences in the volume remaining within the reservoir; that is, they represent the volumes produced in the interval of time between the readings on pressure. Now if the numerator and denominator on the left are equal, those on the right must likewise be equal. The equation under these circumstances indicates the following law: *Equal amounts of fluid are produced for equal amounts of decline in the square of the pressure.* As a law it possesses restrictions. It is correct only for reservoirs wherein the volume varies as the square of the pressure, as in Capillary Control,[7] and it is furthermore correct only as applied in accordance with paragraphs (*b*) and (*c*) in section 111. Where reservoir systems do not possess like dimensions, we cannot compare them on the basis of equality, but we can, however, compare them on the basis of proportionality.[8] The numerator and the denominator of both members in the equation between differences simply do not bear the relation of 1 to 1 in this case. We are permitted to say that in general, with one or more reser-

[6] The six equations following appear in the same order as their correspondents in Volumetric Control. (See § 111.)

[7] It is also correct for the V-shaped tank. (See § 135, Equation 334, p. 364.) The tank, however, has no other relations in common with Capillary Control.

[8] The dimensions of reservoirs in Capillary Control are discussed in § 174.

voirs in Capillary Control, *proportional amounts of fluid are produced for the same proportional amounts of decline in the square of the pressure.*

Equation 487 may be changed to read as follows:

$$\frac{(P_1 + P_2)(P_1 - P_2)}{(P_2 + P_3)(P_2 - P_3)} = \frac{Vo_1 - Vo_2}{Vo_2 - Vo_3} \quad \ldots\ldots\ldots\ldots (488)$$

*Equal or proportional amounts of fluid are produced for equal or proportional products of the sum and difference of pressure.* In so far as the difference between two values of the pressure is concerned, it is immaterial to which of the possible zero-points the measurements of pressure refer, for such values when subtracted, one from the other, give the same result whether pressure is measured from absolute zero, atmospheric zero, or potential zero. Clearly this is not the case, however, with the sum of the two values. Here the pressures must be expressed in potential units. Thus on comparing Equation 212 (p. 277) with Equation 488, we see that it is necessary to be more particular in regard to our units of pressure in Capillary Control than in Volumetric Control. The situation can be corroborated by comparing Figures 81 ($b$) and 146 ($b$). In the former, wherever $p_1 = p_2$, always $v_1 = v_2$, while in the latter, wherever $p_1 = p_2$, then $v_1$ and $v_2$ are only related in accordance with their respective distances from the $X$ axis.

We should obviously question the practical utility of these laws for reservoirs in Capillary Control.[9] They are here stated for the purpose of emphasizing the fact that the principle of equal production per pound decline is erroneous as applied to these reservoirs. In places of such laws as these, which involve the squares of values, or the products of their sums and differences, we should prefer to use the relative curves, for they obviate the necessity of computing with exponents.

The laws of equal and relative expectation, as given in section 112, hold without modification in the present control. Equations 214 and 215 must, of course, be written as follows:

$$P_1 = K_1 Vo_1^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (489)$$

and

$$P_2 = K_2 Vo_2^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (490)$$

These equations do not require change in the wording of the laws. The fact that the pairs of equations differ in the two finite controls is significant, however, with respect to these very laws. It shows us that *under no circumstances is it proper to compare the expectancy of a reservoir in Capillary Control with that of a reservoir in Volumetric Control.* Granted that two such reservoirs possess, say, equal values of potential pressure and potential volume at some

[9] The fact that they are correct can be easily verified in the case of the Zoar Storage Field pressure-volume curve, Fig. $I_2$, Appendix I.

instant in their respective lives, then it is impossible that they can have had previously, or that they can have subsequently, greater or smaller equal values of these functions, respectively,[10] for the differing exponents prohibit like variation in the values of the functions.

Laws of equal or relative expectation may be based either upon pressure-volume relations or upon velocity-time relations in this as well as in Volumetric Control. These laws are subject to the same restrictions as before. Such restrictions were discussed in section 112.

We can appreciate the freedom from restrictions when we base forecasts upon the relations between pressure and volume. The results of computations are accordingly the most accurate that can be obtained. It would likewise be most satisfactory to forecast the values of other functions by using pressure as a basis for computations. As in Volumetric Control, so in the present one; all functions can be expressed in terms of pressure. Corresponding to the list given in section 113 we have the following:

$$Vo = KP^2$$

$$Ve = KP^{3/2}$$

$$Ac = KP$$

$$E = KP^3$$

$$Po = KP^{5/2}$$

to which we may add, as before,

$$T = KP^{1/2}$$

A chart which shows the relative curves for these equations is given in Figure 161. This may be compared with Figure 93. I believe it is not necessary to enter into details concerning the features of these curves, in so far as they may be used as a basis of forecasting. As before, the curves bear the usual consistent relations between themselves. We now note that by integrating

$$Vo = KP^2$$

we obtain

$$E = KP^3$$

and that by integrating

$$Ve = KP^{3/2}$$

we obtain

$$Po = KP^{5/2}$$

[10] The statement pertains to finite values, and not to the final value zero.

The interpretation given the corresponding equations in section 113 applies with respect to these.

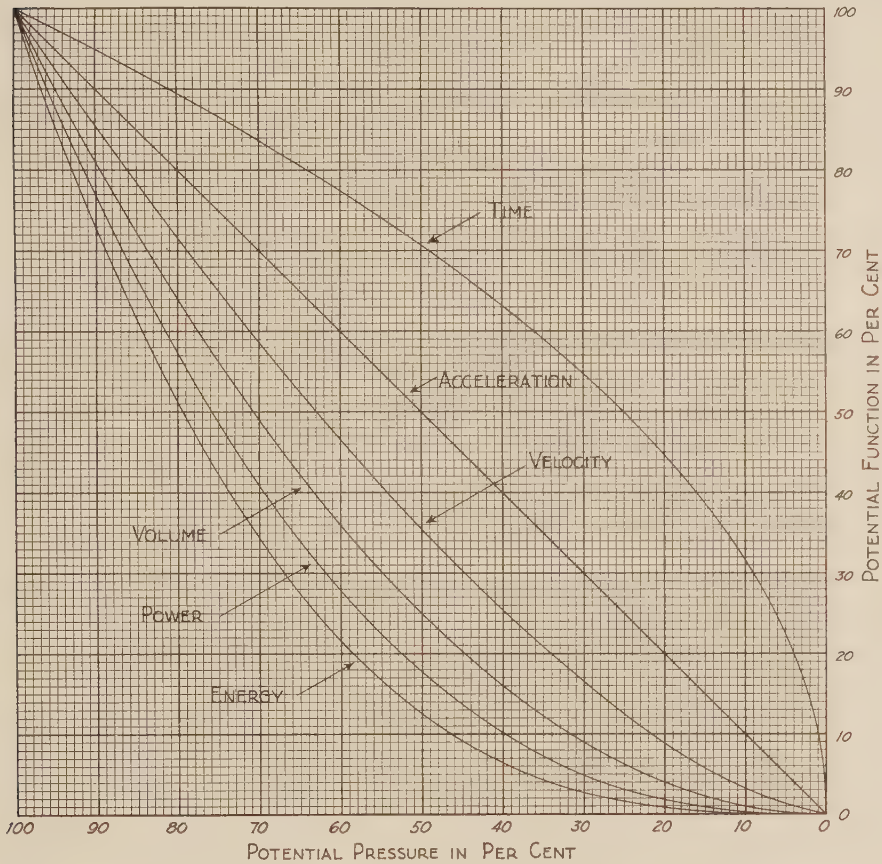



Fig. 161

**162.** *The combination reservoir.*—Our Jamin capillary tubes produce either gas alone or liquid and gas together, depending upon circumstances which were described in section 153. If both fluids are produced, we say that the system constitutes a combination reservoir. We are now to investigate such a system.

In section 114 we noted three possible cases in connection with a combination reservoir of the closed type in Volumetric Control. These cases referred to the amount of gas present in comparison with the amount of liquid. There can be, so we found, an insufficient amount, just a sufficient amount, or more than a sufficient amount of gas within the system to produce the liquid. But this is a physical concept of the reservoir system. We refer to the system as a container of fluid rather than as a producer of fluid. For precision we are

required to refer to the system as a potential reservoir, this having been defined in section 19. With respect to the potential reservoir there is always just sufficient gas to produce the liquid, the amount of liquid being determined by the area subtended by the velocity-time curve.[11] Now the situation with respect to a system composed of a Jamin capillary tube is the same, except for restrictions to be noted. We have accepted the fact that so long as *R,* the radius of action of *P,* does not exceed the length of the tube, the reservoir is one of the closed type, as explained in section 157. If we cling to a physical conception of this reservoir, we must say that there is always insufficient gas to produce the liquid within it; but if we maintain our mathematical conception of the reservoir, we must say again that there is always just sufficient gas to produce the liquid, the amount of liquid being determined by the area subtended by the velocity-time curve.[12] In the physical sense there cannot be just a sufficient amount of gas, or more than a sufficient amount of gas, to produce all liquid within the tube, since this would imply the destruction of all the globules and the production of their liquid, a circumstance which is prohibitive so long as the reservoir is to be one in Capillary Control.[13] In this respect the closed types of reservoirs in the two controls differ.[14]

As between reservoirs of the closed type in the two finite controls there is yet another difference to be noted. The soda-siphon bottle, as an example of a reservoir in Volumetric Control, possesses rigid glass walls. The space-volume of this system is constant. On the other hand, the Jamin capillary tube, although possessing rigid glass walls on a plane perpendicular to its axis, provides a variable dimension for the potential reservoir along the axis. This dimension is *R,* a length that is increased on filling the tube and decreased on producing from the tube.[15]

Whether the tube produces gas alone or liquid and gas in combination, *R* is a length precisely determined by the "sufficient amount of gas." In the case of the tube being a combination reservoir we can say that the proportional amount of gas being what it is, the radius *R* must be what it is. It is clear that if we are to investigate the gas-time relations for production from

[11] As we know, the velocity-time curve refers only to the rate of production of the liquid from a combination reservoir. The situation, as here described, is clearly a matter of definition, one that is advantageous on account of the precision it lends to our computations.

[12] The amount of liquid referred to is obviously the potential volume of liquid within the system.

[13] In the capillary tube of smooth bore some of the globules certainly are produced.

[14] If all the globules are to be destroyed, and their liquid produced, the system must first be converted into Volumetric Control, and in this control, as we have said, there can be more than sufficient gas to produce all such liquid.

[15] This circumstance is to be fully investigated in connection with the secondary functions of performance.

such a tube, we are limited to that portion of the container within the radius *R*, since the potential reservoir is confined to this distance, and since our primary function relations are confined to the potential reservoir.[16]

The amount of gas dissolved in the liquid, and therefore the amount of gas produced with the liquid from an ideal combination reservoir in this control, is dependent upon the static pressure of the reservoir in accordance with Henry's Law. Reservoirs in Capillary Control cannot differ from others in this respect. Here we should prefer as our type the Jamin tube with contractions and expansions which are distributed in a perfectly uniform manner throughout its length. The system is ideal; the "texture" is perfectly homogeneous, and the action of breaking and making on the part of the globules is repeated continually at uniform intervals of time. The tube contains liquid in excess of the amount required to fill the contractions.[17]

If the static pressure continuously diminishes during the process of production from a reservoir—a circumstance whose truth is unquestioned in regard to this control—the amount of gas per unit volume of liquid issuing from the system must also diminish. In fact the amount of gas per unit volume of liquid and the static pressure diminish continually in the same proportion. Figure 76, although originally designed for Volumetric Control, serves in this control. No modifications need be made, either in the drawing or in its description to be found in section 115. By taking *S* per cent of the gas, corresponding to the static pressure, as 100 per cent, we have the amounts of *A* per cent, *P* per cent, *C* per cent, *RS* per cent, and *RC* per cent, as before. *P* per cent, the potential gas, varies in accordance with the equation

$$P\% = (T\%)^2$$

where *T* per cent represents time remaining in life.

**163.** *Gas-time relations.*—According to sections 74 and 116 the gas velocity, or the rate at which gas is produced from a combination reservoir, is equal to the proportional production of gas multiplied by the velocity for the liquid. We found the latter quantities constant in their values throughout the process of production in Hydraulic Control, and variable in Volumetric Control. They are also variable in Capillary Control, but variable in accordance with laws that differ from those in Volumetric Control.

If we were to discuss gas-time relations in the present control, we would find it logical to repeat the argument given in section 116. I therefore pro-

[16] Nevertheless our investigation of conditions beyond *R* is in no way restricted by this fact. Our secondary functions of performance pertain to distances less than, and greater than *R*.

[17] See §§ 150 and 153. I believe the system with contractions and expansions more nearly approximates the natural system with its porous formation.

pose that we substitute Figures 162 and 163 in place of Figures 96 and 97, respectively, and interpret them strictly in accordance with that argument.

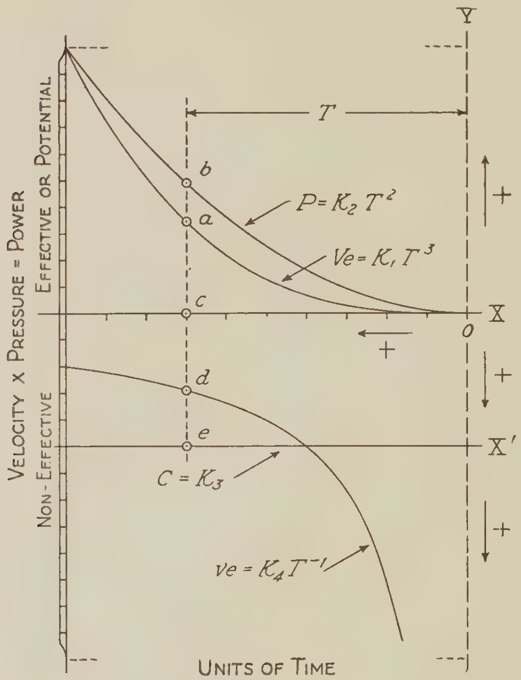



Fig. 162

In section 116 we developed sets of gas equations which pertain to reservoirs in Volumetric Control. While we found the appropriate method for that control to differ somewhat from the method for the preceding control, the differences in fact arose simply because we were dealing with a finite control, and not an infinite one.[18] Capillary Control, like Volumetric Control, is finite; consequently the methods of procedure for these two are alike. Inasmuch as we have once followed the procedure very carefully, I believe it is unnecessary to do more at present than to present the list of equations that pertain to the present control.[19]

Following is the list of equations:

$$Ve = K_1T^3 \qquad (491)$$

$$P = K_2T^2 \qquad (492)$$

$$C = K_3 \qquad (493)$$

$$ve = K_4T^{-1} \qquad (494)$$

$$G_{Pp} = K_2T^2 \qquad (495)$$

$$G_{Cp} = K_3 \qquad (496)$$

$$G_{Sp} = K_2T^2 + K_3 \qquad (497)$$

$$G_{RSp} = K_2T^2 + K_5 \qquad (498)$$

[18] We recall that these differences lay in the method of treating the conceptual velocity *ve* and in the method of apportioning the gas.

[19] See footnote 11, § 116, page 293. Our present investigation is again based on the assumption that there is no by-passing of the gas within the reservoir. In practice certain constants $K'_1$ and $K'_2$ would presumably be determined for a slow rate of production, and changed to $K_1$ and $K_2$ in accordance with Case 1, theoretic performance, for the faster rate. Now the equations indicate the quantities of gas for the minimum by-passing realized at the slow rate, and the measured quantities of gas can be compared with them. The indicated quantities of gas obviously serve as the goal of efficient operation in this control, for the gas with its pressure is the source of energy in these systems.

$$Po_a = PVe + CVe + Pve + Cve \quad \text{(499)}$$
$$Po_a = K_1K_2T^5 + K_1K_3T^3 + K_2K_4T + K_3K_4T^{-1} \quad \text{(500)}$$
$$E_a = PVo + CVo + Pvo + Cvo \quad \text{(501)}$$
$$E_a = \tfrac{1}{6}\,K_1K_2T^6 + \tfrac{1}{4}\,K_1K_3T^4 + \tfrac{1}{2}\,K_2K_4T^2 + K_3K_4 \quad \text{(502)}$$
$$Po = PVe \quad \text{(503)}$$
$$G_{Ve} = K_1K_2T^5 \quad \text{(504)}$$
$$E = \tfrac{2}{3}\,PVo \quad \text{(505)}$$
$$G_{Vo} = \tfrac{1}{6}\,K_1K_2T^6 \quad \text{(506)}$$
$$Po' = CVe + Pve \quad \text{(507)}$$
$$G'_{Ve} = K_1K_3T^3 + K_2K_4T \quad \text{(508)}$$
$$E' = CVo + Pvo \quad \text{(509)}$$
$$G'_{Vo} = \tfrac{1}{4}\,K_1K_3T^4 + \tfrac{1}{2}\,K_2K_4T^2 \quad \text{(510)}$$
$$E'' = Cvo \quad \text{(511)}$$
$$G''_{Vo} = K_3K_4 \quad \text{(512)}$$

If for Capillary Control we confine our attention to energy and gas displaced or produced from the reservoir, the equations between power and gas velocity, on the one hand, and time, on the other, possess the following form:

$$\left.\begin{matrix}\text{power}\\ \text{gas velocity}\end{matrix}\right\} = k_1t^5 + k_2t^3 + k_3t$$

Similarly, the equations between energy and gas volume, on the one hand, and time, on the other, possess the following form:

$$\left.\begin{matrix}\text{energy}\\ \text{gas volume}\end{matrix}\right\} = k_1t^6 + k_2t^4 + k_3t^2$$

For the retained quantities we would add the following terms to the respective forms:

$$k_4t^{-1} \quad \text{and} \quad k_4$$

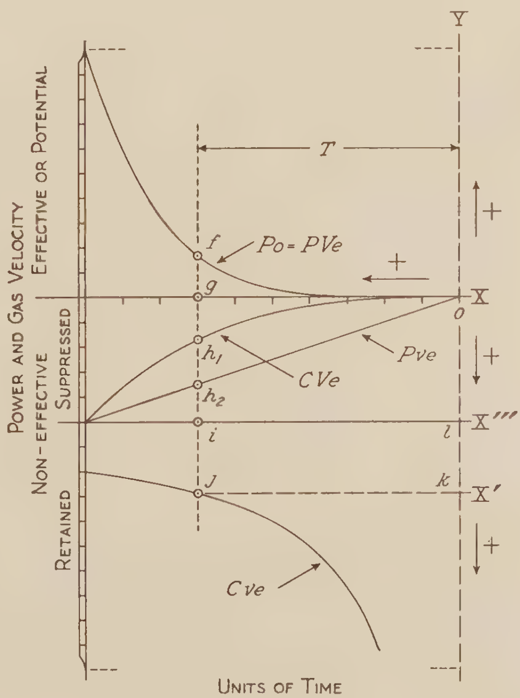



Fig. 163

As in Volumetric Control, the terms do not reduce to the simple form which we found in Hydraulic

Control. Once more it is essential that we discriminate between potential and suppressed functions.[20]

**164.** *Paths on producing from and into reservoirs.*—It is as necessary to make a distinction between percentage rate production and constant rate production in Capillary Control as it is in Volumetric Control. Ideal performance, as heretofore described, is percentage rate production. The reservoir system is considered to be ideal, and it is equipped with an orifice the size and physical condition of which remain perfectly constant during the process of production. We may replace Figure 98 by Figure 164 and apply thereto the argument of section 117 without modification.

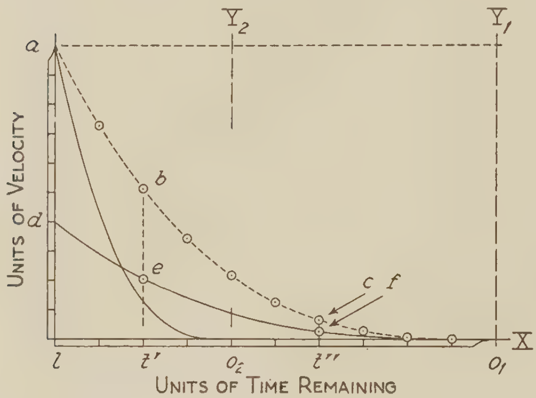



FIG. 164

Constant rate production we found to be frequently dictated by our own needs or by restrictions placed upon us by others. Where we may require a given amount of fluid per day, per week, or per any other unit of time, and where others may only be able to transport, sell, or consume a given amount of fluid per unit of time, we permit our reservoir to produce accordingly. Velocity is constant with time, and it is necessarily less than the maximum percentage rate for a given period of time. If we proceed as in section 117, we obtain the following equations:

$$Ac = \text{zero} \qquad (513)$$

$$Ve = K \qquad (514)$$

$$Vo = KT \qquad (515)$$

$$P = KT^{1/2} \qquad (516)$$

$$E = KT^{3/2} \qquad (517)$$

and

$$Po = KT^{1/2} \qquad (518)$$

The relation between pressure and volume is not altered in any way; consequently Equation 516 follows from Equation 515. The curves for these equations are shown in Figure 165. It is of interest to note that the pressure-time curve is convex toward the upper section of the plat. It appears to be "bowed the wrong way" in comparison with the usual decline curve. Where production from a reservoir in Capillary Control takes place at a constant

[20] See § 116, last paragraph.

rate, the curve between pressure and time has this appearance; and, conversely, where the curve between pressure and time has this appearance, the reservoir is one in Capillary Control and production is taking place at a constant rate.[21]

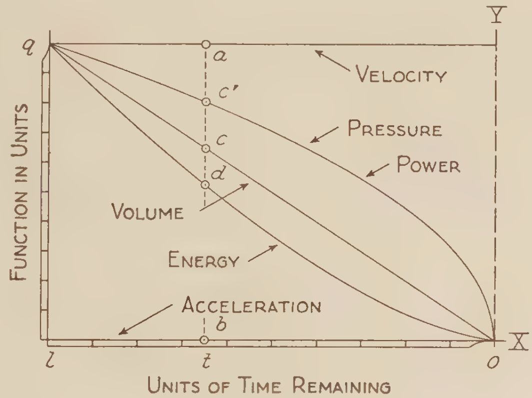


Fig. 165

A reservoir system normally of Capillary Control when in the process of production may be placed into communication with a reservoir that contains fluid under a greater pressure than that within the system. Under the circumstances the system may be said to fill with fluid, or it may be said that production takes place into the system. By proceeding as we did in section 118 we obtain the following equations:

$$P = KT^2 \quad \text{.......................} (519)$$

$$Vo = KT^4 \quad \text{.......................} (520)$$

$$Ve = KT^3 \quad \text{.......................} (521)$$

$$Ac = KT^2 \quad \text{.......................} (522)$$

$$E = KT^6 \quad \text{.......................} (523)$$

and

$$Po = KT^5 \quad \text{.......................} (524)$$

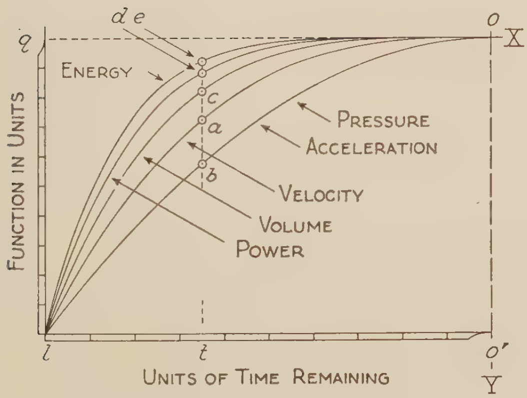


Fig. 166

These are our normal relations between functions. The curves of accline are the inverted curves of decline, as shown in Figure 166. It is immaterial whether the reservoir of higher pressure is in Hydraulic, in Volumetric, or in Capillary Control.

Let us suppose now that an operator fills a reservoir system of this control by means of some sort of a pump which runs at a constant speed. We have the following equations:

[21] The curve is in agreement with experience in gas production. It would no doubt constitute good evidence of the existence of Capillary Control in oil production.

$$Ve = K \quad \text{.......................} (525)$$

$$Vo = KT \quad \text{.......................} (526)$$

$$Ac = \text{zero} \quad \text{.......................} (527)$$

$$P = KT^{1/2} \quad \text{.......................} (528)$$

$$E = KT^{3/2} \quad \text{.......................} (529)$$

and

$$Po = KT^{1/2} \quad \text{.......................} (530)$$

These equations are precisely the same as the ones we found for constant rate production. Figure 167 for the present situation is in fact the image of Figure 165 in a mirror.

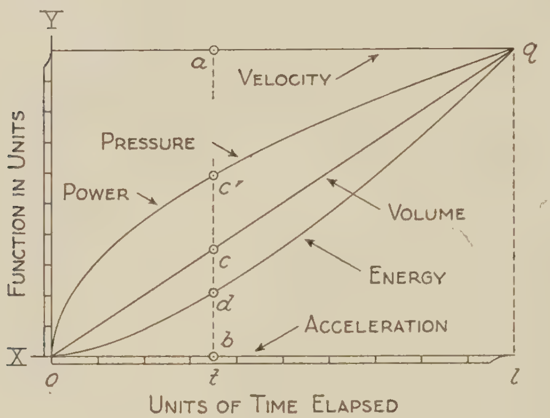



Fig. 167

If, as explained in section 118, in this process of filling, the capacity of the plant furnishing energy to the pump is reached, the curves of Figure 167 break into those of Figure 168. Power, the rate of displacement of energy, is now constant. As a consequence we have the following equations:

$$Po = K \quad \text{.......................} (531)$$

$$E = KT \quad \text{.......................} (532)$$

$$P = KT^{1/3} \quad \text{.......................} (533)$$

$$Vo = KT^{2/3} \quad \text{.......................} (534)$$

$$Ve = KT^{-1/3} \quad \text{.......................} (535)$$

and

$$Ac = -KT^{-4/3} \quad \text{.......................} (536)$$

To obtain Equations 533 and 534 we simply write

$$P = KT^{m}$$

and

$$Vo = KT^{n}$$

where the exponents $m$ and $n$ are to be determined. We know from Equation 532 that $m + n = 1$, and we also know from pressure-volume relations in this control that $n = 2m$; therefore $m$ and $n$ are 1/3 and 2/3, respectively.

The curves for velocity and acceleration are hyperbolic. The former is positive, while the latter is negative. To be mathematically exact the acceleration-time curve should be placed by itself, below the $X$ axis. I have put it in the quadrant with the other curves merely as a matter of convenience.

The hyperbolic curves in the figure are exaggerated in the vertical direction for the purpose of more clearly displaying the properties of variation in these functions. All ordinates for velocity have been multiplied by 32.31, and all for acceleration have been multiplied by 969.3.[22]

Figures 165, 166, 167, and 168 correspond to Figures 99, 100, 101, and 102 in Volumetric Control, respectively. In the description of the latter certain points indicated on the drawings were mentioned. Now similar points are indicated in the present drawings, and their significance is in each case the same as before. They can be interpreted in the light of the argument given in sections 117 and 118.

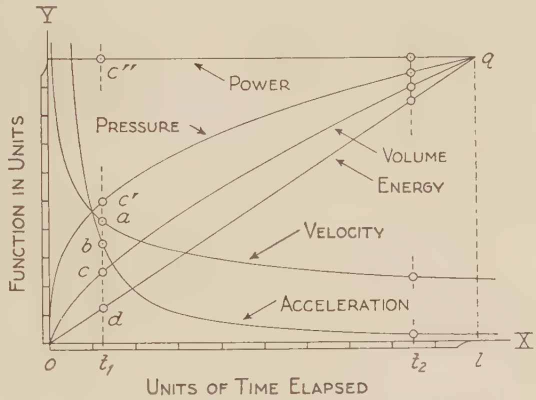


Fig. 168

[22] No particular significance is to be attached to these numbers. They merely cause the present figure to display the symmetry of Fig. 102.

## CHAPTER XXVIII

# Theoretic Performance

"A rule, reached by the observation of facts, cannot possibly embrace the entire fact, in all its infinite wealth, in all its inexhaustible manifoldness; on the contrary, it can furnish only a rough outline of the fact, one-sidedly emphasizing the feature that is of importance for the given technical or scientific aim in view. What aspects of a fact are taken notice of will consequently depend upon circumstances, or even on the caprice of the observer. Hence there is always opportunity for the discovery of new aspects of the fact, which will lead to the establishment of new rules of equal validity with, or superior to, the old."—Ernst Mach

165. *A pressure diagram.*—We have hitherto established the laws for the delivery of fluid from reservoirs in Capillary Control. These laws, as we know, are based upon an ideal; the Jamin tubes, as physical containers, are perfectly ideal, and likewise are the imposed conditions upon their performance. No act originating either with Nature or with us disturbs the mathematically defined paths of the primary functions.

In our study of reservoirs in Volumetric Control we found it convenient and advantageous to determine the laws of delivery by devoting our principal attention to accurately defined artificial reservoirs. Without doubt our line of investigation at the time was guided by the problems that are met in observing the performance of natural reservoirs, though these did not receive their full consideration until the subject of primary functions had been thoroughly treated. Now we are proceeding in the same manner with regard to reservoirs in Capillary Control. I believe we profit by this method, for the primary functions of both artificial and natural reservoirs are identical. We temporarily avoid matters which arise in connection with the secondary functions—a feature that appears to be particularly desirable in the present control. In the same way as before, however, we are continually guided by the problems that are presented by natural reservoirs.

The performance of reservoirs in Hydraulic and Volumetric controls becomes easily understood when it is illustrated by means of pressure diagrams. Certain principal lines, *K, J, N,* and *I,* with auxiliary lines, *M* and *L,* define ten pressures with which such performance is concerned. These lines have appeared heretofore in several of our figures which are, in fact, space diagrams, such as Figures 47, 60, 94, 111, and 112. Some of them—the principal ones—either actually appeared in our pressure-time diagrams or their pres-

ence was inferred.[1] Their associated pressures were fully described in sections 62 and 71.

We must be already aware of the fact that our pressure-time diagrams in Capillary Control are concerned with *K, J, N,* and *I,* for these diagrams are identical in this and in Volumetric Control. Can we construct a proper space diagram for the present control? I believe we can. Figure 169 appears to represent accurately the conditions as we know them to be. I have indeed

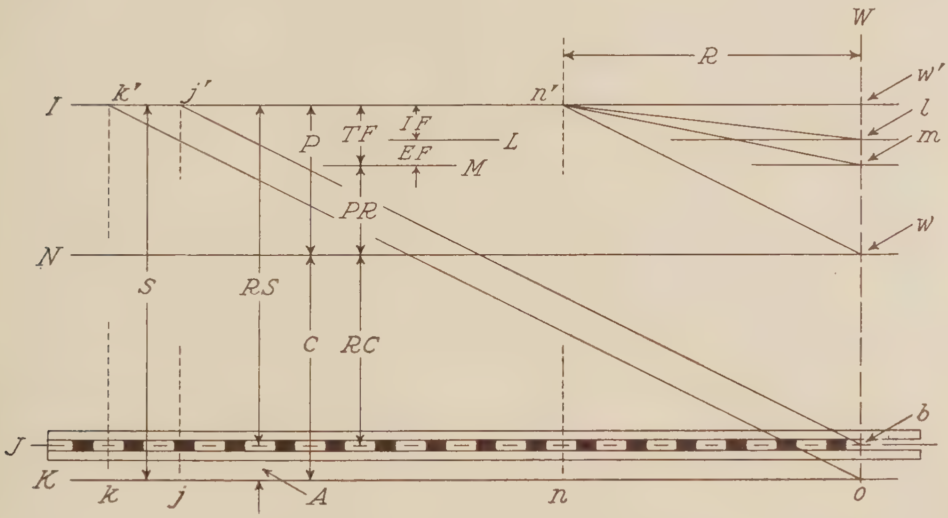



Fig. 169

designed this figure in accordance with the earlier ones.[2] The reservoir system consists of a Jamin capillary tube of smooth bore, although one with contractions and expansions might as well have been selected. The tube can be imagined to extend indefinitely toward the left, or, if we will agree that under no circumstances will the potential pressure *P* be allowed to be in excess of the sum of the *f*'s of the globules shown, it can be imagined to be closed at the left end. *Certainly it is immaterial whether it is sealed here with solid material or with further globules of liquid that are not to be disturbed in the least during the process of production.*[3] Any accessory apparatus to the right of the drawing is omitted. The point *b* denotes a Position 0 as of Figure 148 (*a*), for some globule which now lies at its left.

[1] In particular the line *K* corresponds to the absolute axis *X′*, the line *J* to the atmospheric axis *X″*, and the line *N* to the potential axis *X*.

[2] The line *I* now takes the place of *Q* for porous-filled combination reservoirs in Volumetric Control, such reservoirs having been converted to Capillary Control. We are no longer concerned with the fact that the container did or did not serve as a reservoir in Volumetric Control before serving as one in Capillary Control; consequently, for the sake of uniformity, we resume the original notation in the line *I*.

[3] The statement is verified by experiments with the tubes.

The tube is shown to be in equilibrium, ready to produce, at a static pressure $S$, greater than $A$, the pressure of the atmosphere. Production is to take place against a constant back pressure $C$, greater than $A$, but less than $S$. Thus the system is provided with a potential pressure $P$.

In section 92, we noted particularly that in the diagram of Figure 47 the lines $K$, $J$, $N$, and $I$, as drawn, pertain only to $W$ under the given conditions of production from it. We extend these lines outward horizontally for the purpose of conveniently designating the dimensions of the spaces between them. *They have little, if anything, to do with any part of the reservoir system other than with that which we choose to consider the orifice.* Now we choose the point $b$ as our orifice for analytical reasons which must be evident from our description of the experiments with tubes in sections 151 and 152. The four lines pertain only to this point $b$.

In a system of this sort there is, for a given potential pressure $P$, a radius $R$, beyond which distance the globules and bubbles remain perfectly undisturbed during the process of production and at the establishment of final equilibrium, and within which distance but one-half the "mobile" fluid is to be produced by the time equilibrium is established, as explained in section 153. Although the potential pressure has a value $P$ at $b$, it is zero at a point $n$, distant $R$ from $b$, since $P$ is progressively diminished by $f$'s existing within $R$. Then in our diagram we accordingly draw a straight inclined line from $w$ to $n'$ to indicate this fact. This line we recognize to be in agreement with the pressure diagram of Figure 145, and others. Our present pressure diagram is the triangle $w'wn'$, and we know that to this there corresponds a volume triangle having the same base $R$ and an altitude that depends upon $P$ and the cross-section area of the capillary bore at $b$.

At the instant production begins the fluid between $b$ and $n$ starts to move. An internal friction head $IF$ is brought into play. Its intensity at $b$ is represented by the space between the lines $L$ and $I$, and its intensity at $n$ is necessarily zero, for there is no movement of fluid here. We may draw the straight inclined line from $l$ to $n'$ to indicate the progressively diminishing intensity of this friction. Exterior to $b$ there is assumed to be an external friction head the intensity of which is shown by the space between the lines $M$ and $L$.[4] Its effect upon production at $b$ amounts to $EF$, while its effect at $n$ is zero, for the same reason as in the case of internal friction. Together $IF$ and $EF$ at $b$ amount to $TF$, the total friction head, and since its constituents are zero at $n$, it is itself zero at $n$. We may draw the straight inclined line from $m$ to $n'$ to indicate the progressively diminishing intensity of this total friction. Thus we divide the triangle $w'wn'$ into smaller triangles, all having the same base $R$. The triangle $mwn'$ represents the residual pressure. *Now we know at all points along* R *the intensity of the potential pressure, which determines the*

[4] We are assuming an external friction head due to a flow-line exterior to $b$, a flow-line not shown in the figure.

*fact that there shall be flow at these points, and the intensity of the residual pressure, which determines the velocity of flow at these same points.*[5]

If we question the right to draw the lines $ln'$ and $mn'$ straight, we can stop production intermittently in the course of our second experiment, section 152, and measure the successive advances made by the various globules along $R$. These confirm the straightness of the lines.

In ideal performance the lines $J$ and $N$ remain fixed with respect to the line $K$; $I$ lowers its position in accordance with production, finally coming to rest in coincidence with $N$ when equilibrium is established at the given conditions of production. While $I$ lowers its position the triangle $w'wn'$ shifts to the right in accordance with Equation 422 (p. 432): namely, $R = KT^2$. The inclined lines remain at fixed angles with $I$ at $n'$, $n'$ moves toward the intersection of $I$ and $W$ to accommodate the variation in $R$, and the intersection of $N$ and $W$ appears to travel up the line $wn'$. All this is in agreement with observations to be made during experiments with the tubes.

In theoretic performance we cause one or more of the lines above $K$ to shift upward or downward in their positions independently of production. These new positions are omitted from the figure in order to maintain its clarity. It is now obvious that if these lines shift, the triangle $w'wn'$ and its constituents are altered. These alterations will be studied in connection with the usual three cases in this performance.

Before we take up these cases let us recall the fact that in our study of Volumetric Control we established the principle of harmonious percentage variation between the five pressures $F$ that appear above the line $N$, when performance is ideal.[6] The individual bubbles in the present system are admitted to be small reservoirs in Volumetric Control; therefore with each of them taken as individual producers, regardless of their particular positions along $R$, the principle of harmonious percentage variation holds. It holds with the bubble or space at $b$; consequently it holds with the system at large. The cumulative or "integrating" effect of the globules and bubbles, which causes the system at large to be in Capillary Control, has no influence upon this principle.[7]

**166.** *Case 1. Alterations in external friction.*—This case involves, as we recall, alterations in the value of the external friction head. In Figure 169 the lines $K$, $J$, $N$, and $I$ are not affected in any way. While the value of $P$ remains the same, its subdivisions alter their values; consequently the lines

[5] We say that analytically these intensities are known, inasmuch as the values of $P$ and $R$ are assumed to be known.

[6] See § 128.

[7] The two following propositions are in perfect agreement: (1) the principle of harmonious percentage variation holds with respect to the individual spaces, and (2) the lines $ln'$ and $mn'$ in Fig. 169 are straight.

$M$ and $L$ move to new positions. The situation with respect to seven pressures agrees with the table given in section 76.

When $M$ and $L$ shift upward or downward, they do so in a manner fully described in the section just cited. In all controls the points $m$ and $l$ move with these lines. Only in the present control have we such a point as $n'$, and in this case this point is not affected, inasmuch as $R$ for the given system depends only upon $P$. The triangle $w'wn'$, to which we hereafter refer as the potential triangle, is not itself altered; but since $m$ and $l$ are shifted, its component triangles are altered. In particular the residual pressure triangle is increased or decreased, and as a consequence the velocity of production is increased or decreased, respectively, in accordance with the equation

$$Ve = K(PR)^{3/2} \quad \dots\dots\dots\dots (537)$$

a relation which we know to be true from experience with reservoir systems in Capillary Control. As a relative equation this becomes

$$Ve = \frac{1}{10}(PR)^{3/2}. \quad \dots\dots\dots\dots (538)$$

While the potential pressure of the system is not altered, the curve between this pressure and time suffers a change, as shown in Figure 114. This figure serves to illustrate the present case. The corresponding curve between residual pressure and time is shown in Figure 170, and the curve between velocity and time, for this alteration, is shown in Figure 171. These are to be compared with Figures 115 and 116 in Volumetric Control.

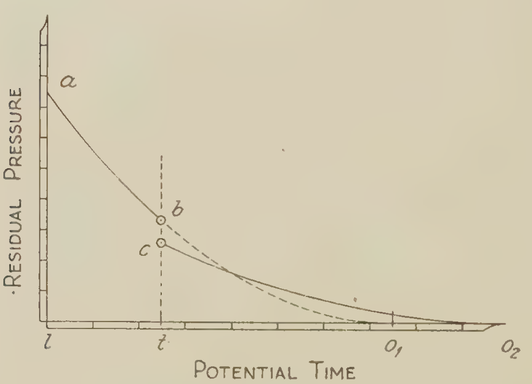



Fig. 170

In either finite control, theoretic performance, Case 1, it is the ratio between the new and old values of the velocity which determines directly the ratio between the old and new values of time remaining; that is to say, time remaining depends upon the velocity inversely. In Figure 171, as in Figure 116, the velocity is assumed to be altered to two-thirds its value; that is, from $tb$ to $tc$. In both figures the area subtended by $bO_1$ is equal to that subtended by $cO_2$. The potential volume is unaltered in Case 1; only the time in which it is to be produced is altered. In both figures the area of the rectangle $tcnO_2$ is equal to that of the rectangle $tbmO_1$, though these are now four times instead of two times the subtended areas. If for the moment we consider abscissas in Figure 159 to represent percentage values of residual pressure, in accordance with

Equation 538 above, and pass a horizontal straight line through velocity at $66\frac{2}{3}$ per cent, we see that the necessary change in residual pressure for Volumetric Control is from 100 to 44.44 per cent, whereas the change for Capillary Control is from 100 to 76.32 per cent. In the present control, as compared with Volumetric Control, the same percentage change in the velocity is attained by a smaller percentage change in the residual pressure, and the same percentage change in the residual pressure produces a greater percentage change in the velocity.[8]

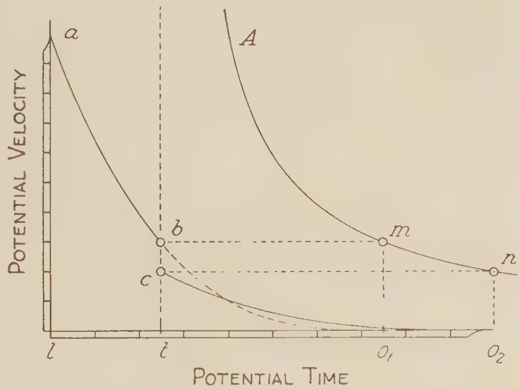



FIG. 171

We can now understand why the distance *cb* is smaller in Figure 170 than it is in Figure 115, while the ratios *tc* to *tb* are the same in Figures 171 and 116.

From what has been said before we know that in this case

$$VeL = 4Vo \qquad (539)$$

where the quantity on the right is constant. Clearly this is the equation of a rectangular hyperbola of the form $xy = k$. *A,* the locus of such points as *m* and *n,* is then a curve of this type. Its asymptotes are the vertical line at *t* and the *X* axis. The situations in the two finite controls are quite alike.[9]

To determine the relation between velocity and acceleration in this case we have Figure 172 (p. 484) replacing Figure 117 for Volumetric Control. There are shown three of an infinite number of possible velocity-time curves for a particular reservoir. All subtend equal areas representing a potential volume of 500 units of fluid.[10] We know from section 129 that alterations in

[8] While this converse situation is a logical sequence of the first one, it can also be seen in Fig. 159. For the first situation we pass a horizontal line through the desired percentage value of velocity, and for the second we pass a vertical line through the desired percentage value of residual pressure. In virtue of the relation between velocity and time remaining we may replace the word "velocity" by the words "time remaining" in the above statement. Thus identical percentage alterations in the frictional back pressure cause more intense effects upon velocity and life in Capillary Control than in Volumetric Control. The same will be found to be true with regard to identical percentage alterations in the constant back pressure.

[9] See footnote 16, § 129, page 341. It is of interest to compare the curvatures of the parabola and the rectangular hyperbola in Fig. 171. Their middle portions can hardly be distinguished by eye. Of course their distinguishing features are to be found in their relations to the axes of co-ordinates.

[10] See footnote 17, § 129, page 342.

accordance with Case 1 cause alterations in acceleration. Acceleration is now a variable; it is not a constant as in Volumetric Control. Nevertheless we may proceed. The method is the same as before, though we cannot abbreviate it as we did in the cited section. First we determine the specific equations for the given curves. This we can easily do, for we know the general velocity-time curve for this control, and we can determine its constant $K$ in each instance by inserting the values for any one point, other than the point for equilibrium, on each curve. Thus we have

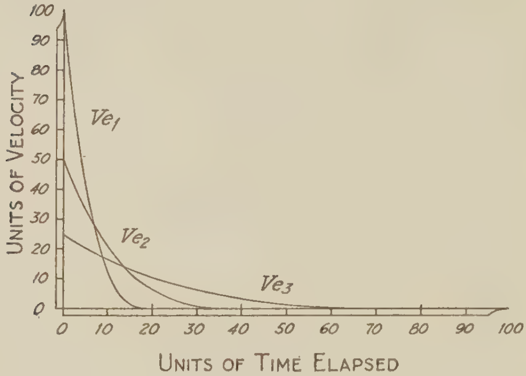



Fig. 172

$$Ve_1 = \frac{1}{80}T^3 \quad \text{(540)}$$

$$Ve_2 = \frac{1}{1{,}280}T^3 \quad \text{(541)}$$

and

$$Ve_3 = \frac{1}{20{,}480}T^3 \quad \text{(542)}$$

Here $T$ represents time remaining, as usual.[11] The figure shows a scale representing time elapsed merely in order to avoid a separate scale for each curve.

Next we differentiate these equations for the purpose of obtaining the specific equations between acceleration and time. Thus

$$Ac_1 = \frac{3}{80}T^2 \quad \text{(543)}$$

$$Ac_2 = \frac{3}{1{,}280}T^2 \quad \text{(544)}$$

and

$$Ac_3 = \frac{3}{20{,}480}T^2 \quad \text{(545)}$$

Now we can write down the following table:

| | | |
|---|---|---|
| $T_1 = 20$ | $Ve_1 = 100$ | $Ac_1 = 15$ |
| $T_2 = 40$ | $Ve_2 = 50$ | $Ac_2 = \frac{15}{4}$ |
| $T_3 = 80$ | $Ve_3 = 25$ | $Ac_3 = \frac{15}{16}$ |

[11] The data for the equations are taken from Fig. 172, wherein abscissas are previously changed to time remaining for the individual curves.

The values for $T$ and $Ve$ are the initial points as shown by the curves, and those for $Ac$ are obtained by Equations 543, 544, and 545. These are three sets of values which might be established with a given reservoir system by means of applying the proper external friction against production. By inspection we see that

$$\frac{Ac_1}{Ac_2}=\frac{Ve_1^2}{Ve_2^2}, \quad \text{and} \quad \frac{Ac_1}{Ac_3}=\frac{Ve_1^2}{Ve_3^2}.$$

In general then

$$Ac = KVe^2 \qquad (546)$$

where $K$ is another constant, obviously 15/10,000 in the present case. The value of acceleration for all instants during the remainder of time in life depends upon the square of the velocity at the instant of an alteration in the external friction head. *The relation between velocity and acceleration in Case 1 is the same in both finite controls, notwithstanding the fact that acceleration is constant and variable, respectively, in the two.*[12]

Since the potential pressure and the potential volume of a given reservoir are not altered by changes in the external friction head, the potential energy of the reservoir is not altered. The curves for these three functions separately plotted with time of course vary with the alteration, inasmuch as time itself is altered. Their mutual curves, however, being independent of time in every way, are identical, regardless of these alterations.

The situation with respect to potential and suppressed power, and therefore with respect to the rate at which gas is produced from a combination reservoir, is like that in Volumetric Control. It is only necessary to replace the curves of Figure 118 by the appropriate ones for this control, and the argument of section 129 holds without modification other than that dictated by the new values for the areas subtended by the curves. Figure 118 is founded upon Equation 235 (p. 294) and Equation 239 (p. 295), whereas the revised figure must necessarily be founded upon Equations 504 and 508 (p. 473).[13]

On the logarithmic plat an alteration in the external friction head causes a shift in the position of the "straight line." In Figure 119 the line with a slope of 1 to 1 should now be replaced by one with a slope of 3 to 1. The situation meets with the description given in section 129.

**167.** *Case 2. Alterations in static pressure.*—We remember that this case involves alterations in the value of the static pressure, alterations which are eratic with respect to time; that is, they do not take place naturally as a result

[12] We must not confuse the present relation in theoretic performance with acceleration-velocity relations in ideal performance.

[13] The revised figure, according to Fig. 163, must have two curves below the axis $X$ to satisfy the separate terms of Equation 508.

of decline in accordance with production. These alterations pertain to either an increase or a decrease in this pressure.

We are now to refer to our system of Figure 169. The situation with regard to seven pressures agrees with the table given in section 78.

In the preceding controls we had occasion to consider both real and apparent alterations in accordance with this case. For reasons to be set forth in detail at a later time we shall say that only real alterations occur in reservoir systems of Capillary Control. *The multiple orifice, as previously conceived, cannot exist for these systems.*[14]

It is a simple matter, of course, to alter the real static pressure of the Jamin capillary tube, for fluid need only be introduced into the system at any time, or withdrawn from the system, independently of our reckoning upon delivery. If we regard one end of the tube as the orifice of the system, then these alterations may be made at either this or the opposite end.[15]

These alterations merely affect the history of performance. On the addition of fluid under pressure the reservoir system immediately assumes a status that existed earlier in its life. By decline in Capillary Control it had performed in a regular manner, only to be set back in its path toward the state of equilibrium. Contrarily, on the withdrawal of fluid the system immediately assumes a status that otherwise would exist later in its life. The system is set ahead in its path toward the state of equilibrium. The situations are illustrated in Figure 173, and the description of Figure 120 in section 130 applies here without modification. The paths themselves are not altered in any manner. The constants $K$ in the primary function equations are the same before and after the alteration. In this Cases 1 and 2 differ in both finite controls.

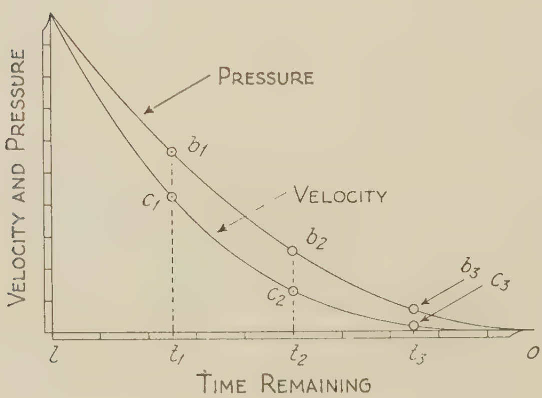


Fig. 173

In Figure 169 alterations in the static pressure shift the line $I$ upward or downward with respect to the lines $K$, $J$, and $N$, which remain fixed in posi-

[14] We are to find that each well producing from a given formation which serves as a natural reservoir in Capillary Control possesses its individual area of drainage. As we already know, all wells producing from a given formation which serves as a natural reservoir in either Hydraulic or Volumetric Control possess an area of drainage in common.

[15] It is preferable to regard the tube with contractions and expansions in these alterations.

tion. $M$ and $L$ shift concordantly with $I$, so that all spaces between the lines above $N$ increase or decrease in the same ratio. The point $n'$ shifts to the left or right, increasing or decreasing $R$ in the same ratio as $P$. Thus the triangle $w'wn'$ immediately assumes a status otherwise normal at an earlier or a later time in the life of the reservoir system.

Given the values of the various primary functions at an instant immediately preceding an alteration in accordance with this case, these being known either by direct observation at the instant or by calculation from earlier values with the use of the relative curves in ideal performance, new values to be assumed by the functions upon the alteration are easily determined from the relative curves by making the proper vertical cut. Thus if we know the new and old values of the static pressure, and consequently the percentage ratio between the new and old values of the potential pressure, we need but locate this ratio on the relative pressure-time curve of Figure 158, or on the horizontal axis in Figure 161, pass a vertical line through the point, and read off the percentage ratios between the new and old values of the remaining functions at the intersections of the line with their curves. Now these two figures cover only the ratios between 0 and 100 per cent. For general use we may conveniently prepare a chart as outlined in Figure 174 (p. 488). The description of this figure follows that given to Figure 121 in section 130.

To illustrate the vertical cut we shall suppose that the alteration in the static pressure is such as to reduce the potential pressure to one-third, or say 33.3 per cent, of its value. Find this ratio on the pressure curve at the point $a$ and draw through it the vertical line $A$. At the intersections made by this line we find the following ratios for the remaining functions:

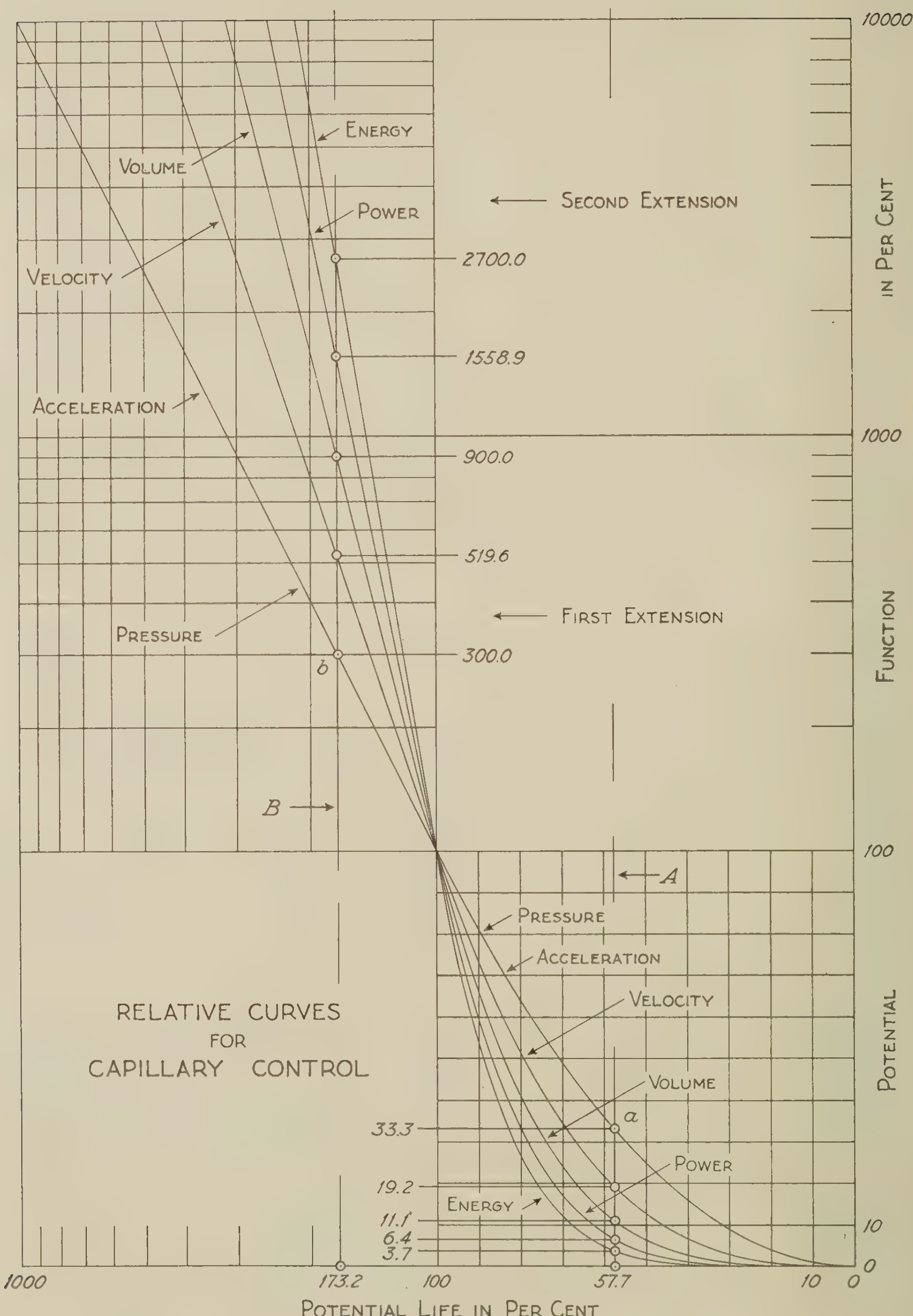
| | |
|---|---|
| Volume | 11.1 per cent |
| Velocity | 19.2 per cent |
| Acceleration | 33.3 per cent |
| Energy | 3.7 per cent |
| Power | 6.4 per cent |
| Life | 57.7 per cent |

all these being the ratios between the new and old values.

We shall next suppose that the alteration in the static pressure is such as to increase the potential pressure to 300.0 per cent of its value. This ratio on the pressure curve is found at the point $b$, and through it the vertical line $B$ is drawn. Now we find the following ratios for the remaining functions:

| | |
|---|---|
| Volume | 900.0 per cent |
| Velocity | 519.6 per cent |
| Acceleration | 300.0 per cent |
| Energy | 2,700.0 per cent |
| Power | 1,558.9 per cent |
| Life | 173.2 per cent |

In comparison with Volumetric Control vertical cuts in Capillary Control give the same ratio in life for the same ratio in pressure, but the ratios in the



10000
Energy
Volume
Second Extension
Power
Velocity
2700.0
1558.9
Acceleration
1000
900.0
519.6
First Extension
Pressure
300.0
b
B
100
A
Pressure
Acceleration
Velocity
RELATIVE CURVES
FOR
CAPILLARY CONTROL
Volume
33.3
a
Power
19.2
Energy
11.1
6.4
3.7
10
0
1000
173.2
100
57.7
10
0
Potential Life in Per Cent
Potential Function in Per Cent


Fig. 174

other functions show a more intense change in the latter than in the former control. And for the same ratio in pressure the ratios for velocity and volume are now the same as those for power and energy, respectively, in Volumetric Control.

The potential pressure curve serves also for the external friction head, the internal friction head, the total friction head, and the residual pressure. While these all vary in harmonious percentage variation in ideal performance, they for the same reason act concordantly in assuming new values upon alterations in the static pressure.

In virtue of the fact that we are concerned here only with alterations in the real static pressure, the question with regard to the proportional production of gas is comparatively simple. In the ideal systems represented by the two classes of Jamin capillary tubes it is obvious that alterations in the pressure bearing upon the liquid do affect the amount of gas dissolved per unit volume of liquid, and consequently they must affect the proportional amount of gas produced with the liquid. Perhaps the situation is more readily understood if we consider the conditions within a tube having contractions and expansions rather than those within a tube of smooth bore. The two finite controls, with respect to their own energy and power curves, that is, with respect to their own gas volume and gas velocity curves, behave alike, in so far as alterations in the real static pressure in Volumetric Control are concerned. As Figure 118 was treated in section 131, so may its correspondent in the present control be treated.[16] The interpretations are the same in both instances.

**168.** *Case 3. Alterations in constant back pressure.*—This case involves alterations in the value of the constant back pressure. The situation with respect to seven pressures agrees with the table given in section 79. In Figure 169 the lines $K$, $J$, and $I$ remain fixed in position, while the line $N$ is caused to shift downward or upward in accordance with a decrease or an increase in the constant back pressure $C$. $M$ and $L$ shift concordantly with $N$, so that all spaces between the lines above $N$ increase or decrease in the same ratio. The point $n'$ shifts to the left or right, increasing or decreasing $R$ in the same ratio as $P$. If production takes place into the atmosphere, $N$ coincides with $J$, and $n'$ takes a position $j'$. The triangle $w'wn'$ is now in fact $w'bj'$. Again, if production takes place into a perfect vacuum, $N$ coincides with $K$, and $n'$ takes a position $k'$, and the triangle $w'wn'$ is now in fact $w'Ok'$. The angles which $wn'$, $mn'$, and $ln'$ make with $I$ are unaltered in the shifting. In particular, $wn'$, $bj'$, and $Ok'$ are parallel.

It is clear that the value of the potential pressure depends upon alterations in the constant back pressure. In fact a decrease or an increase in this back pressure is equivalent to an increase or decrease in the static pressure. The volume of fluid which the system is able to produce is altered in either case.

[16] See § 166, next to the last paragraph.

Figure 173 serves as well in Case 3 as in Case 2. We have learned before that these two cases are very closely related in their mathematical aspects.[17]

Inasmuch as the alterations in potential pressure might be made equally in accordance with the two cases, the vertical cut illustrated in Figure 174 serves in the present case. The functions take on new values as indicated at the intersections of the vertical lines and the respective curves.[18]

Figure 122, which pertains to Case 3 in Volumetric Control, may be referred to in connection with the present case. This figure illustrates the shifting of the $X$ axis for the pressure-time curve, and, because the relation between pressure and time is the same in the two controls, the locus of the point $O$, denoting equilibrium for any and all possible values of the constant back pressure, is the parabola $A$. In regard to the other functions we may take Figure 158 and revolve it in the plane of the plat through 180 degrees about its center, and see at a glance the loci of the various points $O$ for the various functions, as these are caused to shift in their positions. It is evident that the loci are different in the two controls for all functions other than pressure.[19]

It would be a simple matter to revise Figure 123 so as to accommodate Capillary Control. Curves of the proper equations should replace those given.[20] The subtended areas, and therefore energy and gas volume, are changed in the manner explained in section 132.

The "straight line" on the logarithmic plat of Figure 124 must of course be replaced by one with a slope of 3 to 1 to suit this control. The argument, however, remains the same as given in section 132, provided that for the vertical cut on Figure 121 one on Figure 174 be substituted.

**169.** *The three cases, concluded.*—It was admitted in section 80 that if all our reservoirs were in Hydraulic Control the distinction between the frictional back pressure and the constant back pressure would be, for most purposes, unnecessary. In those reservoirs both may be said to behave as mathematical constants, and consequently the sum of their values can be handled as conveniently, and as accurately, as the values taken separately. The effects of these back pressures upon the performance of the reservoir are identical. In the finite controls, however, this is not true, for one behaves as a mathematical variable, while the other retains its behavior as a mathematical constant. In these reservoirs we find that the frictional back pressure does not affect the volume of fluid to be produced, but only the time in which it is to be produced, whereas the constant back pressure does affect the volume

[17] See § 132.

[18] See footnote 9, § 132, page 353.

[19] The curves as loci for the points $O$ are described in § 132.

[20] The revised figure, according to Fig. 163, must have two curves below the axis $X$ to satisfy the separate terms of Equation 508 (p. 473).

of fluid. In one case the constants $K$ for all primary function equations are altered in value, while in the other they are not altered. We can still say that Cases 1 and 3 are concordant in their action according to the explanation given in the cited section, without inferring any notion that they have the same effect upon performance. We must admit that during short periods of time in the life of a reservoir in either of the finite controls we can make no distinction between the effects of the two cases, if we rely upon observations of performance alone. Of course we would know in any particular instance the nature of the alteration, and it is in virtue of this knowledge that we are able to predict the effects.

As Figure 174 now stands, it has a restricted application in Case 1. The following modifications, however, will enlarge its scope:

*a*) Entitle the straight horizontal line at 100 per cent as potential pressure, volume, and energy.

*b*) Where velocity appears, place power with it.

*c*) Where pressure and acceleration appear, for them substitute residual pressure.

*d*) Replace energy by acceleration.

*e*) Replace "Potential Life in Per Cent" by "Reciprocal of the Cube Root of Potential Life in Per Cent."

*f*) Cancel the volume and power curves.

The first three modifications must be clear in the light of our preceding investigations. The velocity-time curve in the figure has the equation

$$Ve = KT^3 \qquad (547)$$

and from Equation 546 (p. 485), we have

$$Ac = KVe^2 \qquad (548)$$

Now by substituting the value of $Ve$ in Equation 547 into Equation 548 we obtain

$$Ac = KT^6 \qquad (549)$$

and thus we account for the modification in (*d*).[21] Again, from Equation 539 (p. 483), we have

$$VeL = K \qquad (550)$$

or

$$Ve = K\frac{1}{L} \qquad (551)$$

In view of Equation 547 above we must write Equation 551 thus:

$$Ve = K\frac{1}{T^3} \qquad (552)$$

[21] Equations 548 and 549 define the curves of loci for all possible values of $Ac$ in Case 1. After the new value is established, this function proceeds to decline in accordance with the relation $Ac = KT^2$.

where $T$ merely refers to the scale as it appears in Figure 174. This equation may be written as follows:

$$T = K \frac{1}{\sqrt[3]{Ve}} \quad \text{.................... (553)}$$

In this we account for the modification in ($e$).[22] It is easily verified by a test with numbers. For example, if the velocity is in this case reduced to 12½ per cent, then by Equation 550, $L$ or $T$ should be 800 per cent. According to the figure, when $Ve$ has this value, the scale for $T$ indicates 50 per cent, and this is the reciprocal of the cube root of 800 per cent.

The modification in ($f$) is obvious. There is no need for these two curves.

In section 133 we found that Figure 124, although presented previously in connection with Case 3, serves to illustrate an alteration in accordance with Case 2. Likewise, when revised, will it serve to illustrate both cases in the present control. The figure is dependent only upon the alteration in the distance between the lines $N$ and $I$, and it is in fact immaterial whether either one, or both, be shifted in position.

*In all three cases the vertical cut on the relative curves shows more intense effects in Capillary Control than in Volumetric Control.* This fact is corroborated in practical experience. Where reservoirs are in Capillary Control, Case 1 provides more varied rates of production, because the range in values of the external friction that accompanies standardized equipment permits a wider range in the velocity. At the same time this case provides a wider range in the values of life in this control. If alterations in the rate of production have been noted with respect to alterations in the potential pressure, whether by natural decline or by alterations in Case 3, it is possible to say with certainty whether the reservoir is in Volumetric or Capillary Control, in virtue of the more intense effects realized in the latter.

By means of all vertical cuts we can construct the theoretically perfect curves for all functions of performance—notably the velocity-time curves for the liquid or the gas, singly or in combination—on the basis that after the alteration is made performance will be ideal. These curves give us a means of comparing actual performance with the ideal, and thus we may know our own efficiency in operation. The cuts give us the necessary information to construct these curves, for they furnish the new values of all the functions including life. We therefore possess initial and final points for the curves, and between such points there is but one path that is defined by one equation with its particular exponent for $T$.

In section 132 we learned how to calculate the absolute contents of reser-

[22] For the like modification of Fig. 121 in Volumetric Control we have

$$Ve = K\frac{1}{T}$$

consequently the title in ($e$) reads, "Reciprocal of the Potential Life in Per Cent." (See § 133.)

voirs in Volumetric Control. This is done by applying the methods of Case 3. From what we know of the system in Figure 169 we can say that the same methods give us the maximum triangle for potential pressure ($C = 0$), and consequently the maximum triangle for potential volume. Twice this volume represents the absolute contents of the system within a maximum radius $R$. The relative curves do not refer to the portion of the physical container beyond $R$, as we have already noted, and as a consequence a calculation on the absolute contents can pertain only to that space of the physical container within the radius $R$. In virtue of Equation 422 (p. 432), namely,

$$R = KT^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (554)$$

we can place the title "*R*, the Radius of Action of *P*" with pressure and acceleration in Figure 174 for the reservoir systems consisting of Jamin capillary tubes, as these have thus far been illustrated. The curve bearing this title is, like the others, for use in Cases 2 and 3. For Case 1 the title must be placed with potential pressure, volume, and energy in the modified chart.

CHAPTER XXIX

# Secondary Functions of Performance

"We are certainly not to relinquish the evidence of experiments for the sake of dreams and vain fictions of our own devising; nor are we to recede from the analogy of Nature, which uses to be simple, and always consonant to itself."—SIR ISAAC NEWTON

170. *Introduction.*—We have thus far taken three steps toward the solution of our problem concerning the performance of reservoirs in Capillary Control. In the first we observed the existence of new primary function relations in our field data; in the second we provisionally assumed these relations to be accurate and proper, and we proceeded to formulate a hypothesis that might account for them; and in the third we attempted—I believe with success—to fit these accepted relations and a highly imaginative basis for their accompanying secondary functions with known principles in physics. We found that these principles rest upon Jamin's experiments with capillary tubes and porous materials. These experiments have been repeated, as explained in the preceding chapters, and the results have been given an elaborate mathematical interpretation in terms of reservoir performance. As a guide in making appropriate interpretations we have had parallel analyses of performance in Hydraulic and Volumetric controls.

We now undertake the fourth and last step in this investigation: namely, that of deducing conditions and events within the natural reservoir of the present control. Natural reservoirs of oil, gas, and water are, we shall say, physically inaccessible to us except at their orifices, but we shall say further that they are rendered otherwise accessible by mathematical analogy and deduction.

Our capillary tubes possess certain primary functions of performance. Accompanying these are certain secondary functions—the conditions and events within the tubes. These are visible through the transparent walls. The two sets of functions are inseparable; where one is found, there must be found the other. What can be more logical than to assume that the same two sets of functions are inseparable in natural reservoirs? We know that the primary functions are to be found among them; we know that capillary channels exist in their porous formations, and that these channels must contain globules of liquid and bubbles of gas, at least where these two fluids are produced from wells which penetrate the formations. We know further that

where we find these primary functions, there indeed do indications point to a limited drainage radius for the wells, as well as to other phenomena that are yet to be cited. For my own part I cannot refute the assumption. I confess that I have continually attempted to do so, but—if I may use words of Pierre Simon Laplace—"each difficulty which has arisen has become for it a new subject of triumph, a circumstance which is the surest characteristic of the true system of Nature."[1]

*Millions of bubbles of gas, as countless as the sands of the seashore, separate globules of oil or water throughout the porous and permeable reservoir formation. The fluids are thus securely locked in place within the formation, and they cannot move until we relieve the pressure at some point by drilling in a well. The pressure within the reservoir cannot subdue them; they dictate the paths of decline.* These paths define Capillary Control, and by knowing them we recognize conditions and events beyond our vision.

**171.** *The ideal natural reservoir.*—Let us be certain, now, that we understand the circumstances under which one or the other of the two finite controls exists in natural reservoirs. To repeat what has already been said, at least in part, if the hydrostatic pressure within the reservoir, that is, the pressure due to the weight of a column of liquid in the productive formation, is sufficiently great to overpower the resistant action of the globules and bubbles—that which we have called Jamin action—the reservoir is in Volumetric Control. When we say that this pressure overpowers the action, we do not infer that such action is entirely absent. It is indeed present wherever a porous formation contains an intimate mixture of liquid and gas, the latter existing in the dissolved and free states. We mean to say only that the action does not dictate the laws of delivery, as we might know by observing relations between primary functions during the process of production. In Volumetric Control Jamin action is subordinate. Its effects under these circumstances have been fully described in sections 97 and 99.[2] On the other hand, if the hydrostatic pressure—provided there is such pressure present—is not sufficiently great to overpower the resistant action of the globules and bubbles, the reservoir is in Capillary Control. It is necessary to say "provided there is such a pressure present," for it is not essential that the pressure be present. The formation, in so far as it itself, or its porous and permeable part, extends laterally, may be quite horizontal; and obviously, if this is so, there can exist no hydrostatic pressure within it, if we ignore, as we must, the difference in elevation between the bottom and top of the formation. Where a hydrostatic pressure is absent, the natural reservoir containing globules and bubbles is in Capillary Control. In any case Jamin action dictates

[1] Laplace, *Mechanique Céleste,* Paris, 1799. His statement referred to the law of gravitation.

[2] We have learned since reading those sections that the same principles hold with respect to Hydraulic and Volumetric controls.

the laws of delivery, as we might know by observing relations between primary functions during the process of production. In Capillary Control Jamin action is predominant.

In Volumetric Control we encounter only the kinetic effects of Jamin action, whereas in Capillary Control we encounter both its kinetic and static effects.[3]

We must remember that whether a natural reservoir is in one or the other control is not dependent alone upon the pressure. We cannot, for example, classify oil and gas fields as to their control solely upon the basis of initial or subsequent pressures within them. In fact we can say from experience that there are natural reservoirs in Volumetric Control wherein the pressure is less than within others in Capillary Control.

We can list the factors upon which the control does depend. These are as follows:

*a*) The value of the static pressure within the reservoir.

*b*) The value of the surface tension of the liquid in contact with the free gas, and the value of the force of adhesion between the liquid and the solid material of the formation.

*c*) The texture of the formation—the porosity; that is, the size and form of the pores, the number of pores per unit of length, or per unit of space, within the formation, and the diameter and form of the capillary canals which connect the pores. As to the form of the canals we are particularly interested in the contour of the connections between them and the pores, at least at that end of the canals which is closer to the well.

*d*) The solubility of the gas in the liquid at standard pressure and temperature, and the degree to which the liquid is saturated with gas.

It is essential to specify the static pressure because of the fact that the proportional amount of all gas present which happens to be in the free state depends upon this pressure in accordance with Henry's Law. Upon the surface tension, the force of adhesion, and the texture of the formation depends the limiting value of the force $f$ which one globule can offer as a resistance against the pressure of the reservoir, as explained in section 148. The contour of the connection between the canals and the pores determines largely the relative amounts of sliding, with the globules maintaining their two menisci of positive radii of curvature, and of standing, with the globules possessing one meniscus of positive radius and the other of negative radius of curvature. Upon the number of pores per unit of length within the formation depends the possible number of $f$'s present, likewise per unit of length. Each pore will tend to hold but one bubble, and the number of pores actually holding bubbles, per unit of length, will depend upon the solubility of the gas in the liquid, the degree to which the liquid is saturated, the proportional amount of the gas in the free state, and its degree of expansion in accordance with Boyle's Law.

[3] See § 154.

This is the situation with respect to any natural reservoir in Capillary Control. If we are to develop laws concerning the secondary functions of these reservoirs, it will once more be necessary to define the ideal reservoir in this control. With these laws determined we can subsequently make allowances for the actual reservoir as we know it to be in the field. This knowledge of the actual reservoir is, of course, largely the result of our general and specific studies concerning sedimentary formations.

Specifications for the ideal natural reservoir in general were given in section 22. Items (*a*), (*b*), and (*e*) of that section hold without qualifications, while in place of item (*c*) we shall say that

*a′*) The formation must lie as a horizontal mathematical plane throughout an area described by $R$, the radius of action of the potential pressure $P$. The lay of the formation beyond $R$ is immaterial.

And we must obviously restrict item (*d*) to the statement that

*b′*) The formation shall contain one perfect gas and one perfect liquid.

To these it is necessary to add the following:

*c′*) The formation is a consolidated one. If it is composed of grains of sand, for example, it is a true sandstone, and not merely loose sand that is capable of mass movement with the liquid.[4]

*d′*) In conformity with the present item (*a′*) we shall say that with respect to the formation beyond the radius $R$ it is immaterial whether it outcrops at the surface somewhere or not, whether it is in communication with a porous and permeable formation which does so or not, and whether it is possibly in communication with the surface along a fault or fault zone or not. If it meets any of these conditions affirmatively, we must specify the further condition that no water shall enter the formation from the surface.[5]

*e′*) The preceding factors, upon which the control depends, are satisfied in a manner which prescribes the existence of Capillary Control.

The size of this "potential system" is enormously great in comparison with the necessarily accompanying "accessory system" composed of the well itself, with its casing or tubing. Thus the performance of this space within the well proper, as a reservoir in Volumetric Control, does not vitiate observations upon the performance of the natural reservoir to the slightest appreciable extent.[6]

The ideal well in this control is simply one which produces fluid from an

[4] I mean to include here both vibratory and migratory movements of the grains of sand. Laboratory experiments with porous-filled reservoirs indicate the possibility that item (*c′*) might necessarily be placed in the preceding list of factors, upon which the control depends. For the present we shall be content to place it here.

[5] In this we are assured of the fact that there will not be a conversion from Capillary to Volumetric Control. (See § 188 and Appendix J.)

[6] We recall the care taken in our experiments of §§ 151 and 152 to avoid interference between Volumetric Control in the accessory system and Capillary Control in the potential system. There the dimensions of the two systems were of the same order.

ideal reservoir, and at the same time it is one capable of ideal performance; that is, one in which the three cases in theoretic performance are inoperative.

172. *Tubular versus radial systems.*—The reservoir systems that were analyzed in the preceding chapters are tubular ones; that is, flow takes place within them in one linear direction. Natural reservoir systems are radial. Flow takes place within them in all possible directions about the well as a center.

We found these two sorts of systems to exist in the other controls, and now we encounter them here. *In any of the controls the primary function equations and curves are not dependent upon whether the system is tubular or radial, though we must admit that the secondary function equations and curves are quite dependent upon it.* Let us investigate the principal difference between these systems. To do this we shall consider Figures 175 and 176.

In Figure 175 we have a system composed of several Jamin capillary tubes arranged radially about a central chamber. The tubes are identical in

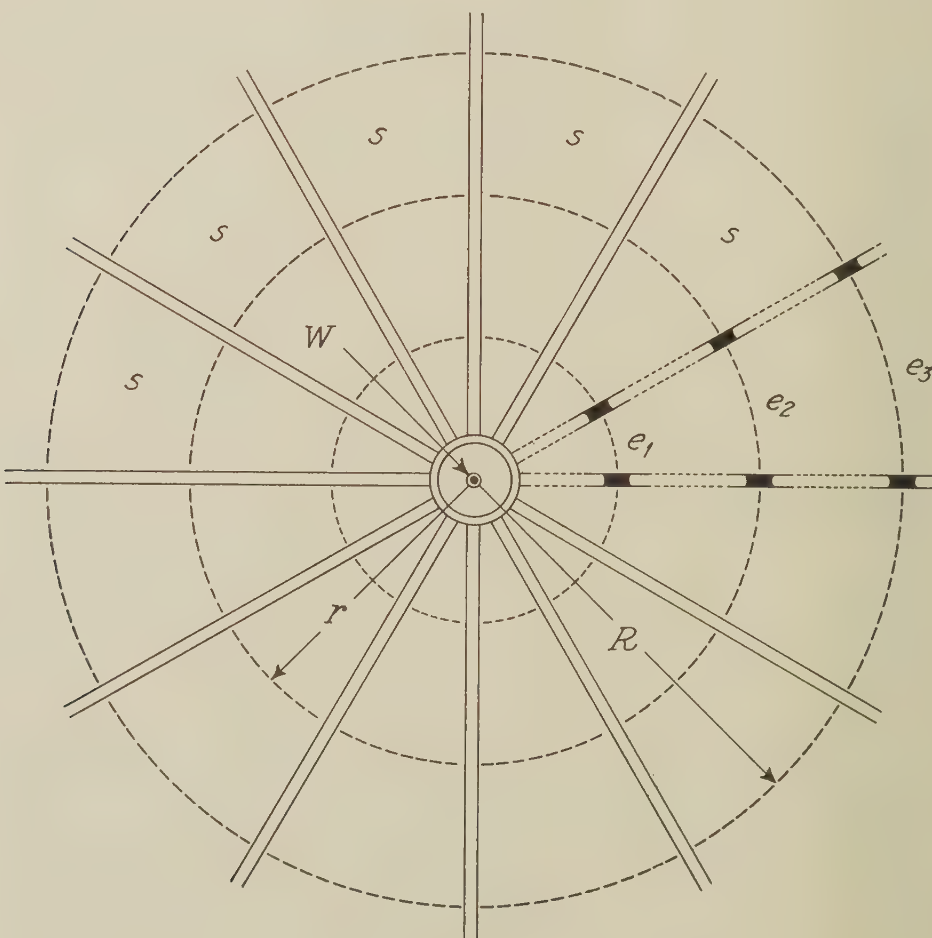



Fig. 175

all respects; they possess like fluids, their capillary bores have the same diameter, the numbers of globules and bubbles per unit of their lengths are the same, and the intensity of the pressure, before production at $W$, is uniformly the same in each. The initial pressure, we shall say, is $P$, to which there corresponds the radius $R$. Because of decline $P$ eventually reduces in value to $p$, to which there corresponds a radius $r$ of a value such that

$$\frac{r}{R}=\frac{p}{P} \qquad (555)$$

in accordance with the relation we have previously found to exist between these functions. It is easily seen that the circle described by $R$, and any such circle as that described by $r$, encounters a number of globules equal to the number of tubes in the system. The latter number is constant; therefore the circles encounter a constant number of globules, regardless of the magnitude of $r$ in comparison with $R$, and regardless of the magnitude of $p$ in comparison with $P$—that is, regardless of decline during the process of production. Evidently we deal here with a system which is no less tubular than that shown in Figure 59.

Now Figure 176 (p. 500) represents a radial system. We see at once a difference between this and the preceding one. We shall say that this system is perfectly homogeneous in all directions, with regard to texture, fluids, and pressure, as before. The number of globules, or the number of pores, encountered by the circles with $R$ and $r$ as radii depends upon the length of these radii. Such spaces as $s$, $s$, $s$, and so on, in Figure 175 are void with respect to the reservoir system, but they are not so in the present system. Let $e$ be the length of arc subtending a fixed angle at the center; then we see that in the former figure the number of globules encountered by $e$ is the same regardless of the position, and therefore the length, of $e$, whereas in the latter figure the number of globules or pores varies directly as the position of $e$ from the center, and therefore the length of $e$, increases. In fact we may write this condition in the following form:

$$e=kR \qquad (556)$$

or

$$e=kr \qquad (557)$$

depending upon the radius in which we are interested. In these equations $e$ is the length of the arc and $k$ is a constant for a given radial system wherein the subtended angle at the center is fixed. As between $e$ and the number of globules and pores it includes, we may write

$$n=k'e \qquad (558)$$

where $n$ is this number and $k'$ is a constant that depends upon the texture of the formation.

It is clear, I believe, that Equation 555 is not correct as applied to a radial system. If $r$ is two-thirds of $R$, $p$ cannot be two-thirds of $P$. Perhaps this

can be seen more readily if we imagine the systems illustrated in the two figures to be in equilibrium throughout at atmospheric pressure and filled with globules and bubbles at a higher pressure. In Figure 175 the radius will vary directly as the pressure, as we already know, but in Figure 176 the radius

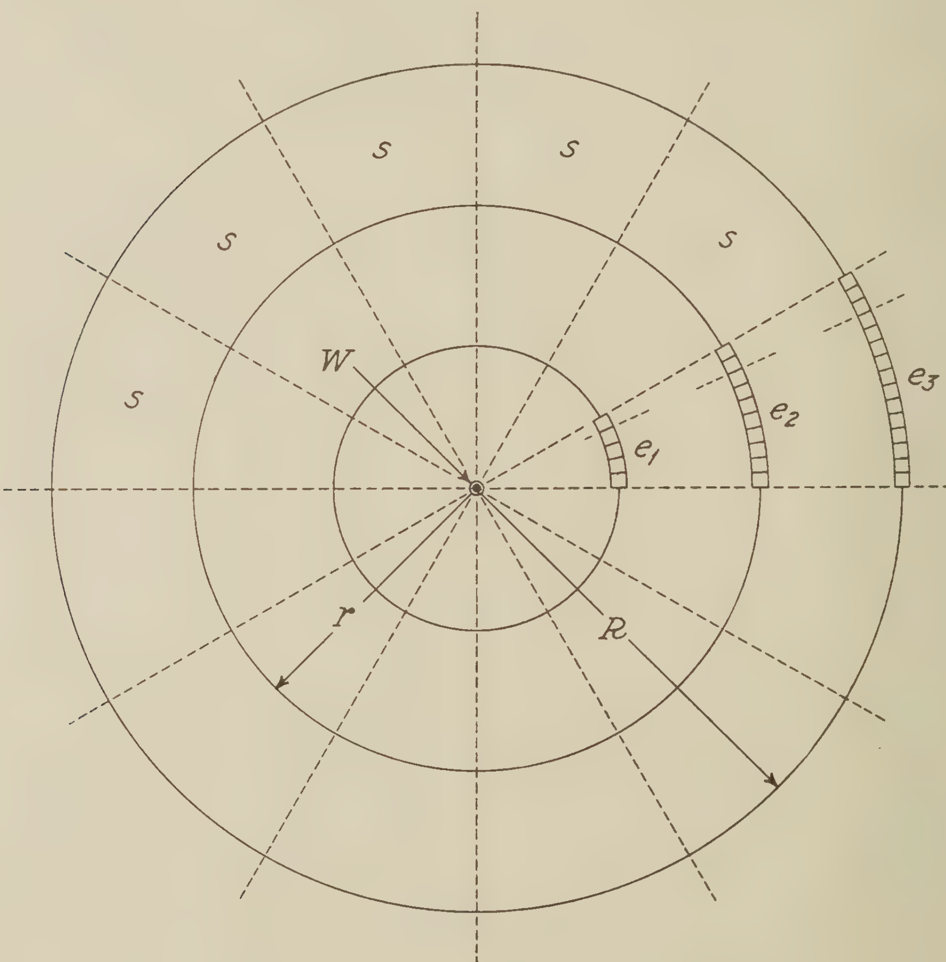



Fig. 176

cannot keep pace with the pressure, while the latter is increased, for the reason that as the number of resistant forces $f$ along any radius is increased, that along the corresponding circumference is also increased. The former, which offset the pressure by their acting in series, are those which determine the radius, while the latter by acting in parallel do not intensify the total resistance to the pressure merely in virtue of their increasing number.[7] Or

[7] The $f$'s in series are cumulative in their effect upon the potential pressure, while those in parallel are not so. We must remember that $P$, or $p$, and the $f$'s are intensive, and not extensive factors.

again, the increasing number of pores brought into play by the increase of the radius takes an increasing amount of the fluids without a directly corresponding increase in the number of $f$'s to offset the pressure. Since Equation 555 will not serve in the present system, we shall determine a proper one. Before we do this, let us consider certain other circumstances that are encountered in a natural system of this control.

173. *Pressure, volume, and energy cones.*—Figure 177 represents a pressure diagram for a natural reservoir system in Capillary Control. It is a section in profile through the well $W$. The formation at depth is perfectly homogeneous in texture in all directions, and it is of uniform thickness

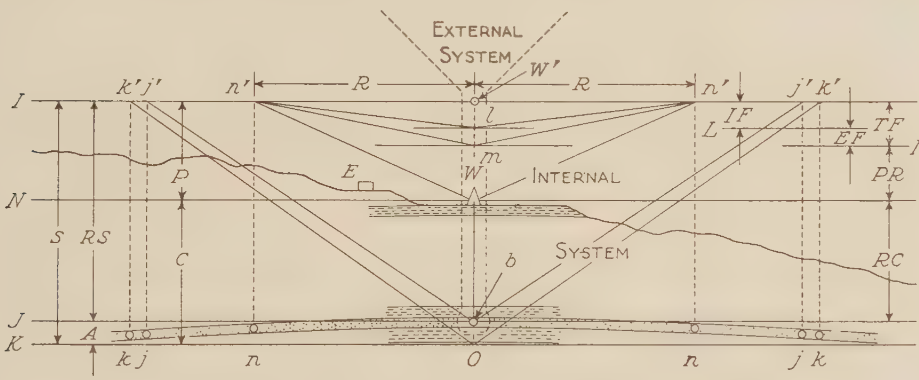



FIG. 177

throughout the portion shown in the drawing. All details comply with our definition of an ideal reservoir except one: namely, the formation is represented as having a slight dip supposedly equal in all radial directions from $W$. This domal structure is assumed merely for the purpose of approaching more closely the actual conditions that are known to exist frequently in the field. The dip is so slight that we may safely ignore any of its effects upon the pressure within the reservoir. It is easily seen that this diagram replaces that of Figure 169, for certainly there are differences between the tubular and radial systems to be accounted for.

We need not be concerned with the lay of the formations beyond the limits of the drawing. It may be that the productive formation outcrops at the surface, as in Figure 47, or it may be "pinched out," or perhaps cut off by a fault. In any case there are no circumstances which interfere with the fact that the system is in Capillary Control.

The major lines, $K$, $J$, $N$, and $I$, with the minor ones, $M$ and $L$, are shown in the usual manner. These define the ten pressures as before. In the light of our description of Figure 169 it can be seen at once that the potential pressure triangle $W'Wn'$, by revolution about the vertical line through $W$, defines a *potential pressure cone,* the base of which is a circle of radius $R$, and the

altitude of which is $P$. Within this cone there are others of smaller altitudes, although of the same base. These might be named in accordance with the pressure head that determines their altitudes.

Each pore within the productive formation is a small reservoir in Volumetric Control; consequently, in accordance with Boyle's Law in potential phase, there is a *potential volume cone* that exists in company with the potential pressure cone. Its base is also a circle of radius $R$, and its altitude, we shall say, is $h_{Vo}$. Of course this is entirely in agreement with Figure 140.

The potential pressure and potential volume cones may be combined. Thus we learn of the *potential energy cone*. Its base is a circle of the same radius $R$.[8] What shall we say its altitude is? For the sake of uniformity among our symbols, let us prefer to say that the altitude of the potential pressure cone is $h_P$. This more clearly corresponds to $h_{Vo}$, the altitude of the potential volume cone. Inasmuch as we agree that potential energy is the product of potential pressure and potential volume, we may conveniently, and consistently, adopt an altitude for the potential energy cone equal to the product of the altitudes of the potential pressure and potential volume cones. Thus

$$h_E = h_P \times h_{Vo}$$

Returning to the consideration of pressures, we see that, if the constant back pressure $C$ is reduced to the atmospheric pressure $A$ for production into the atmosphere at the point $b$ in Figure 177, the radius of the cones is increased. The point $n'$ shifts to $j'$, and $R$, we shall say, takes a value of $R_{RS}$ to correspond with the increase in $P$ to $RS$. Again, if $C$ is reduced to zero for production into a perfect vacuum maintained at the point $b$, the point $n'$ shifts to $k'$, and $R$ takes a value of $R_S$ to correspond with the increase in $P$ to $S$. The situation is quite analogous to the one we encountered in the system of Figure 169, except for the fact that the inclined lines $Wn'$, $bj'$, and $Ok'$ are not parallel, as we shall shortly see.

$R_{RS}$ and $R_S$ define a *registered static pressure cone* and a *static pressure cone*, respectively. They may be said to exist in any natural reservoir of this control at all times, regardless of the fact that $C$ may be greater than $A$, or greater than zero, for the same reason that the pressures themselves may be said to exist always, regardless of the same fact. Corresponding to these

[8] We have agreed that gas is the sole source of energy in this control. There may or may not be a hydrostatic pressure existing beyond the limits of $R$. If there is such a pressure, it is neutralized entirely by the globules and bubbles exterior to $R$, and it therefore has no influence on the system defined by $R$. (See § 154, first three paragraphs.) There is always just sufficient gas to produce the amount of liquid that is defined by the area subtended by the velocity-time curve. (See § 162.) If we should imagine the same system to be provided with more energy, or less energy, then $R$ would be larger or smaller accordingly. The actual amount of energy being what it is, the value of $R$ is what it is.

larger cones there are volume and energy cones. In all we have ten pressure cones, ten volume cones, and ten energy cones in this system.[9]

We shall now proceed to determine the relation between $P$ and $R$ in the radial system.

As a matter of definition it has already been said that the altitude of the potential pressure cone is determined by the value of $P$. This is in agreement certainly with the shifting of the line $N$ in Figure 177. Thus

$$h_P = k_1 P \quad \dots\dots\dots\dots\dots\dots\dots\dots (559)$$

where $h_P$ represents the altitude of this cone, and $k_1$ is a constant, in reality depending upon the value of the resistant force $f$, although introduced here merely for the purpose of writing the expression in the form of an equation.

In agreement with the fact that the individual pores within the formation, including those in direct communication with the well at $b$, perform in Volumetric Control, and in agreement with the relations between pressure and volume in these individual pores, we may write

$$h_{Vo} = k_2 h_P \quad \dots\dots\dots\dots\dots\dots\dots\dots (560)$$

where $h_{Vo}$ is the altitude of the potential volume cone, and $k_2$ is a constant, in reality depending upon the porosity and thickness of the formation, though introduced here for the same purpose as $k_1$.

The volume of any cone is equal to one-third the altitude multiplied by the area of the base, and the volume of the so-called potential volume cone actually represents the potential volume of fluid to be produced from the reservoir under the given conditions of production; that is, under the given conditions in $S$, $C$, and therefore $P$. Concerning this fact we shall have more to say in the next section; for the present I suggest that we adopt it as true, and so write

$$Vo = k_3 h_{Vo} R^2 \quad \dots\dots\dots\dots\dots\dots\dots (561)$$

$Vo$ is the primary function of potential volume; $k_3$ depends upon $k_2$, and includes $\pi/3$; and $R$ is the radius of this cone, as we learned above.

Now production from this reservoir at large is in Capillary Control.[10] We know that we may therefore write

$$P^2 = KVo \quad \dots\dots\dots\dots\dots\dots\dots\dots (562)$$

[9] The order of our lines $K$, $J$, $N$, $I$, $L$, and $M$ in the diagram is purely an arbitrary matter; consequently in any case we can speak of cones the bases of which lie on $I$. As a matter of fact the cones need not have plane bases on $I$. For example, it is immaterial whether we generate a cone with the triangle $mWn'$ in its present position, or first shift $mW$ so that $m$ coincides with $W'$ and then generate a cone. In both instances the geometrical solids so generated are equal in their volumes.

[10] It is of course necessary to distinguish between production from individual pores (Volumetric Control) and production from all pores taken as a group that constitutes the reservoir at large (Capillary Control).

the usual expression for the relation between potential pressure and potential volume in this control.

With these four equations we can solve our problem. To simplify matters we shall run all constants together, designate them as $k$, and pay no attention to their successive values. By substituting the value of $Vo$ in Equation 561 into Equation 562 we have

$$P^2 = kh_{Vo}R^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (563)$$

Next, by substituting the value of $h_{Vo}$ in Equation 560 into Equation 563 we obtain

$$P^2 = kh_P R^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (564)$$

Now by substituting the value of $h_P$ in Equation 559 into Equation 564 we have

$$P^2 = kPR^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (565)$$

This, when divided by $P$, becomes $P = kR^2$, or, as we shall prefer to write hereafter,

$$P = KR^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (566)$$

This is the relation between the potential pressure and its radius of action in the radial system. *The square of the radius depends upon the value of* P, *or the radius depends upon the square root of the value of* P.[11] To double the radius in a process of filling, $P$ must attain four times the value; to treble the radius, $P$ must attain nine times the value; and so on. In a process of production the radius is one-half when the pressure is one-fourth; the radius is one-third when the pressure is one-ninth; and so on.

Inasmuch as the area of the base of the cone depends upon the square of the radius, it is evident that the area of influence of the potential pressure $P$ depends directly upon the value of $P$. This we see from Equation 566.

In place of Equation 555 (p. 499), we must now write

$$\frac{r}{R} = \left(\frac{p}{P}\right)^{1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (567)$$

where the symbols with the significance there ascribed to them pertain to the radial system. In accordance with this equation the inclined lines $Wn'$, $bj'$, and $Ok'$ in Figure 177 have been drawn.

174. *Determining factors of the cones.*—If we admit that the porous formation with its globules of liquid and bubbles of gas behaves in its primary functions as a bundle of Jamin capillary tubes radiating in all lateral directions from the well—and this we clearly must do if we would account for the identity which exists among the primary functions of the formation and the tubes, as we know the latter in the laboratory—then we must further admit

[11] Perhaps this parabolic variation between $P$ and $R$ could have been surmised at the time Figs. 175 and 176 were compared.

that there is in profile section a volume triangle in the case of the formation, as well as in the case of the tubes. We say that this triangle which pertains to the formation generates a cone upon its revolution about a vertical line through the well. The base of the cone so generated is a perfect circle. This is proper when, and only when, the reservoir—the formation with its fluids—is perfectly homogeneous in all directions radiating from the well. But what if the actual reservoir that we encounter in the field is not perfectly homogeneous? *The base of the cone continues to be a closed area;* R *simply possesses different values along the various radial lines; the area is not circular, but irregular, and the amount of irregularity depends upon the heterogeneity that exists within the formation.*

The volume cone is geometrically perfect, then, in the ideal reservoir system described in section 171. There we said that such a reservoir possesses a formation which is homogeneous in all directions, meaning the vertical as well as all horizontal directions. I have ventured to say before that the conditions for the deposition of sedimentary formations are such as to provide a high degree of homogeneity, at least over areas of considerable size, in all directions paralleling the bedding-planes. That we often find heterogeneous strata from the top to the bottom of the formation is readily admitted. Our ideal reservoir therefore differs from the actual reservoir mainly with respect to homogeneity perpendicular to the bedding-planes of the formation. We shall proceed on the basis of the ideal, and subsequently we shall consider the effects of this vertical heterogeneity.

The volume of the so-called volume cone is equal to one-third the altitude multiplied by the area of the base, as stated above, and this volume, we say, actually represents the potential volume of fluid to be produced from the reservoir under the given conditions of production. This is in perfect accord with the mechanics of Jamin capillary tubes in general. We recall the fact that in section 155 we found the potential volume of a reservoir in this control, at any instant of its life, to be equal to one-fourth the potential velocity at the instant, multiplied by the time in life remaining. Now if we consider the initial instant of the life of the reservoir, that is, the instant when $Vo$ represents the entire potential volume to be produced from the reservoir under the given conditions, and when the radius of action of $P$ is $R$ and life is $L$, we may write the following equation:

$$\frac{1}{3}\pi R^2 h_{Vo} = \frac{1}{4} VeL \qquad (568)$$

where $h_{Vo}$ is the altitude of the potential volume cone. Both members of this equation represent potential volume. By rearranging the terms we obtain

$$R = \sqrt{\frac{3VeL}{4\pi h_{Vo}}} \qquad (569)$$

and

$$h_{Vo} = \frac{3VeL}{4\pi R^2} \qquad (570)$$

If the values of the terms on the right are known, those of the terms on the left become known. We shall see that $R$ and $h_{Vo}$ can be measured in the field. Such measurements must agree in accordance with these equations.

Let us see that we understand correctly the position of our potential volume cone with respect to, say, the absolute volume and the mobile volume of fluid within the reservoir. Figure 178 shows a cylinder of radius $R$. The horizontal lines $K$, $J$, $N$, and $I$ indicate at the left the pressures with which we are concerned, particularly $P$ and $C$, and these lines, when carried through to the right, may be used to indicate various portions of fluid contained in the reservoir. The entire cylinder represents the absolute volume of fluid within the potential radius $R$. Its altitude is, we shall say,

$$h_{Vs} = h_{Vo} + h_{Vc} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (571)$$

where the subscripts imply volumes that are related to the static pressure, the potential pressure, and the constant back pressure, respectively. Between $N$ and $I$ we have the potential volume cone, with its altitude $h_{Vo}$, inclosed in a cylinder of the same base and altitude. The volume of this cylinder is precisely that volume of fluid to which we have referred as the "mobile volume."[12] The volume of the cylinder with the altitude $h_{Vc}$ may rightly be called the "immobile volume," inasmuch as it is impossible for this fluid to be produced from $W$, so long as there is a constant back pressure $C$.[13] *We are to note that these quantities of fluid which we term the absolute volume, the mobile volume, and the immobile volume refer to a restricted portion of the reservoir described by the potential radius* R.[14] They in no way refer to the reservoir beyond $R$. Furthermore,

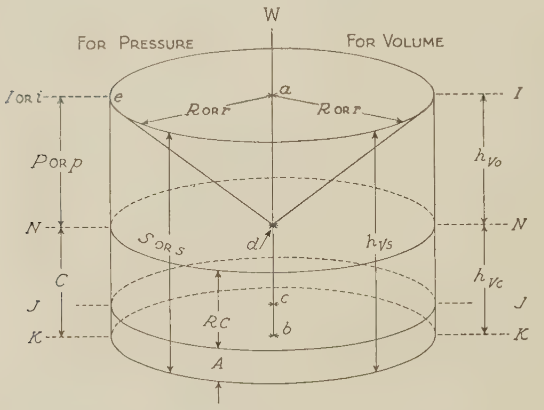



Fig. 178

[12] See § 153.

[13] The term "immobile volume" is in fact the volume *vo* of § 163. Where we have previously called this the "retained volume," it is now better that we give it a new name, inasmuch as the volume of fluid retained by the reservoir really consists of two quantities: namely, the present immobile volume that is retained in virtue of $C$, and the portion of the mobile volume exterior to the potential volume cone that is retained in virtue of Jamin action. The distinction between these retained volumes is now essential.

[14] Fig. 178 is the same as Fig. 66, except for the inclosed cone above the line $N$. It should be particularly noted that $R$ here has a significance quite unlike that in the earlier figure. I believe the present figure clearly indicates the various volumes with which we must hereafter deal. The absolute volume is equal to the sum of the mobile and immobile volumes, and the mobile volume includes the potential volume.

we should observe that the absolute volume is not the same as the volume of the so-called static volume cone, for with the latter the radius $R$ is taken at a larger value $R_S$.

The radius $R$, then, describes the drainage area of the well $W$. *No fluid that lies beyond* R *can either enter the drainage area or be produced from the well.* If $C$ is not zero, $R$ may be increased by decreasing $C$, but if this is done, we need only repeat the statement as applied to a larger drainage area for the well.

The volume of a cone is one-third that of the cylinder with the same base and altitude. The potential volume of fluid is therefore one-third of the mobile volume at the given constant back pressure. To sum up the situation we may say that *"the isolated oil, gas, or water well in Capillary Control produces one-third of the fluid underlying its drainage area, such fluid being mobile at the given constant back pressure which is exerted against production. The quantitative distribution of this produced fluid, irrespective of the thickness of the formation which serves as a reservoir, may be represented by a space diagram in the form of a right circular cone with a base of radius* R."[15]

The altitude $h_{Vs}$ of the cylinder representing the absolute volume of fluid within the radius $R$ is equal to the thickness of the productive formation, multiplied by the factor for porosity. For example, if the thickness is 10 feet, and the porosity is 30 per cent, the value of $h_{Vs}$ is 3 feet. The altitude $h_{Vo}$ of the potential volume cone is the same percentage of $h_{Vs}$ as $P$ is of $S = 100$ per cent. This is in accord with pressure-volume relations in Volumetric Control. As we have already noted on several occasions, whenever we deal with individual pores, we are concerned, not with Capillary Control, but with Volumetric Control. Our capillary tubes are here arranged in a bundle, and the altitudes of the volume cylinders and cone pertain to a vertical column of individual pores, one above the other, the column having a horizontal cross-section equal to the area of an individual pore, as these are located immediately at the well. We may imagine all tubes from the top to the bottom of the formation to be acting independently of one another; consequently, whatever we say of the individual pore applies with equal correctness to the column of pores.

When we say that our porous formation is perfectly homogeneous in all directions we imply the fact that the pores are of the same cross-section on three planes at right angles to each other, and that the canals which connect these pores are of the same dimensions and texture likewise along these three planes. The planes may be so conceived as to define for us three important lines by their intersections. These are as follows:

[15] See Frontispiece. It is necessary to specify the "isolated well" in order to avoid complications which arise by interception among wells. Interception is treated in chapter xxxi. We now understand clearly what is meant by "such fluid being mobile at the given constant back pressure."

*a*) A horizontal line extending outward from the well; that is, a radial line.

*b*) A horizontal line perpendicular to that of (*a*) at some point away from the well; that is, a line tangential to a circumference described by the line of (*a*).

*c*) A vertical line perpendicular to those of (*a*) and (*b*) at their point of intersection.

There are canals, we say, along the course of these three lines. Presumably this is true even with the actual porous formation, although we might expect the canals along the line of (*c*) to differ somewhat, at least for various portions along its length, from the canals along the lines of (*a*) and (*b*).[16]

Flow takes place along the line of (*a*). This is no other line than $R$ itself. What function, if any, may canals along the lines of (*b*) and (*c*) perform? Certainly none, if perfect homogeneity exists along all lines of (*a*). But if it should be that this perfect homogeneity does not exist, then the canals along the lines of (*b*) and (*c*) perform the important function of equalizing the pressure among the pores which lie on the outer surfaces of cylinders which we may imagine to exist within the distance $R$, such cylinders being concentric at the well $W$. They permit the passage of fluid tangentially or vertically in case of obstructions along the radial lines. *They therefore tend to nullify any effects of heterogeneity within the formation, and allow the reservoir within it to perform as though the formation itself were more nearly homogeneous than it actually is.*

We have provided for the measurement of the altitude of the potential volume cone. I believe that in general we should not rely upon this measurement for the determination of the radius $R$ by means of Equation 569, above. $R$ will ordinarily be desired with greater accuracy than is possible by such a computation. The value we determine for $h_{V_0}$ may be unsatisfactory, for the reason that we cannot always be certain of our measurements of the thickness and porosity of our productive formations. We may judge wrongly concerning the total thickness of formation that actually comes into play in the process of production; strata which appear to be very compact may really contribute to production.[17] Furthermore, we may feel, perhaps with some degree of certainty, that although we are satisfied with the measurements at the well, these values do not represent average measurements throughout the drainage area.

We must determine some method of measuring $R$ directly. This matter we shall be able to consider at a later time, when we come to investigate the

[16] This is because of the heterogeneous character of the various strata constituting the productive formation.

[17] In natural reservoirs of this control it is possible that fluids in the more compact strata issue directly into those of a more open texture and thereafter proceed to the well. Compare with the analogous situation in Hydraulic and Volumetric controls, § 141.

principles of the forced drive.[18] Meanwhile it will be sufficient for us to know the factors upon which $R$ depends in a natural reservoir. These are as follows:

*a′*) The value of the potential pressure, which we know to be the value of the static pressure within the reservoir, minus the value of the constant back pressure against production.

*b′*) The value of the surface tension of the liquid in contact with the free gas, and the value of the force of adhesion between the liquid and the solid material of the formation.

*c′*) The texture of the formation—the porosity; in particular, the size and form of the pores, the number of pores per unit of length, or per unit of space, within the formation, and the diameter and form of the capillary canals which connect the pores. As to the form of the canals we are particularly interested in the contour of the connections between them and the pores, at least at that end of the canals which is closer to the well.

*d′*) The solubility of the gas in the liquid at standard pressure and temperature, and the degree to which the liquid is saturated with the gas.

Except for item (*a′*) these are identically the same as the factors upon which the existence of Capillary Control depends.[19] In neither list must we consider the thickness of the formation as a factor.[20] Upon the surface tension, the force of adhesion, and the texture of the formation depends the limiting value of the force $f$ which one globule can offer as a resistance against the pressure of the reservoir, as explained in section 148.

Inasmuch as the size of the potential pressure cone depends upon the altitude and base of the cone, we may say that its size depends upon the following factors:

*a″*) The value of the potential pressure, as the determining factor of the altitude.

*b″*) The value of $R$ in accordance with the foregoing list of items. In these, as we know, the potential pressure appears as a determining factor of the radius of the base.

We say that the potential volume of a reservoir in this control depends upon the size of the potential volume cone; but upon what factors does the size of the cone depend? These are as follows:

*a‴*) The size of the potential pressure cone.

*b‴*) The thickness of the formation and its factor for porosity.[21]

[18] See § 189.

[19] See § 171.

[20] $R$, we say, is dependent upon an intensive factor $P$ in any given reservoir with its fluids. It is immaterial how many pores there may be in the vertical column described above, for any number act in parallel, and not in series.

[21] Volume is extensive; therefore we are here concerned with the number of pores in the vertical column described above.

Obviously the size of the potential energy cone depends upon the size of the potential pressure and the potential volume cones.

If we understand the dependency of the cones upon the various factors, and the interdependency of the cones themselves, we are prepared to investigate the performance of the cones during the ideal and theoretic performances of the reservoir.

CHAPTER XXX

# Secondary Functions of Performance *(Continued)*

"This book is written in the mathematical language, and the symbols are triangles, circles, and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth."—GALILEO

**175.** *Volume differential cylinder.*—Let us go to the well shown in the diagram of Figure 177 two times in its history, descend by way of the casing with an aneroid barometer in hand, and explore throughout the reservoir in all directions as far as we like, for the purpose of learning what conditions exist therein.[1] Our first visit takes place at the moment the well is drilled in. The well is closed at $W$, the casing stands full of oil from $b$ to $W$, and no oil has been allowed to leave the well at $W$. Having set our barometer to read zero on the derrick floor we step into the casing. What pressure will the barometer now register? If we can imagine the casing to be extended upward indefinitely, we might open the valve at the derrick to find the oil rising to a point $W'$. This valve, when closed, must exert a back pressure equal to the weight of the column of oil from $W$ *to* $W'$; therefore on arrival inside the casing our barometer must indicate a pressure equal to the weight of this column; that is, a pressure equal to $P$.

We descend. Certainly the indicated pressure must increase, for the weight of the superincumbent column of oil increases in proportion to the depth. When we arrive at $b$ the barometer indicates a pressure $RS$, since $RC$ is the weight of the column between $W$ and $b$, and $P + RC = RS$.

We walk out from $b$ in many directions, and we find that aside from a slight increase due to the fact that we descend on the dip of the formation our barometer indicates no change in the pressure. We may conveniently ignore the effects of the dip, and proceed on the basis of a formation that lies perfectly horizontal. No matter how far we go in our exploration—presumably at least to some point beyond $k$—all is perfectly normal and invariable. There is no sign of anything we might call a pressure or volume cone; in fact, if we were to give a geometrical interpretation of pressure as indicated by

[1] Our investigations from here onward are continued purely in the light of mathematical deduction. If forces "are exerted in the interior of the earth at depths which will always be inaccessible, mathematical analysis can yet lay hold of the laws of these phenomena. It makes them present and measurable . . . ."—Joseph Fourier.

the barometer and volume as we see it, we would do so by constructing cylinders of uniform altitude and bases of undefined radii. Gas and oil are uniformly distributed throughout the reservoir; each pore of the formation tends to hold but one bubble, and the two menisci of the globules that fill the capillary canals connecting the pores have equal radii of curvature. It is true that there is more oil present than just the amount necessary to fill the canals; in fact we find the excess oil resting within the pores.[2]

Our second visit takes place at the time when the well by production against the unaltered constant back pressure $C$ has reached equilibrium. The casing remains filled with oil, the valve at the derrick is open, yet no more oil flows from the well. We descend again, having noted that our barometer still reads zero on the derrick floor. When we arrive at $b$ the barometer indicates a pressure $RC$, for, inasmuch as $P$ has become zero, $RS$ is reduced to $RC$, equal to the weight of the column of oil in the casing.

Again we walk out from $b$ in many directions. In any direction we take we find that the barometer indicates an increase in pressure directly proportional to the distance traveled, and we find this increase the same in all directions. At the point $n$, and at all points beyond, the indicated pressure is $RS$, that which we found everywhere on the first visit. Now if we were to give a geometrical interpretation of the pressure, we would do so by removing a right circular cone from the former cylinder, a cone with its axis vertical, and with this axis passing through the center of the casing, with its altitude equal to $P$, and with its radius of base $R$ equal to the distance from $b$ to $n$.

In regard to the volume of fluid we see bubbles of gas near $b$ that are now larger than before; but as we go outward these become smaller and smaller, until at $n$, and at all points beyond, they are of the same size as we found them on the first visit. Where the gas occupies greater space we naturally find less oil remaining within the formation. The decrease in the size of the bubbles agrees, of course, with the increase of pressure. This is in accord with pressure-volume relations in Volumetric Control, the control in which the pores, as small hollow reservoirs, individually perform.[3] In giving a geometrical interpretation of the circumstances in which we find volume two methods of construction are open to us.[4] If we wish to represent the actual distribution of the fluids as we see them on this second visit, we do so as in Figure 179 ($a$). White at the well indicates the removal of 100 per cent,

[2] If it were not for this excess there is little probability that the casing would be filled with oil in the manner prescribed.

[3] These pressure-volume relations take into account the passage of the gas into the free state according to Henry's Law, and the subsequent expansion of this gas according to Boyle's Law.

[4] The term "volume" may be interpreted here to mean either the oil withdrawn since the first visit or the increase in space occupied by the gas since the first visit. If the reservoir were a producer of gas only, volume would preferably be interpreted to mean the gas withdrawn since the first visit, as indicated by the aneroid barometer.

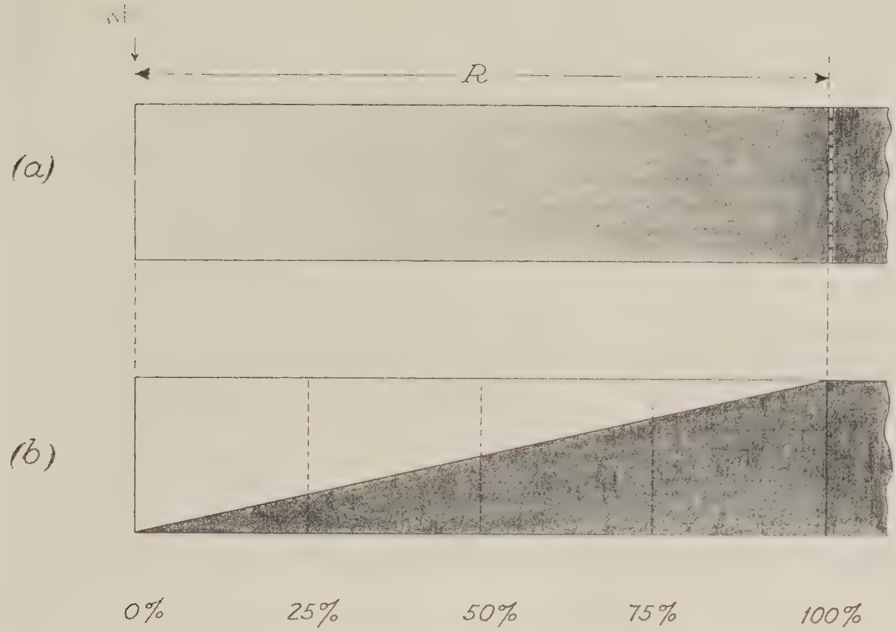



Fig. 179

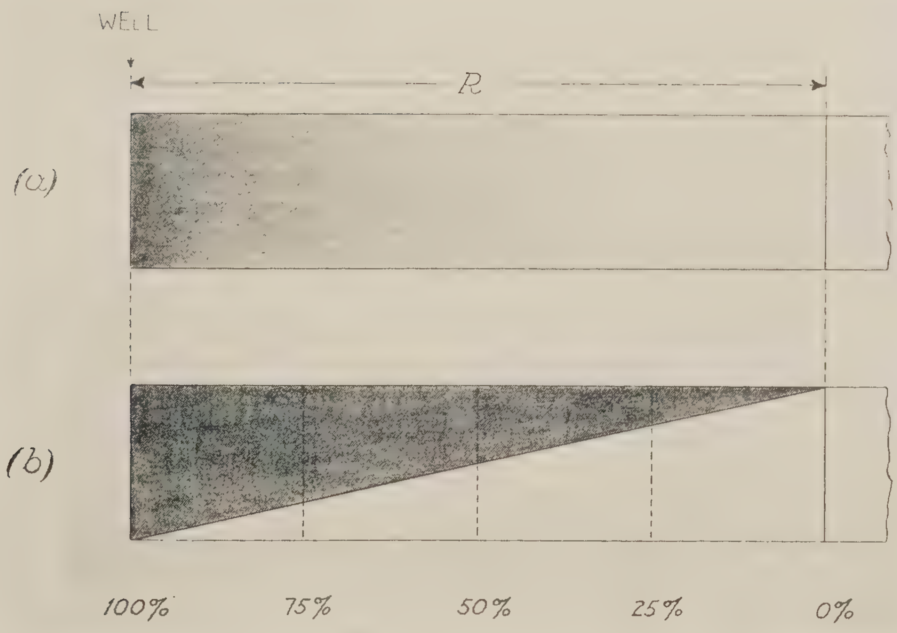



Fig. 180

and therefore a remainder of 0 per cent, of the mobile fluid, while black at the distance $R$ from the well indicates the removal of 0 per cent, and therefore a remainder of 100 per cent, of the mobile fluid.[5] Between these points we have a uniform gradation from white to black. What sort of a geometrical body is this? Outwardly it appears as a cylinder when revolved about the vertical line at the well. Inwardly we see that its density varies from zero at the well to unity at the distance $R$ from the well, the gradation being perfectly uniform. I propose to call this body a *differential cylinder.* In particular this is a "zero-one" differential cylinder. If we desire, we can say that the cylinder found on the first visit is a "one-one" cylinder, thus indicating a uniform density from center to circumference. In both we have strictly correct means of representing the physical distribution of the volume of fluid under the respective circumstances.

If it were within our power to restore the remaining fluid to its former density by pushing it vertically downward to the floor of the formation, this fluid would appear as in Figure 179 ($b$). The white triangle, with its base at the ceiling of the formation, becomes a cone upon revolution about a vertical line through the well. This cone represents the space vacated by the potential volume. Its dimensions agree identically with those of the potential volume cone.

Figure 180 ($a$) represents the distribution of the potential volume within the formation before any fluid has been produced from the well. Black at the well indicates 100 per cent of the mobile volume to be removed, while white at the distance $R$ from the well indicates 0 per cent of the mobile volume to be removed. Between these points we have a uniform gradation between black and white. Upon revolution about the well this geometrical body becomes a "one-zero" differential cylinder. Obviously if from a one-one cylinder a one-zero cylinder is withdrawn, a zero-one cylinder remains behind in the formation.

If it were possible for the well to produce its fluid only from the space contiguous with the ceiling of the formation, the space to be thus vacated would appear as in Figure 180 ($b$). This black triangle upon revolution about the well becomes the potential volume cone.

It should appear from this discussion that our representation of volume by cones is purely a mathematical one. It is indeed strictly quantitative. Such a representation conforms to pressure-volume relations expressed by Boyle's Law in potential phase; therefore so long as we choose to represent pressure conditions by a cone, we may do likewise with volume conditions. As a matter of fact the volume cone and the volume differential cylinder have identical mathematical properties, as we shall observe later. We may suit our convenience, or our imagination, by using either one. Perhaps the cone is simpler to visualize in mathematical operations, while the differential cylinder

[5] See § 174 for the definition of mobile volume.

is simpler to visualize in problems which involve the physical state of the reservoir interior, apart from quantitative considerations.

On the two visits which we have made underground we have encountered extreme conditions. We need not dwell upon the conditions to be found on a later visit, when the well is at equilibrium with $C$ reduced to $A$, for in such a case $P$ is merely again zero after having been restored to a value $RC$, and the point $j$ merely replaces $n$ in a physical and mathematical aspect. If we should do this we should only repeat what has already been said. However, between extremes there is much of interest in regard to events that take place during the process of production. These we now investigate.

176. *Active cones in production.*—Before the well is drilled into the formation that is to serve as a reservoir in Capillary Control the fluid, we say, is securely locked in place by bubbles of gas disseminated throughout the mass of liquid. Neither the gas nor the liquid can move until the pressure is relieved by the opening of the well. Then such fluid that happens to lie within a predetermined radius[6] $R$ becomes active, or mobile, and of this quantity of fluid only one-third actually is produced by the time equilibrium is established at and for the well. No fluid beyond $R$ is rendered mobile by the opening of the well. R *as an active radius exists only for an instant, the initial instant of production from the well.* Immediately thereafter the active radius is shorter than $R$; it is, we shall say, of a value $r$, which, during the entire process of production, possesses all values ranging from $R$ to, and including, zero. The radius $r$ declines in value simply because the potential pressure $P$ declines in its value. If we say that $P$ after the initial instant assumes a declining value $p$, then the relation between the four functions is expressed by Equation 567 (p. 504): namely,

$$\frac{r}{R} = \left(\frac{p}{P}\right)^{1/2} \quad \text{.......................} (572)$$

Now we can write, in accordance with Equation 566 (p. 504),

$$p = Kr^2 \quad \text{.......................} (573)$$

where $K$ is the same constant as that in the equation between $P$ and $R$. Of course, we may also write

$$r = kp^{1/2} \quad \text{.......................} (574)$$

where $k$ is the reciprocal of the square root of $K$. Obviously this equation might be written down as a consequence of the relation in Equation 572.

[6] The radius is predetermined by the four factors ($a'$) to ($d'$) listed in § 174.

By means of Equations 573 and 574 Figure 181 ($a$) and ($b$) are constructed. In ($a$) we have profiles of the pressure cones at ten equal intervals of $r$ between $R$ and zero, and in ($b$) we have profiles of the same cones at ten equal intervals of $p$ between $P$ and zero. All cones with a radius of base $r$ and an altitude $h_p$ in accordance with the value of $p$ are to be called *active pressure cones*. Only the fluid within them is undergoing a change—a decline—in pressure in the process of production. *The space within the reservoir, lying between the active pressure cone and the original pressure cone with radius of base* R *and altitude* P, *is in equilibrium under stress.* This stress is due to the distorted condition of the globules that are separated by the bubbles. The fluid within this space undergoes no further decline in pressure during production. It is continually increasing in size while the active cone is decreasing in its size. When equilibrium is finally established at the well, we have the conditions which we found on our second visit.

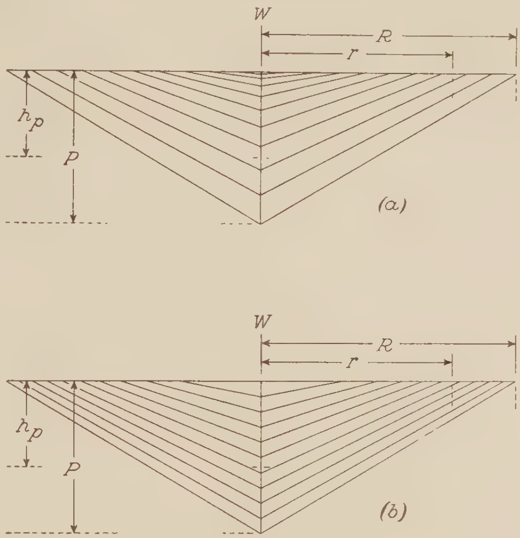



FIG. 181

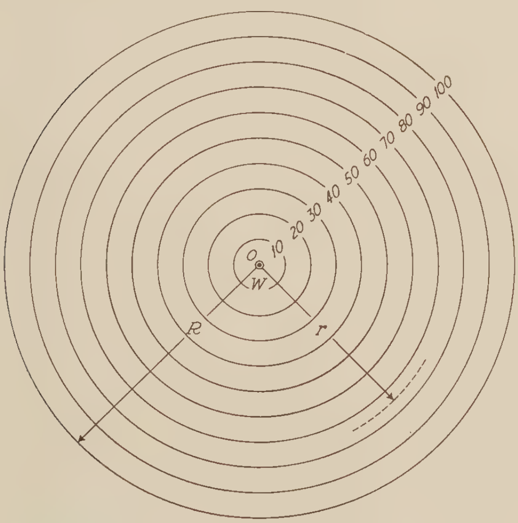



FIG. 182

Corresponding to the active pressure cone we have the *active volume cone* or *active differential cylinder*, and also the *active energy cone*. The radius $r$ serves as the radius of base for all of these, while for volume and energy the altitudes are $h_{vo}$ and $h_e$, respectively.

From our studies of the primary functions we know that $P = KT^2$, and since $p$ is the value of the potential pressure subsequent to $P$ we may say that

$$p = Kt^2 \quad \text{.........................(575)}$$

[ § 176

where $t$ is the value of time remaining subsequent to $T$. With the right-hand members of Equations 573 and 575 we obtain

$$r = kt \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (576)$$

The value of the constant $k$ is determinable from the $K$'s of the original equations. Here we see that *the radius of the active cones varies directly with time remaining.* The bases of the active cones for ten equal intervals of time are shown in Figure 182 (p. 515). It is clearly the horizontal plan of Figure 181 ($a$).

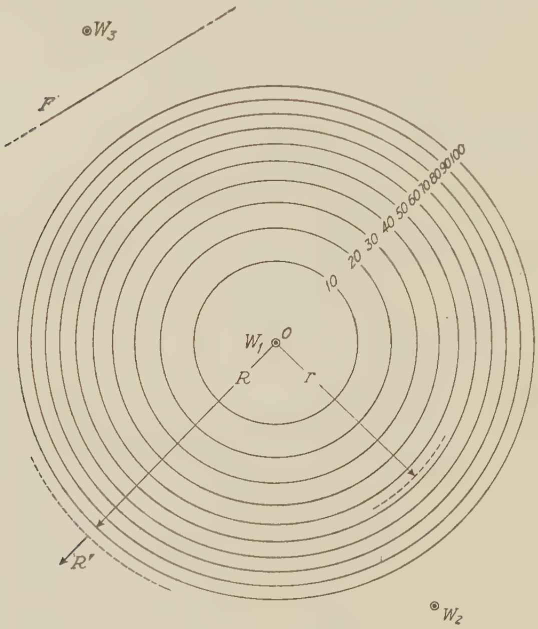



Fig. 183

A corresponding plan for Figure 181 ($b$) is shown in Figure 183.[7] Since Equation 573 may be changed to

$$p = ka \quad \ldots (577)$$

where $a$ is the area of the bases of the active cones, and $k$ is a constant whose value is $K/\pi$, the areas in this figure bear the ratios

$$10 : 9 : 8 : \ldots . 1 : 0$$

to conform to ten equal intervals of pressure. And if $p_1$ and $p_2$ are two of these successive values of $p$, while $a_1$ and $a_2$ are the corresponding values of $a$, we see that in accordance with Equation 577

$$p_1 - p_2 = k(a_1 - a_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (578)$$

The area of the circle designated 10 and all areas of the surrounding rings are equal, inasmuch as the intervals in $p$ are equal.

177. *Primary functions and* r.—We find the radius of the active cones diminishing with time remaining and we know, of course, that all primary functions of performance are declining with time remaining. In virtue of Equation 576 and of the primary function relations in Capillary Control, as given in section 159, we may write down the equations which express the

[7] A few features are included in this drawing for use in sections to follow.

relation between the values of these primary functions subsequent to their initial ones and the radius of the active cones. Thus in all we have

$$p = kr^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (579)$$

$$vo = kr^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (580)$$

$$ve = kr^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (581)$$

$$ac = kr^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (582)$$

$$e = kr^6 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (583)$$

and

$$po = kr^5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (584)$$

The $k$'s are determinable in every equation; in fact we already know that the $k$ in Equation 579 is the same as the $K$ in Equation 573 (p. 514). The others are easily computed by combining this one with the various values of $K$ in the primary function equations.

I have written the symbols above in small letters. Is it not clear that, with the exception of the difference in the meanings of $r$ and $R$, all these might as well be written with initial capital letters in the usual manner? If we hold this in mind we may continue to approach our reservoirs at any instant during life and say: "We here have a reservoir complete in itself. We may not know what this container has accomplished in the past. However, let us begin observations now, and adopt the present conditions as 100 per cent with the assurance that the reservoir will yet make a full sweep through the relative curves."[8] Now we can draw inclined straight lines on the charts of Figures 158 and 174, from 0-0 to 100-100 and from 0-0 to 1,000-1,000 respectively, and entitle them "The Active Radius."[9] If we should know the present value of $r$ and the initial and present values of any one of the primary functions, $R$ may be determined by the proper vertical cut.

This association between $r$ and the primary functions calls our attention to the fact that not only is this radius declining naturally in ideal performance, but also that it is subject to alterations in theoretic performance. In order to approach the study of the latter let us refer to Figure 184 (p. 518). We have again the triangle $W'wn'$ which defines the potential pressure cone. When production begins at $W$ the fluid within the corresponding volume cone or differential cylinder starts toward the well. A point $m$, we shall say, denotes the position of the edge of the active cone at any instant during life. Obvi-

[8] See § 102.

[9] We recall that the curve for $R$ in the tubular system has the equation $R = KT^2$, whereas in the radial system it has the equation $r = kt$. Had we desired to make the distinction between $R$ and $r$ in regard to the tubular system, we would have derived the equation $r = kt^2$, an equation in which the symbols agree with the present ones. The distinction seemed unnecessary in the experiments with the tubes, for we were not particularly interested in the space lying between $R$ and its subsequent values $r$.

ously it begins at $n'$ and ends at $W'$; it is, so to speak, a "running point" during production. Through $m$ we may imagine a vertical line drawn. This line marks the boundary of the active differential cylinder. The boundary, of course, is a "running line," always keeping pace with $m$. But we know $m$ to define in fact a circle; it therefore follows that the boundary line, first at $A$ and subsequently at $B$, defines a *running cylindrical surface*. $B$ runs smoothly in ideal performance, but not so in theoretic performance. We shall now see what happens when alterations are made in accordance with the three cases.

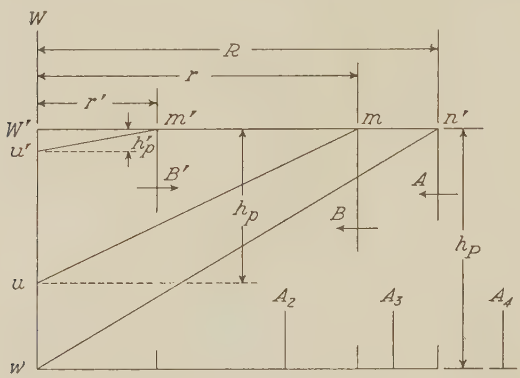



FIG. 184

CASE 1. Let us suppose that when $B$ is in the position shown the external friction head at $W$ is increased. The immediate effect is noted at the well. A small pressure cone with $m'$ and $B'$ originates here, and proceeds to enlarge until $B'$ coincides with $B$. Perhaps $B$ has moved a little in the time it takes $B'$ to reach it. No matter; the important consideration is the fact that the two cones coincide with the loss of the subordinate cone's identity on arrival at its destination. $B$'s velocity of approach is thereafter less in conformity with the decreased rate of production from $W$.

If, on the other hand, the external friction head is decreased when $B$ is in the position shown, there is also a small pressure cone originating at the well, and it enlarges in the same way, losing its identity when $B'$ coincides with $B$.

The two subordinate cones differ in one important respect. The first is a *high pressure cone,* while the second is a *low pressure cone.* These may be compared with the waves which emanate from a point where a pebble is thrown into a pond of smooth water. They are alike in their radial motion,[10] but otherwise they differ. The high pressure cone travels as a crest of the wave, with no trough behind it, and the low pressure cone travels as a trough of the wave, with no crest behind it. With the high pressure cone there is a

[10] Both are circular under ideally homogeneous conditions, and both are traveling outward from the center at a constant lineal velocity. In Equation 576 (p. 516) we have $r = kt$, which by differentiation becomes

$$\frac{dr}{dt} = k$$

*The change in the length of the radius, and therefore the rate of motion for the cylindrical surface defined by the cones, is constant with time.* Of course the velocity of wave motion is a distinct phenomenon, independent of that of the velocity of fluid motion.

"banking" of the fluid at $B'$, a banking which accompanies $B'$ throughout its existence. Fluid exterior to this cylindrical surface possesses momentum which must be diminished.[11] Let us prefer to say that in this instance banking is positive. What shall we say concerning the banking of the fluid at $B'$ with the low pressure cone? To say that there is no banking would not be exactly correct, for by this it would be inferred that banking is simply zero. Indeed it is evident that in this instance banking is negative; that is, less than zero.[12] Fluid exterior to this cylindrical surface possesses momentum which must be increased, and on this increase it actually pulls away from the fluid that is moving in line behind it.

In accordance with Newton's second law a change in momentum means a lineal acceleration in the motion of the fluid. We see, therefore, that *actual banking — positive banking — occurs only when there is a decrease in the momentum of the fluid; only when lineal acceleration produces a decrease in lineal velocity, and only so long as this acceleration of motion lasts.* While fluid is moving normally in accordance with ideal performance, there is neither positive nor negative banking.[13] With theoretic performance there is one or the other.

The size of the active pressure cone, and therefore the size of the active volume and energy cones, is not dependent upon alterations in Case 1. This is in agreement with the fact that the potential functions of pressure, volume, and energy are not altered by changes in the external friction head.

CASE 2. The only way in which an operator can increase the static pressure of the reservoir at an isolated well is to refill it. This he sometimes does for the purpose of storing fluid. At the beginning of this process a subordinate high pressure cone emanates from the well. $B'$ travels toward $B$, and passes it. Whether $A$ is reached or passed depends upon the extent to

[11] By definition the momentum of a unit mass of fluid $m$ having a velocity $v$ is $mv$. The momentum is directed toward the well, opposite to the motion of the cylindrical surface.

[12] If the crest of a wave is taken to be positive, the trough may be taken to be negative, the two representing conditions with respect to a pressure head measured in opposite directions from a datum plane which is defined by the corresponding smooth surface.

[13] This condition may be described as "zero banking." The gas in association with the liquid plays a rôle in banking. The situation can be described briefly as follows:

1. Positive banking accompanies high pressure cones. Gas bubbles at the cylindrical surface are compressed.

2. Zero banking occurs in the absence of high and low pressure cones. Gas bubbles at the cylindrical surface exist under the normal conditions specified by the kinetic pressure gradient. (§ 179.) This gradient is the datum for positive and negative banking at all points outward from $W$. Zero banking defines "steady flow" in connection with Bernoulli's Theorem. (§ 43.)

3. Negative banking accompanies low pressure cones. Gas bubbles at the cylindrical surface are expanded.

Compression and expansion are in accord with Henry's and Boyle's laws.

which refilling takes place. If the original pressure of the reservoir is attained, $B'$, carrying $B$ with it, reaches $A$, but, if this pressure is exceeded, it passes $A$. In this process there is continually a positive banking of the fluid at the cylindrical surface $B'$, for fluid at rest must take up motion, this motion clearly being one directed away from the well.[14]

An operator cannot decrease the static pressure of the reservoir at an isolated well except by way of permitting a natural decline in accordance with production.

I have twice specified an "isolated well" for the purpose of avoiding the matter of interception among wells in reservoirs of this control. The features accompanying interception will be considered in the next chapter. The process of refilling comes up again in connection with the restoration of gas pressure, and also in connection with the forced drive.[15]

Case 3. If at the time $B$ is in the position shown the constant back pressure at $W$ is increased, a subordinate high pressure cone emanates from the well. $B'$ travels toward $B$, reaches it, and upon coincidence the two become stationary. Immediately when this happens a new cylindrical surface $A_2$ is established at a point between $A$ and $W$, the location of which depends upon the amount of increase in the back pressure. The significance of $A_2$ is the same as that previously possessed by $A$; from it there runs a new surface $B_2$ in the continued process of production. For the time during which $B'$ lasts there is a positive banking of the fluid at its surface.

For a decrease in the constant back pressure a subordinate low pressure cone emanates from the well. $B'$ travels toward $B$, reaches it, and passes. If the decrease sets up a potential pressure that is not in excess of its original value, a surface $A_3$ is established somewhere between $A$ and $B$. If the decrease sets up a potential pressure in excess of its original value, a surface $A_4$ is established somewhere outside of $A$. In either case the position of the new surface depends upon the amount of decrease. As before, a new surface, either $B_3$ or $B_4$, runs toward $W$ in the continued process of production.[16]

These are the events with respect to cones and differential cylinders when alterations in theoretic performance are made.

We can see now that it is not correct to say that at the instant $W$ is opened the fluid throughout the volume differential cylinder moves toward the well. It is only proper to say that this fluid assumes a state wherein it is ready to move. A low pressure cone must advance to the boundary of the cylinder

[14] In the description of this process of filling it is assumed that the process is carried on smoothly; that is, without pulsations or partial interruptions.

[15] See §§ 187, 188, and onward.

[16] In Cases 2 and 3 it is clear that the size of the active cones is altered. It is of particular interest in connection with Case 3 to note that we can provide a natural reservoir system with more potential energy, or less potential energy, by altering the constant back pressure, and $R$ becomes larger or smaller accordingly. (See footnote 8, § 173, p. 502.)

before all fluid is actually in motion. Fluid in the immediate vicinity of the well is the first to move.[17]

In the process of production it is certain that time is required for the advancement of the low pressure cone which permits the potential volume to start toward the well. Can we say how long it will take for the boundary of this cone to reach the boundary of the active cone? Perhaps not with accuracy; but what we can say is this: *So long as we can make no distinction at the well between the effects of alterations in accordance with Cases 1 and 3, the boundary of the low pressure cone has not reached the boundary of the active cone.* These cones, so we have learned before, are concordant in their action. For brief periods of time in the course of production we cannot detect any difference in their effects. Over longer periods of time the difference is obvious, in that Case 1 does not affect the size of the active cone, whereas Case 3—like Case 2—does so.

Let us be certain that we understand the effects which ideal performance and Cases 2 and 3 in theoretic performance have upon the volume of the so-called volume cone. If we consider the potential volume of fluid contained in the active cone at any stage of production, we may write, according to Equation 561 (p. 503).

$$vo = kh_{vo}r^2 \quad \dots\dots\dots\dots\dots\dots\dots (585)$$

where $vo$ is the volume of fluid in the cone, $h_{vo}$ is the altitude of the cone, $r$ is the radius of the base of the cone, and $k$ is some constant. For $h_{vo}$ we can substitute its value derived from Equations 560, 559 (p. 503), and Equation 566 (p. 504), in succession. Thus, paying no regard to the values of the various constants, we have

$$h_{vo} = kh_p = kP = kr^2 \quad \dots\dots\dots\dots\dots\dots\dots (586)$$

This upon substitution results in the equation

$$vo = kr^2r^2$$

or

$$vo = kr^4 \quad \dots\dots\dots\dots\dots\dots\dots (587)$$

in agreement with Equation 580, above. *The area of the base of the volume cone, like that of any cone, varies as the square of the radius, and the altitude of this particular cone, in accordance with pressure-volume relations in Volumetric Control, also varies as the square of the radius; consequently the volume of the volume cone varies as the fourth power of the radius.* By a similar procedure we can determine the fact that the volume of the active energy cone varies as the sixth power of the radius, as indicated in Equa-

[17] This is likewise true for wells in reservoirs of Hydraulic and Volumetric controls, where we deal exclusively with "one-one" cylinders. It is evident that we are here considering the effects of inertia, which apply to the initial stages of ideal performance as well as to alterations in accordance with theoretic performance. (See footnote 22, § 141, p. 389.)

tion 583, above. This follows from the fact that the base of this cone varies as the square of the radius, and the altitude varies once as the square of the radius for the pressure component of energy and again as the square of the radius for the volume component; consequently the total variation is as the sixth power of the radius. These variations should perhaps permit us to visualize more clearly the events which take place within the reservoir during ideal and theoretic performances. When, in the latter performance, we cut the relative curves for this control, we give the cones new altitudes, as well as a new base. Volume and velocity for the liquid, and energy and power for the liquid, or volume and velocity for the gas, depend upon these cones. Whereas in Hydraulic and Volumetric controls we deal with cylinders of uniform density, we here deal with cones of uniform density, or cylinders of varying density; consequently we may expect the differences which we observe to exist between the first two controls, on the one hand, and the present one, on the other. These differences form the basis of our tests for the control.

178. *The static pressure gradicnt.*—Granting that the fluid within a natural reservoir of Capillary Control is securely locked in place by bubbles of gas disseminated throughout the mass of liquid, until the pressure is relieved by the opening of a well, we might rightly ask how the fluid might have come to be located in the place we today find it. The fluid must have moved into place, and yet, now that it is there, it cannot move, except as noted. In conformity with the principles of fluid mechanics three ways by which the fluid might have reached its present position suggest themselves. These are the following:

*a*) The fluid might have moved into place under the laws of either Hydraulic or Volumetric Control. In the earlier history of the formation as a container of fluid the pressure might have been sufficient to overpower the effects of Jamin action. In the laboratory we see that this is possible with our capillary tubes. It is the way by which we get a chaplet of globules and bubbles in the tube, and by it we can clear the tube of its chaplet at any time we wish by sufficiently increasing the pressure applied at one end.

*b*) The fluid might have moved vertically into place, perpendicularly to the bedding-planes instead of parallel with them, as (*a*) would infer. If a fine-grained plastic formation, such as some shales, adjoins a coarse-grained competent formation, such as some sandstones, any fluid which the former contains might be forced into the latter, if the formations as a whole are subjected to pressure. This pressure may be due simply to the weight of overlying strata, or to this weight combined with stresses that originate in the processes of deformation of the earth's crust. The fact that fluid will move in the manner indicated may be easily verified by experiment in the laboratory.

*c*) The fluid might have moved into its present place simply by "leakage"

along a fault or fault zone, such features temporarily or permanently placing the present container in communication with an "original" one that is in Capillary Control. We specify this control for the reason that if the original container at the time of the leakage was in either Hydraulic or Volumetric Control, this way would not differ from that of ($a$).

These ways are possible; I would not exclude other ways which may come to light as a result of further studies.

If the fluid entered its present location under the laws of Hydraulic or Volumetric Control, at the subsequent time when the reservoir underwent a conversion to Capillary Control the static pressure gradient was the same as it had been previously; that is to say, it was at that time a perfectly smooth horizontal plane. It is impossible for a reservoir in the first two controls to have a static pressure gradient that is not a perfectly smooth plane, regardless of any irregularity in the shape of the reservoir itself, for the pressure is sufficiently great to overcome any local sources of resistance. *And if the formation in which this fluid now lies has not been subjected subsequently to deformation by lateral or vertical stresses, the static pressure gradient still remains a perfect horizontal plane.* This is the plane $I$ of Figure 185, at a distance $S$ above the plane $K$. After a well $W$ has reached equilibrium as determined by $C$, the distance between the planes $K$ and $N$, the static pressure gradient contains a *conical depression* in the manner shown.

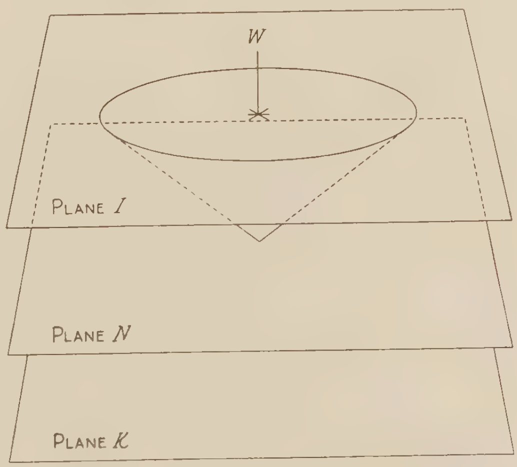


FIG. 185

*If the formation has been subjected subsequently to deformation, the static pressure gradient must be represented by a surface with undulations in accordance with internal stresses that result from the deformation.* The conical depression for a well will show deformity, and this to a degree depending upon the undulations within the radius $R$.

If the fluid entered its present location perpendicularly to the bedding-planes, we might expect the static pressure gradient to be represented by an undulating surface in accordance with any irregularity in the intensity of the stresses over the region. The situation does not differ from that immediately preceding.

If the fluid entered its present location by leakage from a reservoir in Capillary Control, the static pressure gradient must be represented by two inclined planes dipping away from the fault or fault zone. These planes

"feather out" against some horizontol plane at least at a height $A$ above $K$, where $A$ is the pressure of the atmosphere. On the assumption that the inclined planes possess no undulations, the conical depression is the same as that shown in Figure 185, except for the fact that the plane $I$ now cuts it at an angle somewhere between the present location of $W$ and the side of the present circle facing the fault. Wells farther from the fault have smaller cones for a given height of $N$ above $K$. This is in agreement with the fact that the initial static pressure of the wells diminishes as we recede from the fault.[18]

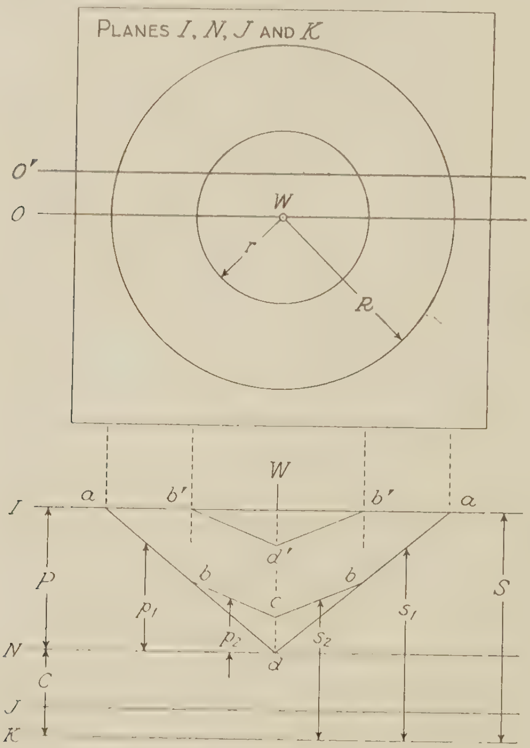



FIG. 186

We recognize the fact, then, that the situation depicted in Figure 185 is ideal. A perfectly smooth horizontal plane[19] represents the static pressure gradient before a well is drilled into the formation, and the same containing a mathematically perfect conical depression represents the gradient at the time the well has reached equilibrium in production, for the given constant back pressure. What can we say of the gradient in the interval between these extremes? In Figure 186 we have a plan of the gradient plane and cone with a vertical profile at the line $O$ passing through $W$. $R$ is the initial radius and $r$ is the radius of a subsequent active cone. The cones which they define are *ada* and $b'd'b'$. Where $S$ is the original static pressure of the reservoir, $s_1$ denotes the static pressure at a point between the distances $R$ and $r$ from $W$. For distances less than $r$ we have $s_2$. Our scheme of representing the intensity of pressures in the profile is an arbitrary one; we therefore conveniently shift the cone $b'd'b'$ vertically downward to occupy the position *bdbc* for the present purpose. Obviously the cone in its new position is exactly equivalent

[18] It seems possible that we may recognize and locate approximately such leakage by faults in our fields of Capillary Control by observing the initial static pressures of the various wells known to be penetrating the same productive formation.

[19] We might say instead, "a portion of the surface of a smooth ellipsoid," if we wish to take into consideration the curvature of the earth.

to itself in the old one. Where $P$ is the initial potential pressure, as measured between the lines $N$ and $I$, $p_1$ and $p_2$ are local potential pressures within the pores which lie at their respective distances from $W$, as measured between $N$ and the line *abcba*. Immediately at $W$ we have, let us say, simply values $s$ and $p$ in accordance with previous discussions concerning this class of reservoirs.

Where a profile along $O$ shows triangular intersections, those along such lines as $O'$ show hyperbolic intersections. The hyperbolas, if projected upon the plane of the triangles, are asymptotic to the inclined sides of the triangles.

**179.** *The kinetic pressure gradient.*—Our active cones pertain to static conditions within the reservoir. It is true that we have pictured their boundaries and vertices to move in the process of production, in order to accommodate decreasing radii and altitudes; but these changes, after all, refer only to changing static conditions. It will be remembered that we studied cylinders of uniform density in connection with reservoirs in Hydraulic Control, and afterward we agreed that the same cylinders must be taken into consideration in connection with reservoirs in Volumetric Control. The only difference to be noted in the two controls concerned the fact that in the former the size, or the volume, of the cylinders remained constant during production, while in the latter they varied mathematically with the functions of performance. These cylinders referred only to static conditions. In fact, after studying them, we immediately proceeded to investigate the kinetic pressure gradients of the systems, and it is to be doubted very much that any confusion arose in regard to the separate identities of the static and kinetic gradients because of our studies respecting the cylinders. Here we are studying a third control —one like the second only in so far as both are finite. We have investigated the static pressure gradient for this control. Are we to be confused at all on finding that there are kinetic pressure gradients associated with the static gradients? Where we formerly had cylinders of uniform density, we now have cones of uniform density, or, as we shall prefer to say for the present, we now have differential cylinders. For all controls the fluid within their respective cylinders is moving toward the well in the course of production. In the finite controls these cylinders are uniformly diminishing in size. For all controls there are kinetic pressure gradients, and in the finite controls these gradients undergo a progressive alteration, in so far as the constants in the equations of their curves live but an instant; that is to say, *the constants are themselves variables.*

We avoided any complexity due to the variable nature of the constants by confining our attention at the moment to Hydraulic Control, a control that we recognized to be a special case of Volumetric Control. Now we have no special case in Capillary Control wherein constants remain constants; nevertheless we can proceed on a basis which we might appropriately have adopted, had we taken up the study of Volumetric Control before that of Hydraulic Control. We can consider the conditions as of one instant—any instant in the

life of the reservoir—and thereafter allow for the variation during successive instants. So let us proceed.[20]

Figure 187 represents a volume differential cylinder. The radius of its base is any possible value $r$, not excluding $R$, and its altitude is, of course, $h_{vo}$. From it we imagine a wedge to be cut in the manner shown, and to this wedge we shall confine our attention for the present. It subtends at $W$ the unit angle

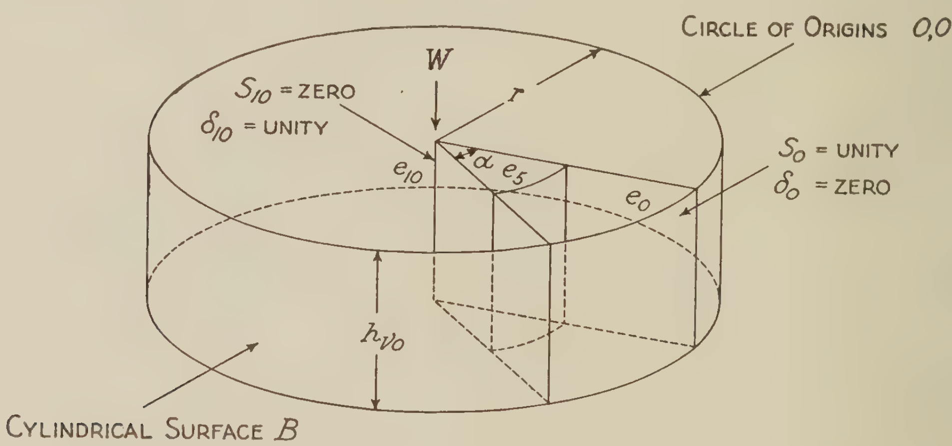



FIG. 187

$a$, and it defines certain cylindrical faces $S$ that correspond to all possible radii equal to and less than the particular value of $r$ for the instant under consideration. $S_0$ is the outermost face. The area of this face is, we shall say, unity, inasmuch as its width $e_0$ and its altitude $h_{vo}$ may each be taken as unity. Since this face lies on the boundary of the one-zero differential cylinder, its density $\delta_0$ is zero. $S_0$ occupies an extreme position with respect to $r$; the other extreme position is occupied by $S_{10}$ at the center. Its area is zero, because its width $e_{10}$ is zero, while its density $\delta_{10}$ is unity, being at the center of the cylinder. All other surfaces and densities vary between these extremes in direct proportion to their distances from the boundary of the cylinder, the upper edge of which, at its intersection with the plane $I$, defines a circle of mathematical origins of co-ordinates.

Now let $z =$ the decimal parts of unity, such that $zr = x$, in which $x$ is the distance from the origin toward $W$. Since $x$ is always expressed in terms of $r$, to be conveniently taken as unity, we may simply deal with $z$ directly. Let $vo' =$ the potential volume of the fluid within all pores lying immediately on the surfaces $S$, and $ve' =$ the lineal velocity of the fluid $vo'$. It is clear that this velocity is directed toward, and only toward, $W$. Of the six variable

[20] We shall take up directly the gradient for the radial system, without considering first that for the tubular system. The usual relations exist between the gradients for the two systems.

quantities, $S$, $e$, $\delta$, $z$, $vo'$, and $ve'$, we need only consider a sufficient number of values to establish the equation of the gradient curve.[21] We shall consider eleven values of each to correspond with ten equal intervals along the radius $r$. These values are represented by the proper symbols with subscripts 0 to 10 inclusive.

In general the volume of fluid passing a surface perpendicularly, divided by the area of the surface, is equal to the velocity of flow, provided only that the units for volume, area, and velocity correspond. In our particular problem we may write

$$\frac{vo'}{\text{Area of } S} = k've' \qquad (588)$$

wherein the units do correspond, and $k'$ is a constant denoting the porosity factor as applied to the surface $S$.[22]

We can proceed to write down the following table of values. I believe all should be clear in view of the foregoing discussion.

TABLE OF VALUES

| $z$ | Area of $S$ | $vo'$ | $k've'$ |
|---|---|---|---|
| 0.0 | 1.0 | 0.0 | 0.00 |
| 0.1 | 0.9 | 0.1 | 0.11⅑ |
| 0.2 | 0.8 | 0.2 | 0.25 |
| 0.3 | 0.7 | 0.3 | 0.42$\frac{6}{7}$ |
| 0.4 | 0.6 | 0.4 | 0.66⅔ |
| 0.5 | 0.5 | 0.5 | 1.00 |
| 0.6 | 0.4 | 0.6 | 1.50 |
| 0.7 | 0.3 | 0.7 | 2.33⅓ |
| 0.8 | 0.2 | 0.8 | 4.00 |
| 0.9 | 0.1 | 0.9 | 9.00 |
| 1.0 | 0.0 | 1.0 | Infinity |

From these numbers in the first and fourth columns we can, by inspection, write the equation between $z$ and $y'$, where the value of the latter is $k've'$. Thus we have

$$y' = \frac{z}{1 - z} \qquad (589)$$

By taking our values from zero to unity we have obtained an equation of limited scope. In an actual case we of course would have quantities as ordinarily expressed in their proper units. The present equation, however, may

21 The quantities $\delta$ and $vo'$ are in reality identical in so far as our analysis is concerned.

22 The equation is based upon a special application of the principle of continuity. (See footnote 7, § 87, p. 182.) The region $S$ may be taken as one bounded by $S_a$ and $S_b$, two parallel surfaces at an infinitesimal distance apart. Thus $S$ is provided with the necessary three dimensions. Time is infinitesimally short, being one instant in duration. To conform to the principle we say that the density of the fluid between $S_a$ and $S_b$ is not changing in this instant of time.

be changed to agree with any that might have been obtained had particular units been employed. This is done by introducing a constant $k''$ into it. Thus

$$y' = k'' \frac{z}{1 - z} \qquad (590)$$

is the general equation. It is the equation of the curve for the velocity gradient within the differential cylinder. Within the pores there are pressure-velocity relations in accordance with Volumetric Control. Therefore, if $y_1$ is the pressure drop within the cylinder, due to velocity alone, we can say that

$$y' = k'''y_1^{1/2} \qquad (591)$$

in accordance with Torricelli's Theorem.[23] From these two equations it follows that

$$y_1 = k\left(\frac{z}{1 - z}\right)^2 \qquad (592)$$

This is the equation of the kinetic pressure gradient due to velocity alone. It corresponds to Equation 121 (p. 186). If desired, $k$ may be evaluated by substituting the values which pertain to the edge of the casing; that is, by substituting $y = P$, when $z = r - m$, where $m$ is the radius of the casing. The curve is shown in Figure 188 as the line $B$.

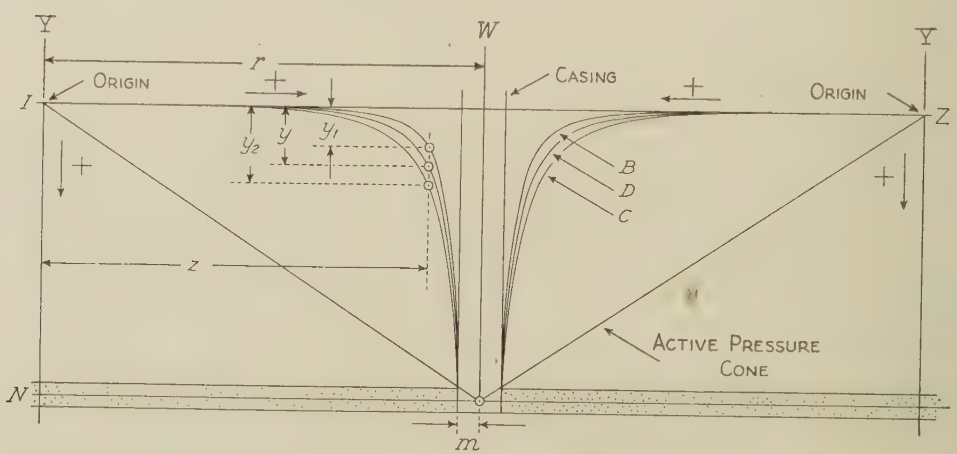



FIG. 188

In order that this equation be fulfilled the fluid must present no resistance to flow. This is obviously far from the possible truth in natural reservoirs of Capillary Control, where there are not only appreciable effects due to the viscosity of the liquid or that of the gas, either being taken to the exclusion

[23] We deal with conditions in the individual pores of the formation; consequently we must use velocity-pressure relations in Volumetric Control.

of the other, but there are also those effects of Jamin action that are analogous to the effects of viscosity—effects that are necessarily present in all porous-filled reservoirs of this control.[24] These we have learned to be far greater in their intensity than any effects due to ordinary viscosity. It is essential that we do not neglect the kinetic pressure gradient due to friction alone in the present system.

Friction, or resistance to flow, is therefore acknowledged to be real.[25] The effects of friction are cumulative along the path of flow, in the present case along the path extending from the circle of origins to $W$. Our procedure after Equation 592 parallels that of section 88. We therefore may be brief.

By integrating Equation 592 with respect to $z$ we have, on previously dropping the constant $k$,

$$y'' = \frac{1}{1-z} + 2\log(1-z) + z + \text{a constant} \qquad (593)$$

where the latter is the usual constant of integration.[26] Since $y'' =$ zero, when $z =$ zero, this constant is equal to minus one. Now we may introduce a friction factor $f$, and thereby obtain the following equation:

$$y_2 = \frac{f}{1-z} + 2f\log(1-z) - f(1-z) \qquad (594)$$

This is the equation of the kinetic pressure gradient due to friction alone. It corresponds to Equation 126 (p. 188). If desired, $f$ may be evaluated. The curve is shown in Figure 188 as the line $C$.

Equations 592 and 594, when combined, give us the equation of the kinetic pressure gradient for the system. The former is to be multiplied by $n$ and the latter by $l$, where

$$n + l = 1$$

In this way we should obtain an equation corresponding to Equation 127 (p. 188). A curve for such a combination is shown in Figure 188 as the line $D$.

Figure 188 corresponds to Figure 60. Perhaps it is difficult for the eye to

[24] See § 154.

[25] As usual, by "real" I mean "sensible"; not to be regarded as negligible, even if its value, as expressed in the customary units, is exceedingly small. Friction is here acknowledged to be great on account of the bubbles of gas that are present throughout the medium, at least great in comparison with the friction which would exist in the absence of the bubbles. Friction, as we have agreed, only exists when fluid is in motion; consequently the static gradient is not dependent upon this friction, for its value is then zero. On the other hand, the kinetic gradient is very much dependent upon it. (See § 154.)

[26] In the operations of calculus any logarithmic terms resulting from integration are "natural"; that is, they refer to the Naperian base $e$. The quantity $(1-z)$ ranges between the limiting values of zero and unity. A table of natural logarithms covering this range is given in Appendix E.

distinguish the curves of these figures. There is, however, one important point that is not to be ignored. *The present curves are not asymptotic to the line of the static pressure gradient; they in fact have contact with this line at the boundary of the differential cylinder.*

For $z$ in Equations 592 and 594 we can substitute its value in accordance with our definition of this function: namely, $z = x/r$. These equations would now be expressed in terms of the usual quantities $x$ and $r$.

In a succession of instants $r$ is a function of time; therefore its value, in accordance with the relation $r = kt$, can be substituted in its place in the preceding equations. By this means the final equation of the kinetic pressure gradient is made to refer to instants of time in sequence.[27]

From here it is possible to proceed with our development in the same manner as we did before. By revolution the equation of the gradient curve becomes the equation of the gradient surface. The radial slipping of fluid is possible.[28] *If this slipping takes place, it extends to the boundary of the differential cylinder. It cannot extend to distances beyond this boundary.*

With respect to volume alone it will be convenient to classify all individual drainage areas in productive formations according to the nature of the fluids. As thus to be classified they will contain the following:

*a*) Either oil or water, one to the exclusion of the other, and this one necessarily accompanied by gas.

*b*) Both oil and water, these necessarily accompanied by gas.

In either case the fluid produced may be liquid with gas or gas alone, as we have already learned. Where both are produced, the gas passes into the free state in accordance with Henry's Law, and thereafter it expands in accordance with Boyle's Law. We therefore are again confronted with a gas distribution gradient and a gas expansion gradient.[29] By-passing of the gas is possible.[30] The properties of foam are not to be ignored.[31]

Where gas alone is produced, the liquid which forms the globules may be water, oil, or condensed hydrocarbon vapors. It is clear that among reservoirs in formations of the same porosity and texture, and in such having gas at the same intensity of pressure, the value of $R$ will differ appreciably in accordance with the three liquids which they may individually possess throughout their drainage areas, for the values of surface tension are at variance.

The case in which both oil and water are present, and in which both are subject to production, will be considered in a subsequent section.

[27] Apparently little is to be gained by doing this. As in many other instances, it is interesting, and important, to know that these generalizations can be made, for we can then better appreciate the scope of the restricted equations.

[28] See § 89.

[29] See §§ 95 and 96.

[30] As we know, the gas alone may slip or by-pass the liquid, or both gas and liquid may slip. In the latter case the two fluids need not slip to the same degree.

[31] See § 133.

CHAPTER XXXI

# Secondary Functions of Performance *(Concluded)*

"Nothing can be known to exist except by the help of experience. That is to say, if we wish to prove that something of which we have no direct experience exists, we must have among our premises the existence of one or more things of which we have direct experience."—BERTRAND RUSSELL

**180.** *The nature of interception.*—In Hydraulic and Volumetric controls all wells that produce from the same formation, that is, from the same physical container, possess potential reservoirs which intercept each other. It is impossible for one well to produce from such a container without having some effect upon the others. While we may not be able to detect any difference in the velocity of production at one well upon the opening or closing of another, we can rest assured in the case of oil and gas, wherein these fluids are confined to pools which are not to be replenished in our day, that every barrel of oil, and every cubic foot of gas, produced from one well means one barrel or cubic foot less to be produced from all other wells at large. In Capillary Control the situation is perhaps the same, or at least similar, in some respects, yet in most particulars it is radically different. Among these particulars we must include features of greatest importance in the economics of oil and gas production.

We say that the radii $R$ and $r$ in reservoirs of this control are of limited length. *All wells that produce from the same physical container, the initial cones and differential cylinders of which do not intercept, possess potential reservoirs which themselves do not intercept.* All such wells as these produce without having the slightest effect upon each other.[1] Their drainage areas do not overlap.

While we are obliged to accept the fact that the limit of resistance $f$ which one globule of liquid, in contact with bubbles of gas, can exert against the pressure of the reservoir is real, even if small, and the fact that, if the number of such globules $n$ is sufficiently great, the product of $n$ and $f$ can repre-

[1] Each of these wells performs as if the others were not present. In Capillary Control we can appreciate the distinction between the physical concept of a productive formation serving as a container of fluid and the mathematical concept of a potential reservoir within the formation.

sent a resistance equal to the pressure of the reservoir, we—or at least some of us—may yet feel that any movement permitted one globule will be transmitted in part to the next, and the movement of this in part to the next, and so on indefinitely, until the boundary of the physical container is reached. If we feel this way, it is undoubtedly because we have a particular inappropriate variation in mind: namely, that of the exponential function, where, say, one value must be multiplied by one one-hundredth to obtain the next value, this by one one-hundredth to obtain the next, and so on.[2] We shall acknowledge this variation to be inappropriate if we can appreciate one or two physical properties of fluids. These are the following:

*a*) When subjected to pressure, the gas dissolves in the liquid, and when relieved of pressure, the gas passes into the free state.

*b*) A globule can offer a resistance less than $f$, say the one-thousandth part of $f$, without the slightest deflection of curvature at its menisci or movement in its position.

The acceptance of either one of these is sufficient to discredit variation in accordance with the exponential function.[3] The transmission of movement is limited to the radius of action of the pressure; the movement itself is exactly zero at a distance from the well equal to the radius.

Non-interception in tubular and radial systems is illustrated diagrammatically in Figure 189. $W$ and $L$ are two orifices which are separated by distances greater than $2R$. The globules within $R$ are distorted; they are offering resistance to the movement of fluid toward the orifices. Where $n_1$ and $n_2$ denote the extent of $R$, the globules between them remain undisturbed in their original equilibrium.

Interception occurs where the orifices are separated by distances less than $2R$. The first of two situations is illustrated in Figure 190 (p. 534). Here the radii overlap. *All globules* m *equidistant from* W *and* L *are equally relieved of a definite amount of pressure on their two sides; consequently they do not move toward either orifice.* The curvature of their menisci denotes a continued state of equilibrium. *All globules not equidistant from* W *and* L *undergo a greater relief of pressure on one side than on the other; they consequently tend to move in the direction of the nearer orifice.* They become distorted, and offer resistance to the movement of fluid toward the respective orifices. In the radial system all globules $m$ define a common chord $M$ for the two areas of drainage. Along this chord there are, of course, alternating globules

[2] To obtain the value zero an infinite number of these multiplications is necessary. For any finite number of them the value is real. We would thus arrive at the limits of the physical container with a value greater than zero, even though very small. This situation is in agreement with the fact that the exponential curve is asymptotic to the axis of abscissas.

[3] We have unimpeachable evidence of the truth of both propositions in laboratory experimentation. The first is obviously in accord with Henry's Law, and the second agrees with Dixon's statement (§ 154) concerning the transmission of tension by liquids.

and bubbles. Like the globules the bubbles are equally relieved of a definite amount of pressure—the same amount as in the case of the globules—on their two sides. The gas expands equally, then, in two directions. The two adjoining globules which lie on the radii of their respective wells are pushed equally

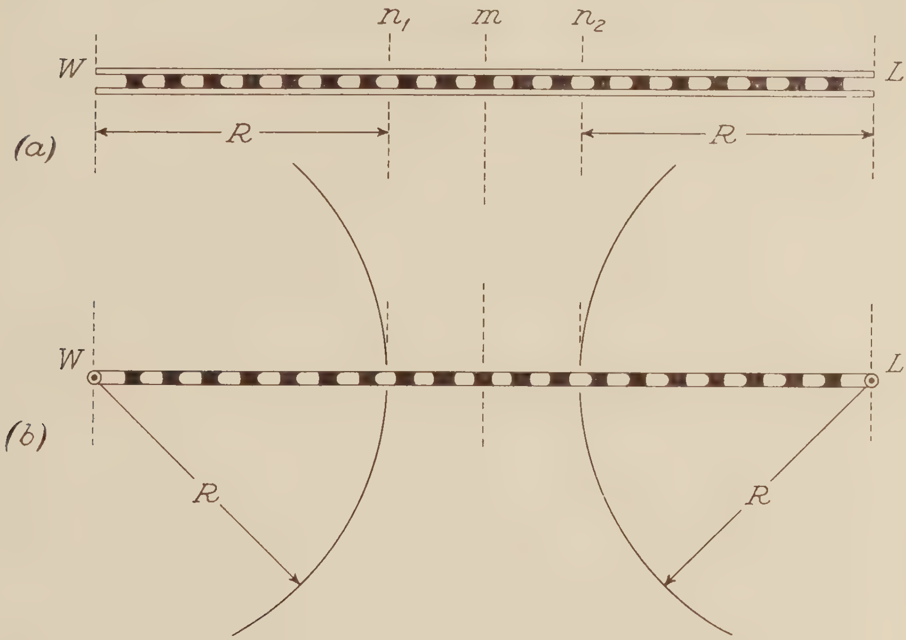



FIG. 189

toward those wells. *If we could conceive a molecule of gas to lie exactly on this chord, upon the expansion of the bubble this molecule would remain stationary.*

This common chord signifies the existence of a *neutral plane* cutting the volume cones or differential cylinders vertically. We can say that it is a neutral plane because no fluid can pass from one side of it to the other. In particular, no fluid in $A$ can enter $B$, either to remain there or to pass on to $W$, and no fluid in $B$ can enter $A$, similarly to remain there or to pass on to $L$. If $L$ were not present, fluid at $q_1$ would move toward $W$, and, if instead $W$ were not present, fluid at $q_2$ would move toward $L$. As both are present, the fluid in the region $AB$ is divided equally between them. $L$ gets the same amount of fluid from $q_1$ whether $W$ is absent or present, and, similarly, $W$ gets the same amount of fluid from $q_2$ whether $L$ is absent or present.[4]

[4] The neutral plane evidently limits the drainage area of a well as effectively as $R$ does without interception. This limitation pertains to the by-passing of gas and to the radial slipping of liquid as well as to normal movement of the fluids toward the well. The multiple orifice does not exist in Capillary Control. (See § 167.)

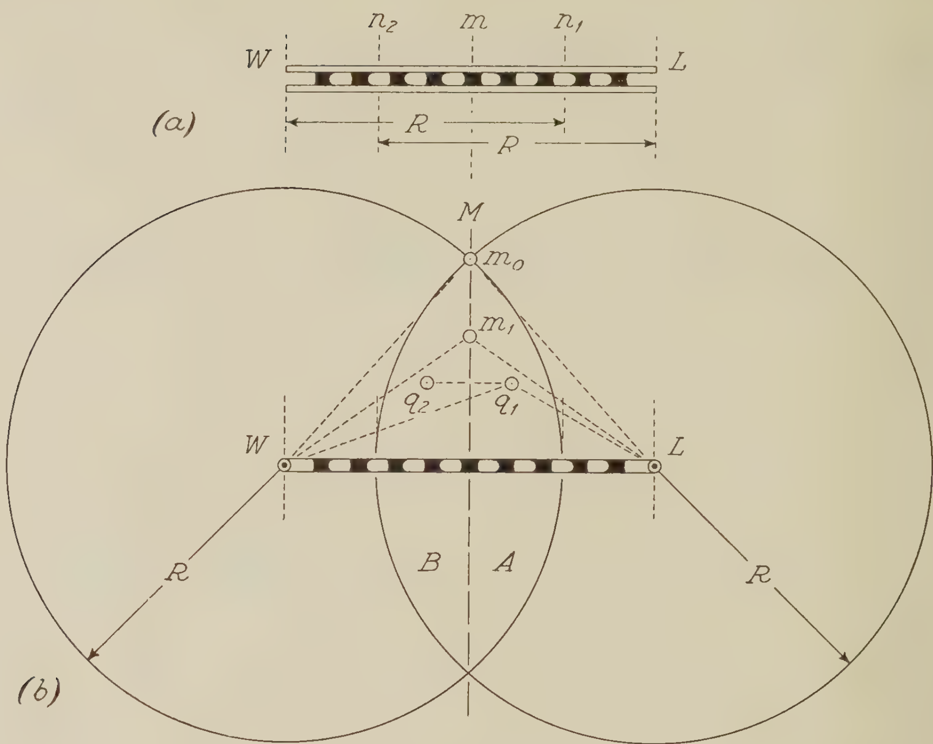



FIG. 190

**181.** *Active cones on interception.*—We have tacitly assumed thus far that $W$ and $L$ are two wells brought in simultaneously. As a matter of fact we well know that this condition is but one of two extremes, and that between the two there are an infinite number of possible stages. We have, so we shall say, the three following conditions:

*a*) $W$ and $L$ are brought in simultaneously.

*b*) $W$ is in equilibrium at the given constant back pressure when $L$ is brought in.

*c*) $W$ is at any intervening stage of production when $L$ is brought in.

For the first condition, in view of what has already been said, I need only add a few remarks concerning the behavior of the active cones during production. In Figure 191 the situation is the same as that shown in Figure 190. The cones start with a radius $R$ which immediately thereafter assumes a decreasing value $r$. The radius $r$, as we know, defines the base of the active cones. During production we see these cones continually cut at the neutral plane through $M$, so long as $r$ is greater than one-half the well-spacing $WS$. For $R$ the cones intercept at $m_0$, and for $r_1$ they intercept at $m_1$. Not until $m_c$ is reached are the active cones intact. For production between $R$ and $r_2$ the primary function equations possess constants $K$ which in reality pertain to fictitious values of the radius. These values are average values; they are

values of intact cones having volumes equal to those of the larger intercepted cones. At the time $r_2$ is attained, the $K$'s change their values; they now pertain to real values of $r$ for intact cones. Production now takes place in a manner entirely independent of interception.

So much for the situation under the conditions of item $(a)$. All remaining situations may be explained by first considering a special case under

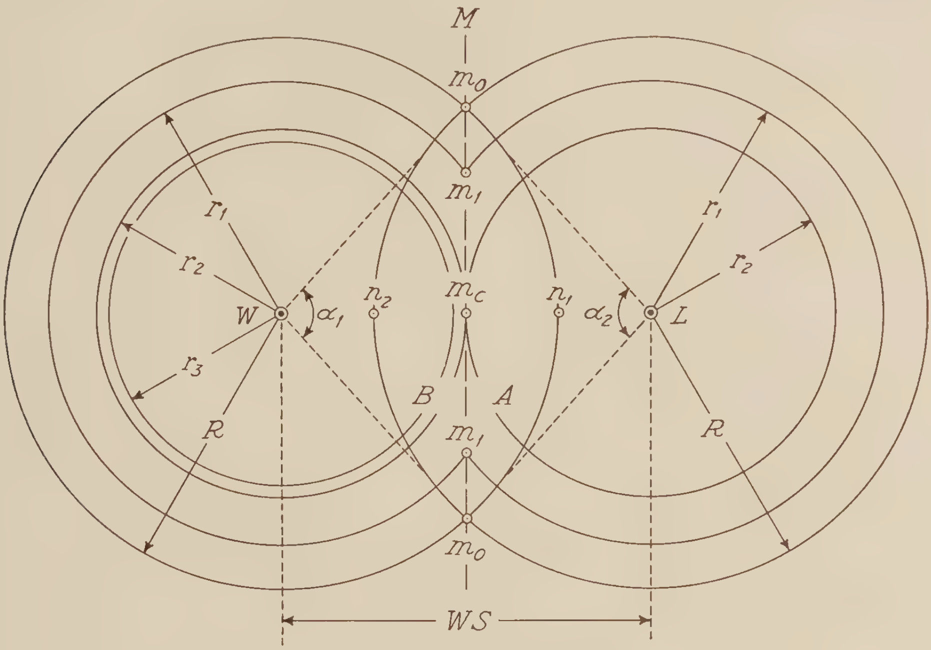



FIG. 191

item $(c)$. $W$ is brought in, and it produces alone during the period in which the radius decreases from $R$ to $r_2$, when $L$ is brought in. It is clear that $W$ drew fluid from a large intact cone before $L$ appeared on the scene. It had, in fact, drawn all the fluid from $A$ that it was ever to receive from this particular restricted region. Had $L$ been present from the beginning this fluid would have belonged to it, instead. It is our problem to know what happens at the time $L$ is brought in.

It will be remembered that we found a state of tension to exist in the space between the cones of $R$ and $r$. The globules are in a distorted condition, thus offering a resistance against, and offsetting, the pressure of the reservoir that exists at and beyond the radius $R$. As long as $L$ is not present, this pressure exists along the boundary $m_0n_1m_0$ in the same manner as it does along the remainder of the boundary, since all sections of the boundary are alike. And as long as $L$ is not present, the state of tension in the region $A$ is the same as that in all other like regions surrounding $W$. But when $L$ is brought in, the pressure along $m_0n_1m_0$ is immediately diminished. The globules in $A$, no longer having so great a pressure to resist and offset, move out-

ward on radial lines from *W*. All the fluid which *W* drew from *A* is replaced at the expense of *W*. As soon as *L* is opened the boundary of a low pressure cone diverges from it in all directions, and that section of the boundary pertaining to the sector $\alpha_2$, on arriving at the common chord *M*, sets up a converging low pressure cone in the sector $\alpha_1$. In this way the conditions in *A* and *B* assume the status which would have existed if the two wells were brought in together.

Whence comes this fluid which *W* places in *A* at the disposal of *L*? When the converging low pressure cone arrives at *W* the boundary of a low pressure cone diverges from this point in all directions not included in the sector $\alpha_1$. The radius $r_2$ shrinks to a value $r_3$, such that the difference in the volumes of cones for these radii is equal to the volume of the intercepted portion of the original intact cone in *A*.

The *K*'s in the primary function equations for *W* are the same before and after *L* is brought in, although during the time in which the low pressure cones are in action they are temporarily different. There is, however, a gap in the paths of decline for *W*; it is as if the static pressure at *W* had been decreased independently of production. The situation at this well is somewhat analogous to that of wells in Volumetric Control, theoretic performance, Case 2, where alterations are made in the apparent static pressure by bringing in an adjoining well. I prefer to say, however, in the present case, that the alteration is one in the real static pressure, inasmuch as each well has its own individual potential reservoir in spite of interception.

*L* produces in exactly the same way as it would, had it been brought in simultaneously with *W*. On continued production *W* reaches equilibrium before *L* simply because the latter is the younger well. It possesses a radius *R* at the time *W* possesses one of $r_3$.

Now let us suppose in the extreme case that *W* has reached equilibrium before *L* is brought in. The situation with respect to the region *A* and the low pressure cones in the sectors is the same as before. *W* replaces fluid in *A* at the disposal of *L*. But *W* has no active cone to become shrunken. This cone assumed zero dimensions at the time of equilibrium. Whence comes this fluid which *W* places in *A*? The sector $\alpha_1$ must take air from the atmosphere sufficiently to allow the replacement.[5] This requires *W* to be open to the atmosphere. What if it is closed? A partial vacuum is created at *W*; consequently the replacement in *A* is incomplete. There is some replacement, because of the expansion of the gas, but such replacement as this is false. It is in *A*, though not at the disposal of *L*. The partial vacuum prevents the establishment of a common chord *M*.[6] The total production of *L* is diminished by that from both *B* and *A*.

[5] This is a case of negative production. (See § 8.) I can cite no instance in practice where this has been noted by the operator. However, the Jamin tubes display the phenomenon.

[6] This phenomenon is likewise displayed by the Jamin tubes.

At any stage of production at $W$ the opening of $L$ decreases the production. Depending upon the size of $W$'s active cone at the time $L$ is brought in three situations may arise, as follows:

*a*) $W$'s production diminishes to a smaller positive value. The active cone contains more fluid than that required by $A$.

*b*) $W$'s production diminishes to exactly zero. The active cone contains just sufficient fluid to satisfy $A$.

*c*) $W$'s production is diminished to a negative value. The active cone does not contain sufficient fluid to satisfy $A$.

**182.** *The degree of interception.*—Our investigation concerning interception is proceeding on the basis that the radii $R$ for the wells $W$ and $L$ are equal. In this we are assuming the following:

*a*) The reservoir is perfectly ideal within the extent of the combined drainage areas.[7]

*b*) Operations at the wells are uniform. Good practice demands the same value of the constant back pressure at both wells.

In introducing Figure 190 it was said to represent the first of two situations in interception. While $W$ and $L$ were separated by a distance less than $2R$, they were in fact shown to be farther apart than $R$. In Figure 192 we

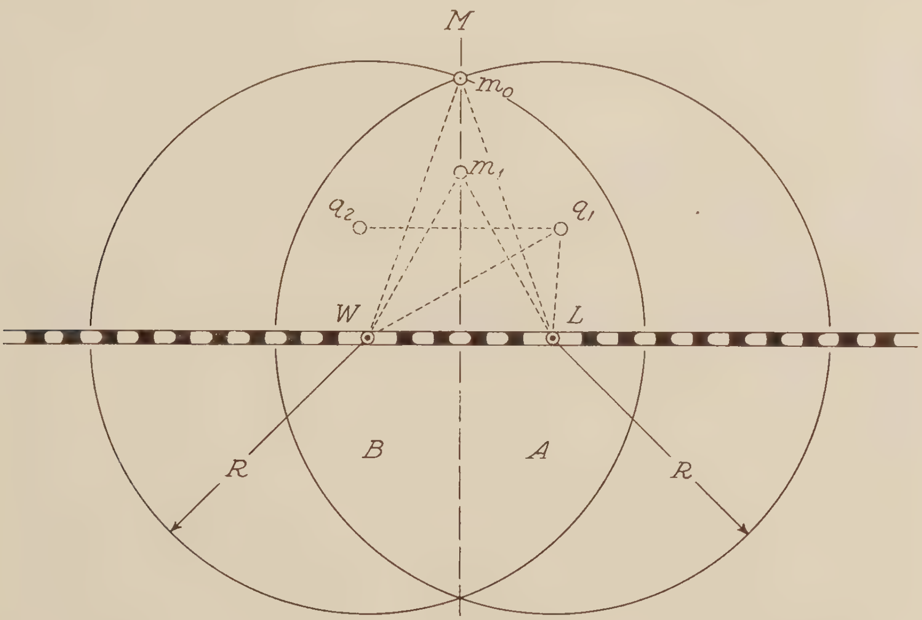



FIG. 192

[7] The assumption of lateral homogeneity is of more importance than that of vertical homogeneity. That the former exists within our natural reservoirs is demonstrated by the fulfillment of the law of equal expectation in fields of this control. Vertical heterogeneity will be considered briefly in § 184.

have a second situation. $W$ and $L$ are separated by a distance less than $R$. The drainage area of either well includes the point at which the other is located. For the same values of $R$ the regions $A$ and $B$ are considerably larger than before. It is to be particularly noted that $L$ lies within $A$, while $W$ lies within $B$.

If the two wells are brought in simultaneously, there is nothing to be noted in addition to what has already been said. The common chord $M$ at once defines the position of the vertical neutral plane, and production proceeds in the usual manner for interception.

If $W$ has been producing for a time before $L$ is brought in, fluid in $A$ is but partially replaced. The extent of replacement depends upon the progress $W$ has made in its process of production and the degree of interception. The movement of such fluid which is replaced is radial, and it is confined, as before, to a sector of $W$'s active cone. Fluid immediately to the right of $W$, Figure 192, should not be expected to pass to the right of $L$, since to do so it would necessarily pass $L$. Again, fluid immediately to the left of $W$ should not be expected to pass to the right of $W$ for the same reason. There is a region extending on both sides of the straight line that connects $W$ and $L$, for the distance included within $A$ and $B$, wherein the replacement of fluid is incomplete. The neutral plane is established without $A$'s complete refilling. The effects of this partial replacement will be noted in a subsequent section.

Obviously the effects of interception on both wells are more intense when the spacing is closer. The regions $A$ and $B$ are larger; the former requires a larger active cone at $W$ to satisfy it, and $W$'s production is diminished to a greater extent. $L$'s initial cone is deficient to a greater extent, for the plane at $M$ cuts off a greater portion of it.

*The quantitative effects of interception are entirely dependent upon the amount of interception.* It will therefore be necessary for us to devise a convenient method of designating the degree of interception, if we would—as we shall—pursue our investigation. This is easily done in the manner of Figure 193. $W$ is a well, and $L$ is a location, the latter being subject to a change in position with respect to the former. The line connecting $W$ and $L$ determines a convenient diameter of $W$'s drainage area upon which a scale of interception may be placed. This scale reads from 0 per cent at the right to 100 per cent at the left. When $WS$ is greater than $2R$ we may say, to be exact, that interception is negative; when $WS$ is equal to $2R$ the drainage areas touch, and the scale indicates 0 per cent interception; and when $WS$ is less than $2R$ the extreme left-hand point on the circumference of $L$'s drainage area indicates directly by means of the scale a single positive percentage value of interception. We see that when $L$ is at the extreme right-hand point of $W$'s drainage area interception is 50 per cent. The drainage circle of each well passes through the other well. Interception of 100 per cent is purely of mathematical interest. It represents the limit of positions for $L$ as this loca-

tion is moved toward $W$. It is clear that 100 per cent interception requires $L$ to be exactly coincident with $W$. In the limit two wells merge into one.[8]

It is important to note the fact that when we give a number which represents the percentage of the diameter intercepted in any particular case, this same number represents the percentage of the radius intercepted by the neutral plane at the common chord of drainage areas.

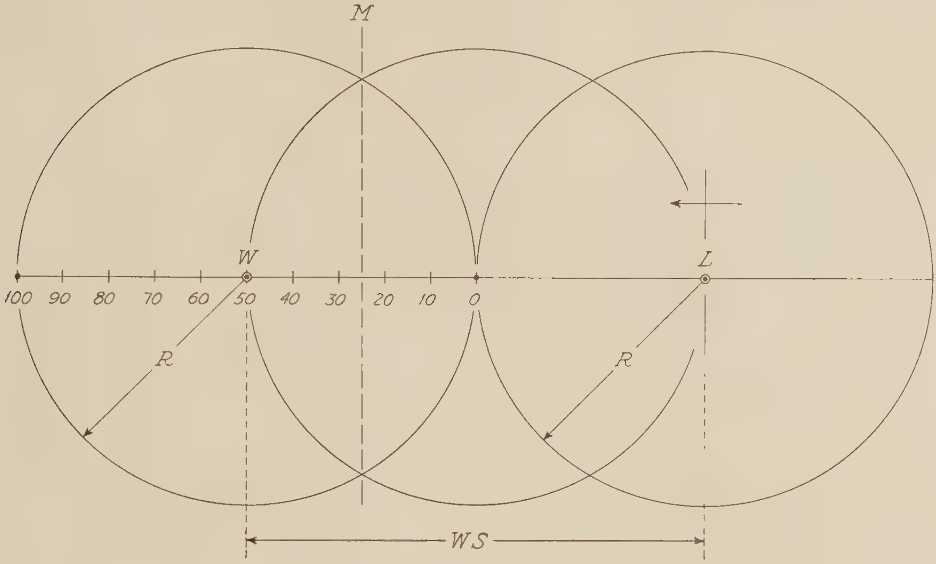



FIG. 193

If we suppose $WS$ to be fixed in value, while the radii are equally variable, the degree of interception is rendered variable. Let us consider for the moment a special case wherein $R$ is the radius with which we have heretofore dealt, one that we may take as a unit radius; $I$ is zero interception; $R'$ is a new radius that is brought into action by a change in the constant back pressure; and $I'$ is the corresponding new interception. We shall assume the constant back pressure to be decreased, in order that $R'$ and $I'$ may be greater than $R$ and $I$, respectively. As a matter of convenience for the present we shall express $R'$ and $I'$ in natural numbers rather than in percentage numbers. Now it is possible to write down the following table of values that are easily verified by means of Figure 193:

| $R'$ | $I'$ |
|---|---|
| $1 (= R)$ | $0 (= I)$ |
| $1\frac{1}{4}$ | $\frac{1}{8}$ |
| $1\frac{1}{2}$ | $\frac{1}{4}$ |
| $1\frac{3}{4}$ | $\frac{3}{8}$ |
| 2 | $\frac{1}{2}$ |

[8] Interception in Fig. 190 is $33\frac{1}{3}$ per cent, and that in Fig. 192 is $66\frac{2}{3}$ per cent.

By inspection we determine the following equation:

$$I' = \frac{R' - R}{2} \quad \text{.......................(595)}$$

In accordance with the relations expressed by Equation 572 (p. 514) we may write

$$\frac{R'}{R} = \left(\frac{P'}{P}\right)^{1/2} \quad \text{.......................(596)}$$

consequently by combining the two equations we can determine the new value of interception for the change in the constant back pressure. But of course Equation 595 suits only the very special case at hand. Let us make it perfectly general. First we substitute the value of $WS/2$ for $R$ in the equation, inasmuch as its significance is more general, in fact, completely so. Secondly, and lastly, we express both $R'$ and $WS/2$ in terms of $R$; that is, we simply divide both by $R$. Thus we finally have the following equation:[9]

$$I' = \frac{2R' - WS}{4R} \quad \text{.......................(597)}$$

Now it is immaterial whether the value of interception is negative, zero, or positive before the change in $R$, and it is furthermore immaterial whether the constant back pressure is decreased or increased. In any case $I'$ has the value to be obtained by means of the equation. If the result is negative, we may be assured that interception is negative—the drainage areas do not overlap for the new radius. Thus the present equation is perfectly general in its application.

**183.** *Fundamental curve of interception.*—The only effect of interception upon the volume cone or differential cylinder is that they are cut vertically by the neutral plane. They are otherwise not impaired in any way. *It is consequently evident that the potential volume of fluid to be delivered from a well in this control is dependent upon the degree of interception.* Why not determine what percentage of decrease in volume the cone or differential cylinder suffers upon being cut in this fashion at any and all positions of the neutral plane along the radius $R$? For example, let us take the cone with its mathematical setting as in Figure 194. There are three axes of co-ordinates, $X$, $Y$, and $Z$, at right angles to one another; the vertex of the cone is at

[9] The equation following is of the form $yz = x - k$. It is the equation of a surface with properties identical with those of Fig. 9. For our present purpose we might replace $T$ by $R'$, $P$ by $R$, and $V$ by $I'$.

the origin, and its axis coincides with $Z$. The equation of the cone in this position is

$$x^2 + y^2 = \left(\frac{R^2}{h_{vo}^2}\right)z^2 \quad \text{.................... (598)}$$

wherein $x$, $y$, and $z$ are the co-ordinates of any point lying on the surface. This cone we shall imagine cut by a plane $U$, the plane being passed from the extremity of $R$ at the boundary of the base toward the center, always maintained perpendicular to $R$ and parallel to the central plane $W$. The problem is simply to determine the ratio between the section of the cone to the right of $U$ and the entire cone. It does not matter what dimensions the cone possesses; therefore we may conveniently assume that $R$ and $h_{vo}$ are equal. The equation then reduces to

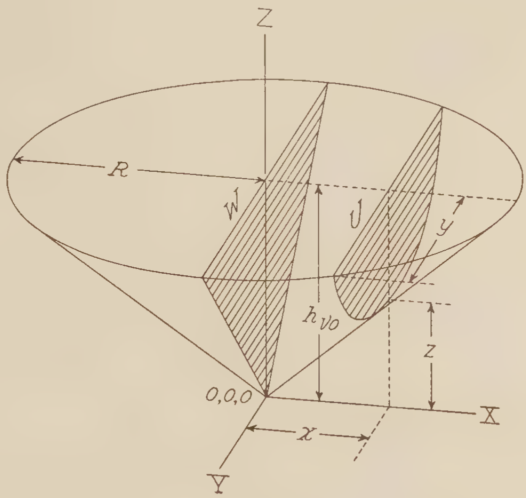



FIG. 194

$$x^2 + y^2 = z^2$$

or

$$y = \pm \sqrt{z^2 - x^2} \quad \text{.................. (599)}$$

The process from here involves double integration. We first set up the following differential equation, giving $x$ a particular constant value of $x_1$:

$$ydz = 2\sqrt{z^2 - x_1^2}\, dz \quad \text{.............. (600)}$$

The constant 2 is inserted in place of the signs plus and minus.[10] By integrating this expression with respect to $z$, and substituting into the result the limiting values of $z$, namely, $z = x_1$ and $z = h_{vo}$ we have

$$\text{Area } U = A - B + C \quad \text{.......................... (601)}$$

wherein

Area $U$ = the complete area of the hyperbola on the plane $U$

$$A = h_{vo}\sqrt{h_{vo}^2 - x_1^2}$$

$$B = x_1^2 \log\left(h_{vo} + \sqrt{h_{vo}^2 - x_1^2}\right)$$

and

$$C = x_1^2 \log x_1$$

[10] For the plus sign we should obtain the half-area of the hyperbola on $U$ in front of the $XZ$ plane ($y$ positive), and for the minus sign the half-area behind this plane ($y$ negative). By inserting the number 2 we obtain the entire area at once.

Had we started with the equation of the one-zero differential cylinder, we should have obtained the same Equation 601. As stated in section 175, the cone and the differential cylinder have identical mathematical properties.

For the second integration we return to a variable $x$ by dropping subscripts, and we set up the following differential equation from the above:

$$\text{Area } Udx = Adx - Bdx + Cdx \quad \ldots\ldots\ldots\ldots (602)$$

The integration of this expression with respect to $x$ gives[11]

$$\text{Volume } V = D + E + F - G \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (603)$$

wherein

Volume $V$ = the volume of the portion of the cone between the planes $W$ and $U$

$$D = \tfrac{2}{3}\left(h_{vo}x\sqrt{h_{vo}^2 - x^2}\right)$$

$$E = \tfrac{1}{3}\left(h_{vo}^3 \operatorname{arcsine} \frac{x}{h_{vo}}\right)$$

$$F = \tfrac{1}{3}\left(x^3 \log x\right)$$

and

$$G = \tfrac{1}{3}\left[x^3 \log\left(h_{vo} + \sqrt{h_{vo}^2 - x^2}\right)\right]$$

Now we can take $R$ and $h_{vo}$ each as unity, and substitute a series of values for $x$ in decimal fractions of $R$ into Equation 603. This substitution will give us values of the portions $V$ between the planes $W$ and $U$. Therefore let

$\frac{1}{2}Vo$ = the volume of the half-cone to the right of the plane $W$, and

$Vo'$ = the volume of the portion to the right of the plane $U$, then

$$\tfrac{1}{2}Vo - V = Vo' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (604)$$

$Vo'$ is the portion of the potential volume cone cut off by the neutral plane.

Furthermore, let

$$x' = 1 - x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (605)$$

in order to have the position of the neutral plane expressed in terms of the distance from the boundary of the base of the cone. The actual substitution of the values into Equation 603, and the transformation of the results in accordance with Equations 604 and 605, enables us to possess the following

[11] The first integration may be made in one step, while the second requires several steps, including those known as "integration by parts." Forms 124, 125, and 151, in Peirce, *A Short Table of Integrals,* may be used. Logarithmic and arcsine terms appearing on integration are "natural." In evaluating the terms by means of the table in Appendix E all percentage numbers are first changed to natural numbers; that is, to decimal fractions with the limits zero and unity.

table in $x'$ and $Vo'$. Final results are conveniently converted from decimal fractions to percentage values.

| $x'$ | $Vo'$ |
|---|---|
| 00.0 | 0.000 |
| 10.0 | 0.214 |
| 20.0 | 1.221 |
| 30.0 | 3.275 |
| 40.0 | 6.514 |
| 50.0 | 11.006 |
| 60.0 | 16.753 |
| 70.0 | 23.692 |
| 80.0 | 31.699 |
| 90.0 | 40.572 |
| 100.0 | 50.000 |

To change the values of $x'$ so that they may read in terms of the diameter of the base they must be divided by 2. With these half-values as abscissas, and the values of $Vo'$ as ordinates, the curve appears as in Figure 195. The

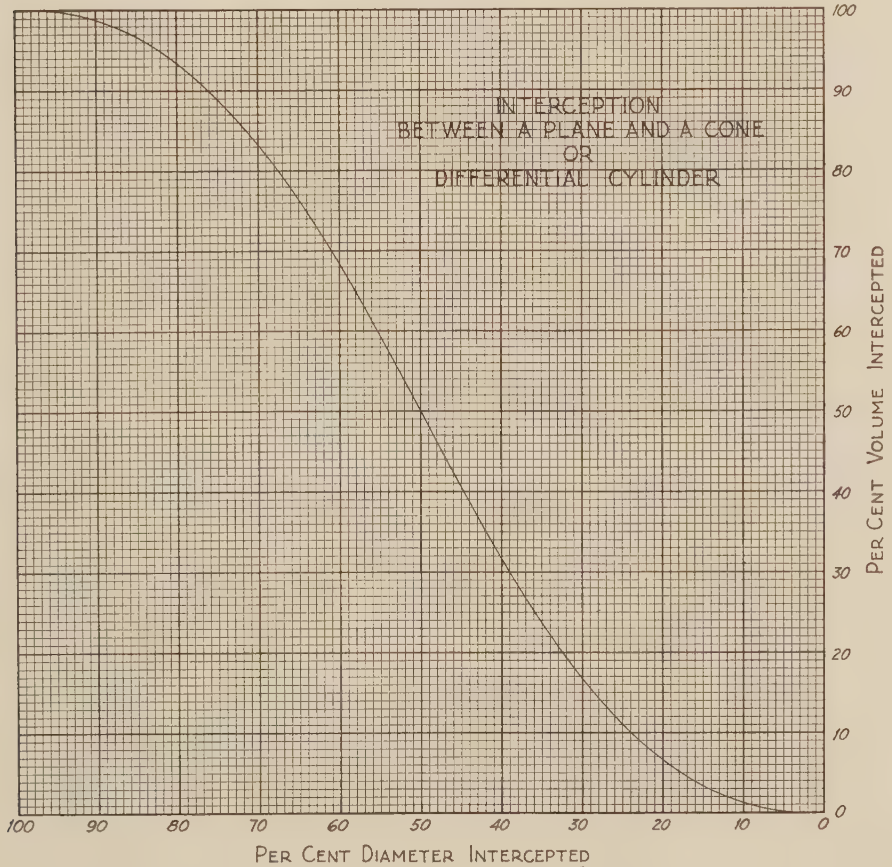



FIG. 195

series of points is plotted from the lower right-hand corner of the plat to the center, the plat is then revolved through 180 degrees in its own plane, and the series is again plotted in the similar position. This is the fundamental curve of interception.

If we take the lower half of the curve, stretch it so that all abscissas possess twice the values shown, and invert it by revolving it into the upper half of the plat, the result is the curve $AB$ of Figure 196. The stretching pro-

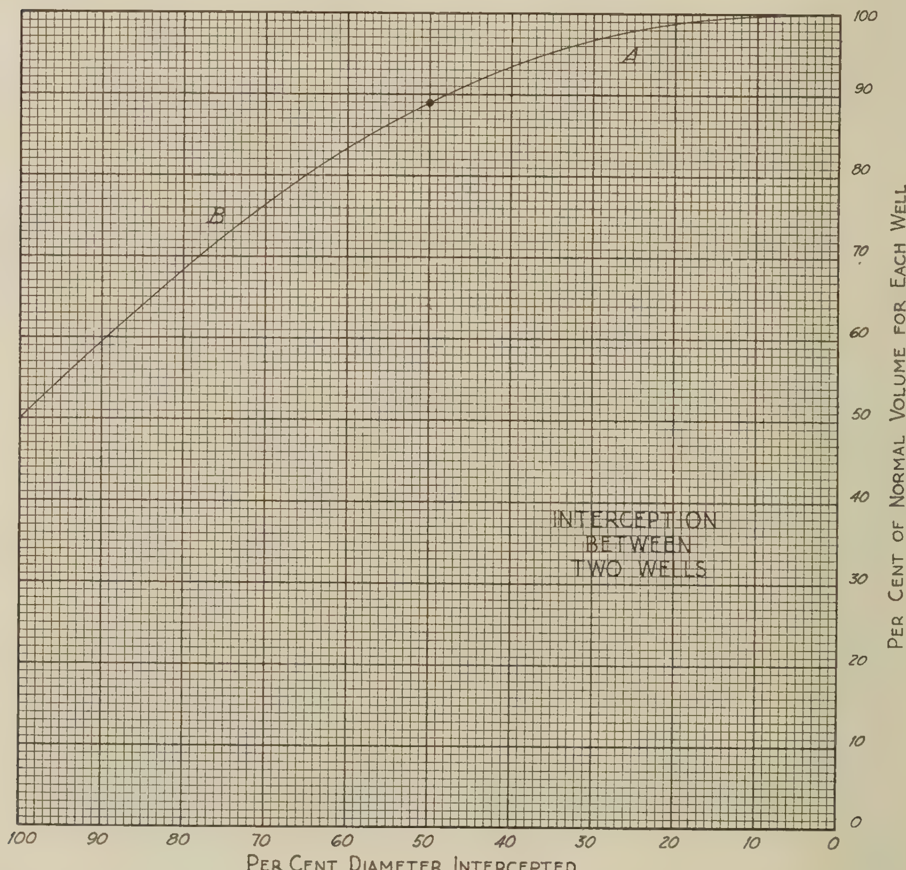



FIG. 196

vides for the fact that as $L$ is caused to approach $W$ from 0 to 100 per cent interception, the neutral plane passes from 0 to 50 per cent of the diameter. The 50 per cent position on the diameter in Figure 195 corresponds to the 100 per cent position for the interception of wells, according to the manner we have designated this interception in Figure 193. By inverting the curve we convert values of $Vo'$, the volume in percentages of $Vo$ lost to the wells, to values in percentages of $Vo$ remaining to the wells in spite of interception. One hundred per cent of $Vo$, that is, the volume of an intact cone, may

be taken as the normal volume to be produced by either well without interception. *AB* then shows the percentage of this normal volume to be produced upon interception of any degree between 0 and 100 per cent.

The two sections of the curve, *A* and *B*, hold on condition that the reservoir is perfectly ideal within the extent of the combined drainage areas, and that the operations at the wells, with respect to the constant back pressure, are uniform, as stated in the preceding section. The section *A* is independent of the time between bringing in the wells, and it is independent of the values of the external friction head which the wells might offer against production. On the contrary, section *B* depends upon these two factors. It is based upon the supposition that *W* and *L* are brought in simultaneously, and that the external friction head in percentage values of the potential pressure during production is the same at both wells.

If *L* is brought in subsequently to *W*, or if the external friction head at *W* is less than that at *L*, *W* produces a greater volume than *L*, yet less than its normal volume except on the one condition that it has reached equilibrium at the given constant back pressure before *L* is brought in. In this case we say that *W* performed without interception. In any case the section *B* of the curve for *L* remains fixed at the right, while it drops to a lower position at all other points. Its new position at the extreme left depends upon either the stage of production which *W* has reached or the relative values of the external friction head at *W* and *L*, or obviously, both. In the case where *W* has reached equilibrium the position of *B* for *L* at the extreme left rests at zero per cent of normal volume to be produced.[12]

*B* drops in this way because of the fact that the replacement of fluid in *L*'s drainage area is incomplete when interception exceeds fifty per cent.

**184.** *The square pattern for wells.*—We have thus far considered the mutual effects of interception with two wells. Let us carry our problem to areas of land covered by wells. At once we recognize the fact that there are three geometrical patterns according to which the surface of land may be homogeneously covered. These are the square, the equilateral triangle, and the hexagon. No other symmetrical figures can completely cover an area. For us the equilateral triangle and the hexagon are the same, inasmuch as a contiguous group of six of the former, arranged about a point, forms the latter. Of the resulting two distinct patterns but one is sufficiently interesting to hold our attention here, namely, the square, for the technical advantages of the triangle appear to be slight in comparison with its economic disadvantages.[13] So long as property lines at right angles are given favor, it seems safe to assume that the square pattern for the location of wells will in turn

[12] This is obvious for the simple reason that the fluid has already been produced by *W*.

[13] For a discussion concerning the triangular pattern see Appendix H.

be given favor. At any rate, we shall confine our present investigation to the square pattern in consideration of the fact that the principles of interception are identical in either case. The only difference to be noted between the patterns is invested in the values of constants.

$W$ is a well, while $L_1$, $L_2$, and $L_3$ are locations that are subject to a shift in position toward $W$. We are to imagine the shift to be uniform and simultaneous, always to maintain a square. Furthermore, we are to imagine the land surrounding this square for an undefined distance on all sides to possess similar equidistant locations which are to shift harmoniously with the given $L$'s. Interception is mutual and identical in degree throughout the field, for each is on the same footing as all others. Interception can possess any value between 0 and 100 per cent in virtue of the fact that the locations can be shifted at will.

Figure 197 shows the square for interception at $16\frac{2}{3}$ per cent. The neutral planes at $M_1$ and $M_3$ divide this square into four quarters, *a, b, c,* and *d.* The fluid to be produced from each quarter will be produced only at the well in possession of the quarter. No fluid can pass the neutral planes to the well beyond.

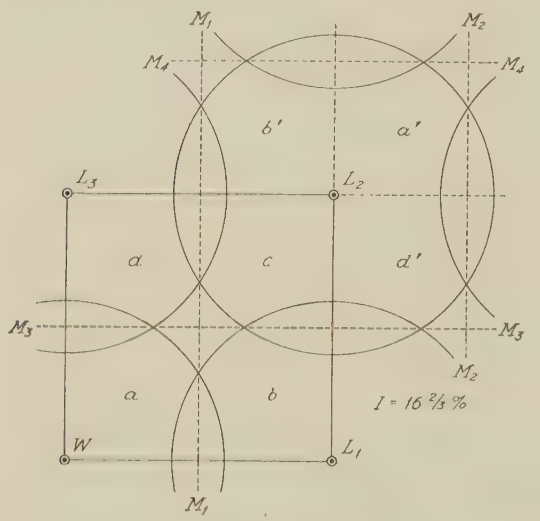



FIG. 197

The situation can be visualized more easily, perhaps, in the square having $W$ or an $L$ at its center. Thus the square about $L_2$ has the quarter $c$ in common with the principal square, $a'$ is identically like $a$, $b'$ like $b$, and $d'$ like $d$. Whatever we find to be true for this square we know to be true for the principal square. The quarters now belong solely to $L_2$. The volume cone for this location is shown to be cut on four sides.

Let us shift the locations in order that interception will be 29.29 per cent. This situation is shown in Figure 198. Now the volume cones touch one another at the center of the principal square. The four-sided "star-shaped" area, left entirely intact in the preceding case, is reduced to zero. A truncated pyramid of conical sides, representing fluid to remain unproduced at $16\frac{2}{3}$ per cent interception, is cut down to a true pyramid of like sides, with an altitude of the same height. The cone for $L_2$ is cut in such a way that the drainage area is a perfect square.

A further shift in the locations is shown in Figure 199. Here interception is 50 per cent. The cones of diagonal wells intercept, but this fact has no

effect upon the situation because a neutral plane at $M_5$ is inoperative so long as those at $M_1$ and $M_3$ are operative. At the center of the principal square there is still a pyramid, geometrically similar to the preceding one, but lower in altitude. Of course, no sign of this exists in the plane of our drawing, for it is buried so that its base is coplanar with the vertices of the original cones. The drainage area of $L_2$ is still perfectly square; the planes on $M_1$, $M_2$, $M_3$, and $M_4$ have moved in equally at the sides. It is not difficult to picture the cone trimmed in this fashion.

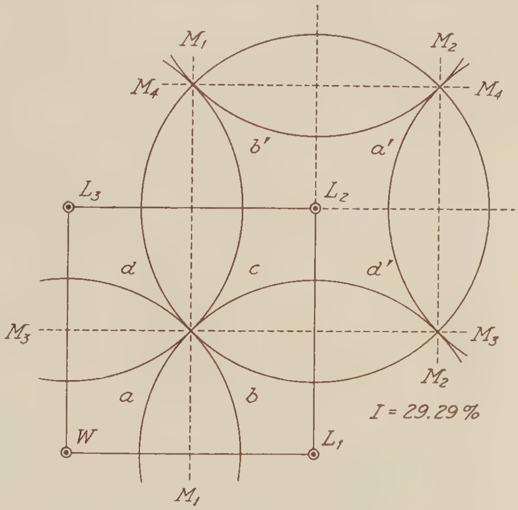



FIG. 198

We can shift the locations still farther, if we like, and draw our figures accordingly. It is obvious that after interception of 50 per cent we get into a maze of circles. Regardless of this fact the situation remains qualitatively the same up to 100 per cent interception. *For the range between 29.29 and 100 per cent the drainage area of* $L_2$, *and consequently that of each well constituting the pattern, is square, with the well itself at the center.* The length of the side of this square is equal to $WS$, the spacing between the wells.

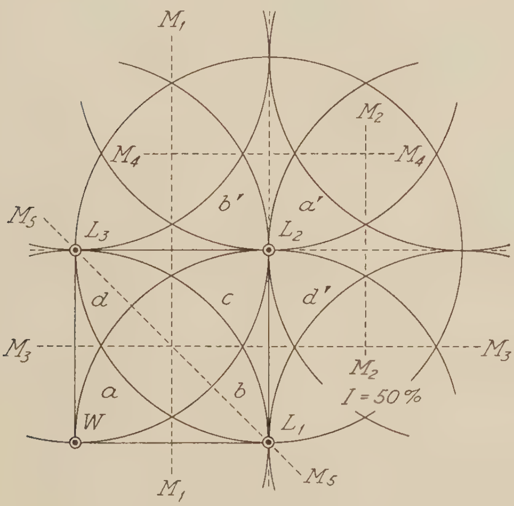



FIG. 199

For any degree of interception between 0 and 100 per cent the length of the side of the squares, $WS$, in terms of the radius $R$ is equal to twice the difference between 100 per cent and $I$ per cent: that is,

$$\frac{WS}{R}\% = 2(100\% - I\%)$$

Thus in Figures 197, 198, and 199, $WS$ is $166\frac{2}{3}$, 141.4, and 100 per cent of $R$, respectively.

Corresponding to Figure 196 we have Figure 200 (p. 548). The curve $AB$,

and its sections $A$ and $B$ taken separately, are either independent of, or dependent upon, the same factors as before. Between 0 and 29.29 per cent interception the drop in $A$ is four times that of $A$ in the earlier figure, in order to accommodate the four cuts shown in Figure 197. From this point the curve is a straight line to the lower left-hand corner of the plat, for after the base is square, successive cuts remove portions of the cone in accordance with an equation of the first degree between $x$ or $x'$ and $Vo'$.[14]

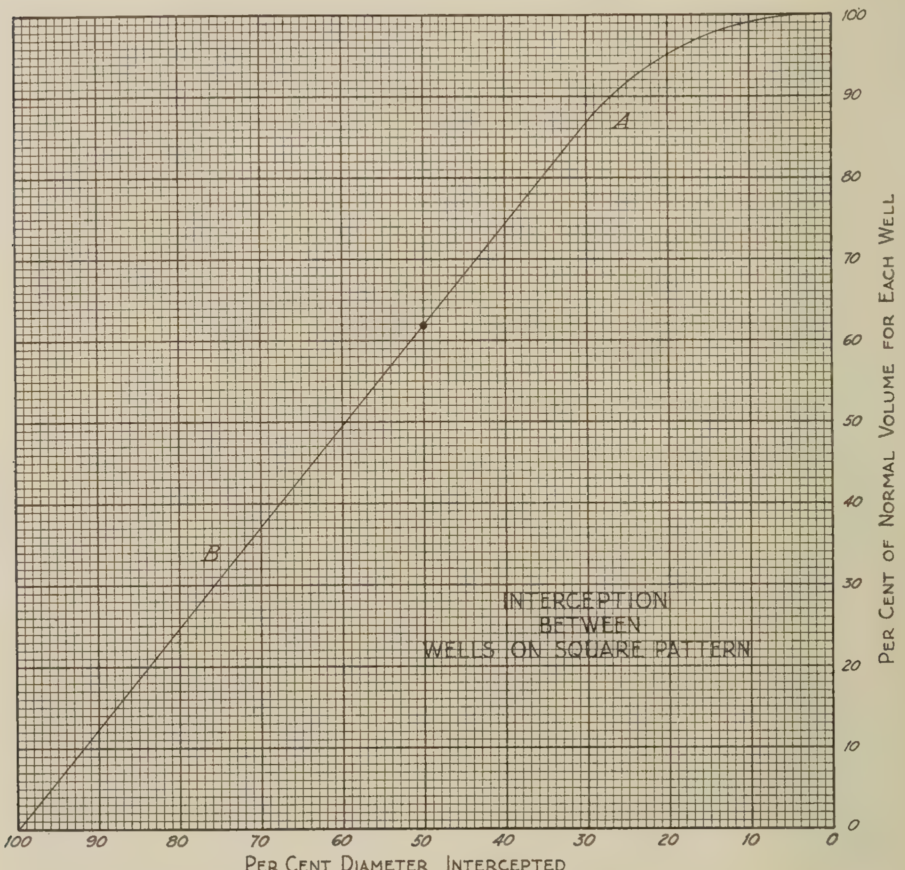



FIG. 200

When the locations are shifted toward $W$ to increase interception, more locations are required in order to cover a given area of land. The extreme point of the curve at the left merely fulfills mathematical conditions. For

[14] This proposition can be proved very easily. The truth of the problem in geometry is dependent upon the fact that the center of the cut coincides with the center of the cone. It is in fact immaterial whether the cut is square, triangular, pentagonal, hexagonal, circular, or of any other regular pattern.

100 per cent interception each well is one of an infinite number required to cover a finite area. If an infinite number of wells are located at one point, each one produces a zero volume in order that the total production from all may be finite.[15]

As the section $B$ dropped in position where there were two wells, so it does now. However, in the present instance both of its extreme points remain fixed, and it sags in the middle. The equation of the first degree is replaced by one of a higher order.

Figure 200 shows us what we may expect from interception under ideal circumstances. The curve is more apt to be fulfilled in $A$ than in $B$ by actual performance. $B$ for any particular well can be raised on the plat only by bringing in this well earlier than others that may intercept in excess of fifty per cent, or by decreasing the external friction head if it is brought in simultaneously with these others. In either case $B$ for the others is necessarily dropped on the plat. In this we assume that the constant back pressure for all wells is not in excess of a most satisfactory low value which is determined by technical and economic considerations in the particular field. If an operator can conveniently give his constant back pressure a lower value than that of his neighbors, it is clear that his cones cut in on those of his neighbors, and as a consequence he raises his curve $AB$ to their disadvantage.

When intercepting wells are not given a constant back pressure of the same value, the neutral plane becomes a curved surface. This surface has its concave side facing toward, and its center farther from, that well having the smaller value of this back pressure.

If the productive formation is composed of strata which differ in texture, $R$ obviously possesses different values in these strata. This is the result of heterogeneity that so frequently exists because of the conditions under which deposition originally took place. The individual strata are quite homogeneous in their lateral extent, while they constitute a heterogeneous formation in its vertical extent. The cones are composite, in that they are made up of a series of horizontal sections of several cones—one for each individual stratum—having different bases of circular area and different altitudes. The percentage value of interception differs for the several sections;[16] yet, on the assumption that the imposed conditions of production are identical at the intercepting wells, there is but one continuous neutral plane midway between each pair of wells.

Inasmuch as the external friction head at intercepting wells affects the fulfillment of the sections $B$ in Figures 196 and 200, we cannot expect the data of performance of these wells to be sufficiently reliable for the practical

[15] Certainly we are not concerned in practice with this limiting situation. However, it is approached in fields of this control, wherein property holdings are of the size of town lots.

[16] This is in accord with Equation 597 (p. 540). $WS$ is a constant for the situation with respect to the individual strata.

purpose of computing either the value of a unique $R$ for the vertically homogeneous formation, or that of the average $R$ for the vertically heterogeneous formation, from the foregoing equations between $WS$, $R$, and $I$. Such a method depends upon data sufficiently accurate to determine $I$ directly from the curves $AB$. Where interception is known to be less than fifty per cent, the sections $A$ might prove to be of some use, at least in check computations if not in original ones.

Offset wells in this control fulfill their purpose admirably, provided they are equidistant from the property line, and provided the imposed conditions of production are identical at the two wells. The neutral plane is thus given the desired location, inasmuch as the property line lies within this plane.[17]

[17] Only the front-line wells serve as offsets. Wells on a second line have no influence beyond the neutral plane which they have in common with those on the first line. This situation is unique in Capillary Control.

CHAPTER XXXII

# Maxima of Production by Natural Flow

"Physical science has by the practical realization of its results transformed the entire life of modern humanity. But, as a rule, these applications appear under circumstances when they are least expected; to search in that direction generally leads to nothing, unless certain points have already been definitely fixed, so that all that has to be done is to remove certain obstacles in the way of practical application."—HERMANN VON HELMHOLTZ

**185.** *A typical contour-map.*—Our investigation concerning the features of a natural reservoir in Hydraulic or Volumetric Control led us, in section 141, to consider the typical contour-map shown in Figure 132. It was specifically pointed out at that time that such a reservoir might be in any one of the three controls. If we now say that it is in Capillary Control, we know that certain features which are involved with a definite drainage radius $R$ must be included. We want to add the necessary symbols for these features, for the purpose of reviewing briefly the circumstances that are peculiar to this control. To do this we must assume a length for $R$. What length shall we arbitrarily select as a reasonable one? In view of indirect computations which I have made from field data I can suggest 3,000 feet—roughly six-tenths of a mile—as a length sufficiently appropriate for the present, with full knowledge of the fact that $R$ is frequently greater, and as frequently less, than this amount in our fields of economic importance.[1] If this is satisfactory, we have our contour-map in Figure 201 (p. 552). The initial cones have circular bases in the manner shown.[2]

If the pool is of oil, completely surrounded by water, part of the particular liquid—whether oil, water, or both—that happens to be included within the drainage areas is produced from the respective wells. Part of the gas within these same areas is necessarily produced with the liquid. On the other hand, if the pool is of gas, completely surrounded by water, we must say that part of the particular fluid—whether gas, water, or both—that happens to be

[1] The data furnished values of $Ve$, $L$, and $h_{ro}$. These on substitution into Equation 569 (p. 505) are assumed to give a reasonable value of $R$.

[2] All wells have been given the same radius, regardless of the fact that both oil and water occur within the formation. We may expect the radius to be greater where there is water. See § 99 concerning the total resistance set up within a unit volume of formation.

included within the drainage areas is produced. Part of the gas in these same areas is produced, regardless of whether water or no water is produced.[3]

The drainage areas of nine of the wells in the figure are intercepted; the tenth well is isolated, in so far as its area is not intercepted. No fluid of any sort can pass the boundaries of the circles, nor can it pass the neutral planes

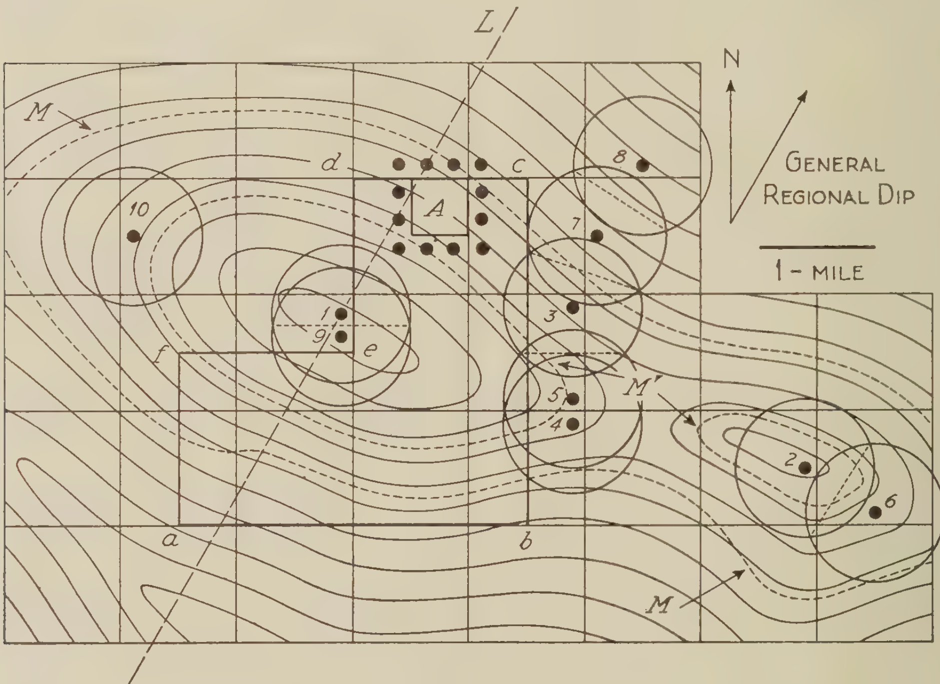



Fig. 201

where these are present, to remain within the drainage areas of the respective wells or to be produced therefrom. The static pressure gradient remains perfectly intact beyond the boundaries of the conical depressions caused by production. When later wells, without restriction as to time, are drilled into the formation, their intensive primary functions—pressure, velocity, acceleration, and power—have initial values independent of the existence of the earlier wells, if their locations are exterior to the drainage areas of these earlier wells. Their extensive primary functions—volume, energy, and life—have initial values independent of the existence of the earlier wells, if their drainage areas do not intercept those of the earlier wells.[4] In other words,

[3] The fluid within the drainage radius being what it is, the fluid produced is what it is. This applies equally to edge water and to bottom water.

[4] The statements here made are obviously a review of events and conditions that are described in the preceding chapters. Although these propositions are based upon mathematical analysis, they are certainly in accord with observations in fields of this control. (See § 146.)

the initial values of the intensive primary functions are the same as they would have been had the earlier wells never existed, provided interception is less than fifty per cent, and the initial values of the extensive primary functions are the same as they would have been had the earlier wells never existed, provided interception is less than zero per cent. For values of interception less than fifty per cent the initial values of the extensive primary functions are independent of the fact that there is an interval of time between the drilling of the earlier and later wells.

For convenience we shall assume that the pool in Figure 201 is of oil, with its associated gas, and consider production from the individual wells. Wells 1, 2, 5, 9, and 10 produce oil and gas to the exclusion of edge water. Wells 3, 4, and 6 will produce oil and gas with only very small amounts of water, since the little water that is included within their drainage areas is located far from the well, on the "feathering edge" of the volume cone. Well 3 will produce still less water in the presence of Well 7, for the two will establish their neutral plane midway between them, thus depriving Well 3 of the portion of its cone beyond the plane. Well 7 will produce only a very small amount of oil. In the presence of Well 3 it will produce still less. Well 8 can produce only water. This water will of course be accompanied by gas which it previously had held in solution. It is impossible for the control to be different within the pool and within the formation exterior to it; gas is the sole source of energy interior and exterior to the pool.

The water-line $M$ remains stationary throughout the formation, except as it may be included within the drainage areas of wells. In case it is included in these areas its movement is restricted to the radii of movement toward the wells. The line $M'$, as a subsequent position of $M$, is void. It has no meaning in this control, for *there is no encroachment of water in the usual sense.*[5] Water troubles in fields of Capillary Control are slight in comparison with those in fields of Hydraulic or Volumetric Control.

A property as at $A$ cannot be completely depleted of oil and gas by neighboring wells in this as in the other controls, although we must admit that a number of wells located in the manner indicated will measurably affect the volume to be subsequently produced from the property. The five wells whose cones extend into the area *abcdef* have little effect upon the total volume of fluid underlying the area, considering the vast amount of this fluid. Effects to any possible extent are completely nullified by offset wells within the area. In order that these may not drain excessively from beyond the boundary of the area they should be at like distances on their side of the property lines. In lieu of drilling these offset wells one would be justified in accepting compensation for the fluid within the sections of cones that are included in the property.

[5] The encroachment of water has been mapped at successive stages in the Coalinga Field (Volumetric Control), while the absence of encroachment has been noted in the Cushing Field (Capillary Control).

*Fields wherein reservoirs are of Capillary Control are admirably suited for reserves, without the necessity of covering the entire structure for protection.*[6]

To forecast the performance of wells in this control we should preferably do so with pressure-volume relations, as explained in section 161. The equal production per pound decline law is inappropriate, while the laws of equal and relative expectation are particularly adapted to this control. The latter, as we know, are based upon the ratio between the $K$'s in Equations 489 and 490 (p. 467). These $K$'s are dependent upon the dimensions of the volume cones; that is, upon

$$\tfrac{1}{3}\,\pi\, R^2 h_{Vo}, \quad \text{or} \quad \tfrac{1}{3}\,\pi\, r^2 h_{vo}$$

where the symbols have the usual significance attached to them. In different formations or strata the $K$'s ordinarily will not be equal. They will be equal in the same formations or strata, provided the conditions of interception between the active cones are identical.

Forecasting should be done independently of any map method.[7] Each well has a potential reservoir that is "self-contained," in so far as it is independent of the productive formation beyond its boundaries. Actual performance more closely approaches ideal or theoretic performance in this control than in either of the others, in virtue of this property of being self-contained. Accuracy of performance is not dependent upon vertical homogeneity of texture within the formation. Our successful experience of the past in applying the law of equal expectation indicates the fact that productive formations are generally more homogeneous in their lateral extent, stratum for stratum, than we might otherwise imagine. This law is dependent upon lateral homogeneity of texture, and in its accurate fulfillment we have evidence of this homogeneity.[8]

The potential energy by virtue of which fluid is produced at the well is invested solely in the gas that is present within the formation. While we may be as careless as we like with the energy possessed by the gas in reservoirs of Hydraulic and Volumetric controls, we must be careful with it in reservoirs of Capillary Control. Its wasteful displacement from these reservoirs unduly shortens the radius $r$ of the active energy cone in the process of production, and this $r$ is of course the same as the radius of the contempo-

[6] See § 141.

[7] In brief summary we then have the following guide in forecasting production from wells in the three controls:

1. Hydraulic Control—map method exclusively
2. Volumetric Control—curve method, supplemented by the map method
3. Capillary Control—curve method exclusively

In forecasting production from the field at large it is obvious that in all controls we must not ignore the map of the field.

[8] The influence of local features in structure is of less importance upon performance in Capillary Control than in Hydraulic and Volumetric controls.

raneous active pressure and volume cones. *It is important to note that wasteful displacement does not affect the radius* R. *It does not affect any portion of the productive formation beyond* R.[9]

In natural reservoirs which produce gas alone the globules break and make, whereas in those which produce oil and gas in combination they break, make, and slide simultaneously throughout the mass of porous formation included within the space occupied by the active volume differential cylinder. This we infer from the behavior of the capillary and porous-filled tubes.

**186.** *Recovery by natural flow.*—In view of the fact that each well in a field of this control has its own potential reservoir it is easily seen that the percentage recovery of oil or gas from a given formation presents a distinct problem in itself. The principles which hold for reservoirs in Hydraulic or Volumetric controls are now inappropriate. Wells 1 and 2 in Figure 132 can produce all the oil within the designated pool, if the reservoir is in Hydraulic Control. The same is true if the reservoir is in Volumetric Control, provided equilibrium at either well is not established before the pool at large, or its separate portions on the two domal structures, can become depleted. It is impossible for Wells 1 and 2 in Figure 201, even with the aid of any finite number of additional wells, to drain the pool of oil completely. *In the first two controls Nature is driving the oil to the wells with water under its own weight, while in the present control she is simply permitting a portion of the oil within a restricted area to move toward the well as an accompaniment to the expansion of gas within this same area.*

Let us investigate recovery in this control. To do this it will first be convenient to recognize recovery under two different circumstances: namely, (*a*) recovery as a result of natural flow, and (*b*) recovery as a result of forced drive. *Each circumstance presents its own peculiarities; recovery in accordance with them can be said to show different maxima of production.*

By natural flow I mean that the line $N$ in Figure 177 may have any possible position between the lines $K$ and $I$, and $W$ is permitted to approach and attain equilibrium on its own accord, and in agreement with such given conditions of production. Thus the constant back pressure may possess any value less than the static pressure of the reservoir. $W$ may produce gas alone or oil and gas in combination. In the latter case $N$ may lie above the elevation at the head of the well, as a result of the well's having to raise its oil to a greater height. $N$ may lie exactly in the position shown, indicating the fact that the oil is delivered by the well itself at the head of the casing. $N$ may lie below the position shown, and at or above the line $J$, in virtue of a lift-pump or a

[9] See footnote 8, § 173, page 502, and footnote 16, § 177, page 520. The reservoir has a certain amount of potential energy depending upon predetermined conditions, and wasteful displacement simply allows this energy, through its component pressure, to escape from the reservoir without carrying its due proportion of liquid to the bottom of the well.

gas-lift installed at the well. Lastly, $N$ may lie at any position between the lines $K$ and $J$ in virtue of a vacuum pump installed at the well. *Under any of these conditions the well performs precisely in the same way.*[10] *The only difference to be noted is a quantitative one, and this difference will be easily reckoned with.*

In case $W$ is a gas well a similar general description holds for its natural flow, although it is evident that the range of positions for $N$ is ordinarily not so great.

The statement was made in section 174 to the effect that the isolated well in Capillary Control produces one-third of the fluid underlying its drainage area, such fluid being mobile at the given constant back pressure which is exerted against production. This statement is exact. It refers, of course, only to a well that is free from interception. Nothing is said with regard to recovery in terms of the static, or absolute, volume of fluid underlying its drainage area, nor is anything said with regard to recovery from the field at large. The latter question would likely not be raised in the case of one isolated well in the field. The former is very easily answered, for by referring to Figure 178 we see that the one-third, or 33⅓ per cent, refers to the fluid in the volume cylinder above the plane defined by $N$; that is, to the fluid which we call the mobile volume. Between this plane and one defined by $K$ lies the immobile volume. The static, or absolute, volume occupies the entire cylinder between $K$ and $I$. We may consequently write

$$33\tfrac{1}{3}\% \times \frac{P}{S} = Q_S\% \quad \ldots\ldots\ldots\ldots\ldots\ldots (606)$$

wherein $P$ is the initial potential pressure at the given constant back pressure $C$, $S$ is the initial static pressure of the reservoir as measured at the well, and $Q_S$ is the recovery in terms of the static volume of fluid underlying the drainage area of the isolated well. The equation is based upon one-one cylinders in pressure and volume; that is, upon pressure-volume relations in Volumetric Control.[11]

The one-third recovery is based upon the assumption that the well penetrates the entire thickness of the formation, and the casing, if present, is perforated throughout the length corresponding to this thickness. If the well merely penetrates the top of the formation, a top-slice of the lower half of a *one-zero differential sphere* replaces the one-zero differential cylinder, and recovery from the drainage area is less than one-third according to the ratio

[10] For this reason alone the term "natural flow" is given its present interpretation. Its usual interpretation is restricted to flow with $N$ in the position shown in the figure. (To avoid the introduction of a new term I have broadened the scope of an old one.)

[11] In Fig. 178 $P/S$ has the value of 5/9.2. Therefore

$$Q_S = 33\tfrac{1}{3}\% \times \frac{5}{9.2} = 18.1\%$$

If initial values of $P$ and $S$ were not observed at the time the well was brought in, they may be computed by means of the relative curves.

between the thickness of the formation and the radius of drainage $R$.[12] We shall continue to assume full penetration in our present investigation.

When wells cover a land area the recovery from the formation at large is less or more than one-third of the mobile volume, depending upon the spacing between the wells and their radius $R$. In Figure 202 ($a$) (p. 558) we have the well $W$ and sets of three locations $L$ in accordance with a square pattern of locations in the field. Let us begin with $L_1$, $L_2$, and $L_3$, forming with $W$ a square whose length of side is $10R$. This is a unit area, we shall say, in the midst of a producing field. The pool below extends indefinitely on all sides, and the entire field is covered by such squares as this. Whatever recovery we have from one square we have from all squares, simply because all are specified to be alike. What percentage of the mobile volume is recovered from this square?

The square contains four quarters of volume cones that are identical; they are equivalent to one complete cone. Of all the mobile fluid under the unit area only that from one cone is recovered. The volume of this cone is

$$\tfrac{1}{3}\,\pi\, R^2 h_{Vo}$$

while the volume of the prism of mobile fluid under the square is

$$(10R)^2 h_{Vo}$$

where $h_{Vo}$ is the height of the cone and the prism. The percentage ratio between these two quantities is 1.047 per cent. Recovery bears this percentage of the mobile volume throughout the field.

Next, let us take the locations $L_4$, $L_5$, and $L_6$. The entire field now contains locations with a spacing of $5R$. The size of the unit area is reduced accordingly. Now the volume of the equivalent cone is the same as before, while the volume of the prism is

$$(5R)^2 h_{Vo}$$

The ratio between the two quantities is 4.19 per cent, and therefore the recovery is 4.19 per cent of the mobile volume.

Lastly, let us suppose the locations to be $L_7$, $L_8$, and $L_9$. The size of the unit area is reduced to a square of sides $2R$. The volume of the prism is

$$(2R)^2 h_{Vo}$$

The ratio and the recovery are 26.18 per cent.

We have proceeded sufficiently to see that we may write the equation

$$(WS)^2 Q_{Pn} = 104.7 \quad \text{......................} \quad (607)$$

wherein $WS$ is the well-spacing in accordance with the square pattern for locations, $Q_{Pn}$ is the recovery in terms of the mobile volume, and 104.7 is the constant which satisfies the conditions in accordance with the square pattern.

[12] For the discussion concerning the differential sphere see Appendix F.

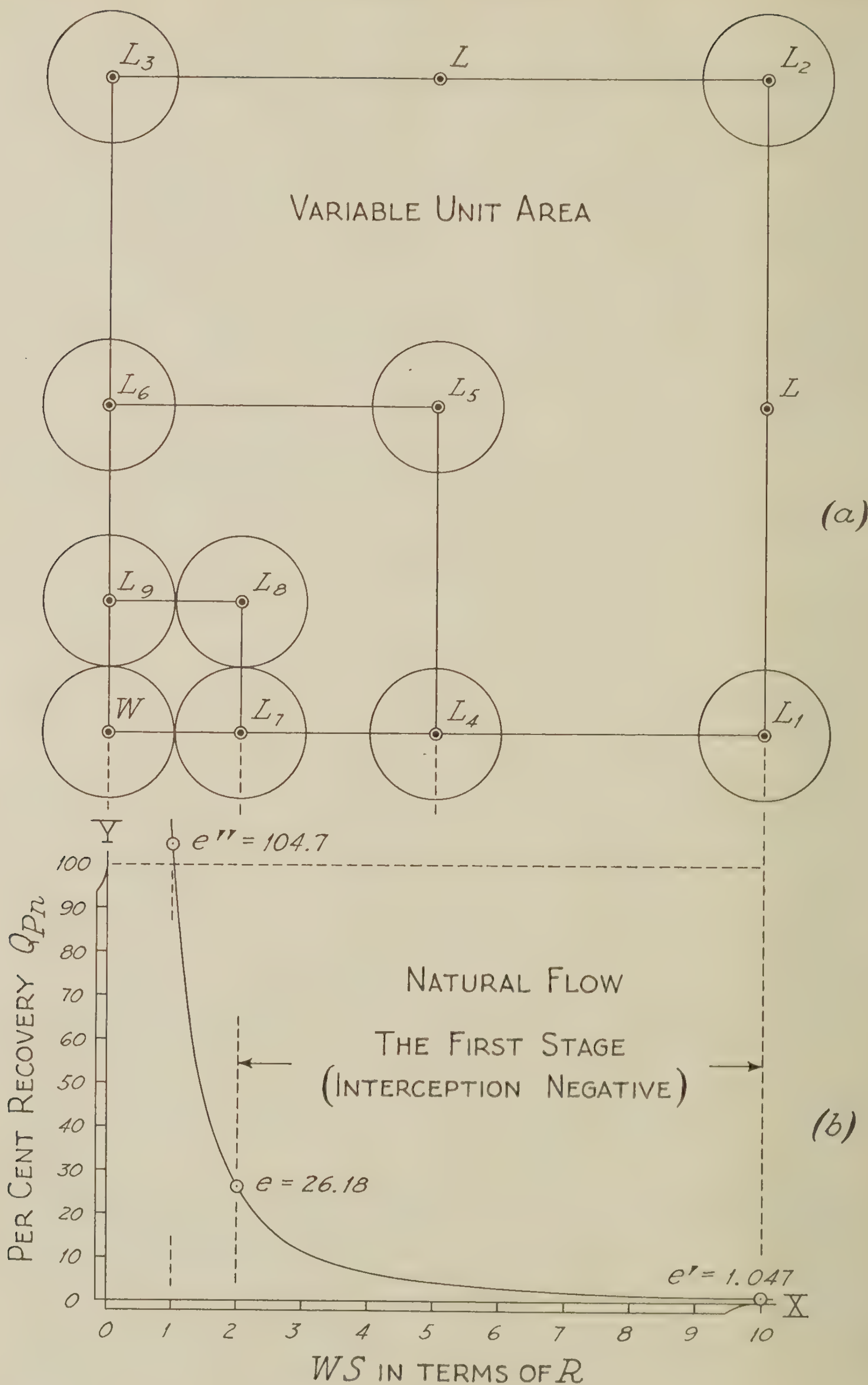

$L_3$
$L$
$L_2$
VARIABLE UNIT AREA
$L_6$
$L_5$
$L$
(a)
$L_9$
$L_8$
$W$
$L_7$
$L_4$
$L_1$
Y
$e'' = 104.7$
100
90
80
70
60
50
40
30
20
10
0
PER CENT RECOVERY $Q_{Pn}$
NATURAL FLOW
THE FIRST STAGE
(INTERCEPTION NEGATIVE)
(b)
$e = 26.18$
$e' = 1.047$
X
0 1 2 3 4 5 6 7 8 9 10
$WS$ IN TERMS OF $R$



FIG. 202

The equation is that of a quadratic hyperbola of the general form $y^2x = k$. It may be written in the form of a variation. Thus

$$Q_{Pn} \text{ varies as } \frac{1}{(WS)^2} \quad \text{.................. (608)}$$

Recovery in percentages of the mobile volume varies inversely as the square of the distance between wells.[13]

If we wish to express recovery in terms of the static volume of fluid within the formation, we can do so in the manner of Equation 606. Thus we write[14]

$$Q_{Pn}\% \times \frac{P}{S} = Q_{Sn}\% \quad \text{.................. (609)}$$

Here $Q_{Sn}$, like $Q_{Pn}$, refers to the fluid underlying a unit area in the field. It therefore refers, again like $Q_{Pn}$, to the field at large.

Now that we have Equation 607, suppose we substitute the value $WS = 1$ into it.[15] We see that $Q_{Pn}$ is then 104.7 per cent, as shown at $e''$ in the figure. This exceeds the amount of mobile volume underlying the unit area, since this is obviously 100 per cent, no more and no less. Again, suppose we substitute the values $WS = 1/2$ and $WS = 1/10$. We see that $Q_{Pn}$ is 419 and 10,470 per cent, respectively. Of course these figures are absurd. They ignore the effects of interception, for after $WS$ becomes equal to $2R$ on shifting the locations toward $W$, the cones are not complete, as explained in the preceding chapter. To assume that they are complete, as implied in the use of the equation for values of $WS$ less than $2R$, is an error. We cannot count our units of volume of fluid more than once. It is quite evident that the count mounts very rapidly as we approach $W$, as shown by the two last substitutions. In fact, when $WS =$ zero, that is, when interception is 100 per cent, the count is infinite.

*The inverse square law for recovery holds for locations in accordance with the square, the triangular, and the hexagonal patterns for locations.*[16] *It holds only when interception is zero or negative.* We can say that this is the law of the *first stage* in the spacing of wells. It covers the region between $WS$ equal to infinity and $WS$ equal to $2R$. For the region between $WS$ equal to

13 For the discussion concerning the triangular or hexagonal pattern for wells see Appendix H.

14 To facilitate the retention of the significance of our symbols in mind let me say that $Q$ stands for recovery (per unit of area), $S$ stands for static quantities (pressure and volume), $P$ stands for potential quantities (pressure and volume), and $n$ stands for natural flow.

15 $WS$ here, as hereafter, is expressed in terms of $R$. The present value is then once the value of $R$.

16 As a matter of fact we can also include in this statement the "five-point" pattern consisting of the square pattern plus a centrally located well.

$2R$ and $WS$ equal to zero we have the law of the *second stage.* This is derived in accordance with the principles of interception.

To derive the law of the second stage we take our plat as in Figure 203. At zero interception we have the initial point for the curve $AB$ from the law

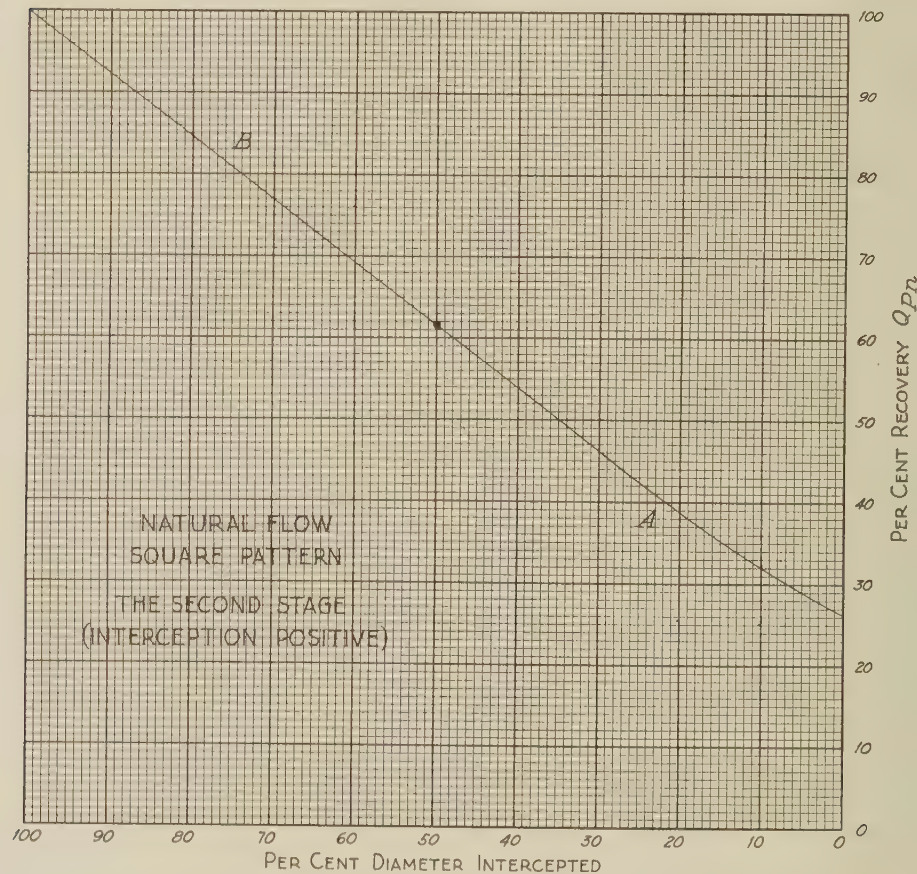



FIG. 203

of the first stage: namely, at 26.18 per cent. From here, to and including interception of 29.29 per cent, points are determined from those of the fundamental curve of interception, Figure 195. The latter points give us the volume of the cone after four sections are cut off, as in Figure 200. On dividing this volume by that of the inclosing prism, and expressing the result in percentage values, we obtain points for this section of the curve. After 29.29 per cent interception the cone is cut to a square base, likewise as in Figure 200. Computed points fall on a straight line that passes through the upper left-hand corner of the plat.[17] This, we shall say, is the law of the second stage expressed graphically. It is impracticable, if not impossible, to

[17] See footnote 14, § 184, page 548.

express the law verbally with any degree of accuracy. We therefore leave it as it is.

Like its predecessor, the law of the second stage refers to the mobile volume of fluid underlying the unit areas and the field at large. At 12 per cent interception $Q_{Pn} = 33\frac{1}{3}$ per cent; therefore, in comparing recovery from the formation in the case of the isolated well with that in the case of a land area covered by wells according to a square pattern, we can say that recovery with the latter is more or less than that with the former, depending upon the fact that interception is more or less than 12 per cent. (By "less" than 12 per cent I mean such positive numbers which are less, zero, and all negative numbers.)

Recovery shown by section $A$ of the curve is independent of any interval or intervals of time in bringing in the various wells. Section $B$ is based upon the assumption that all wells are brought in at the same time. Where there is an interval of time for one, two, or three wells, the curve, with its extreme points fixed, bows upward. Recovery is greater for interception of a given value in excess of fifty per cent when wells are not drilled in simultaneously.[18]

It is to be noted that the recovery of 100 per cent of the mobile volume requires 100 per cent interception. This simply means that in practice the casings of wells must touch each other over the area of the pool in order that this much of the fluid may be obtained by natural flow. *At any distance apart greater than this some of the fluid remains securely locked in place within the formation on account of Jamin action.*

To express recovery in terms of the static volume of fluid within the formation we again apply Equation 609. $Q_{Pn}$ is thus converted into $Q_{Sn}$ in a manner that is independent of negative, zero, and positive interception.

Given any value of interception in a field, where wells are located according to a square pattern, the corresponding ordinate, either in Figure 202 ($b$) for negative interception, or in Figure 203 for positive interception, represents in a mathematical sense the maximum of recovery for natural flow under the conditions of production. *The series of points along the two curves combined represents the locus of all possible maxima by natural flow.*

If we are given the cost of drilling to the required depth, the cost of maintaining wells in operation, the potential volume of the non-intercepted well, and the market value of the fluid, the problem concerning the economic spacing of wells in fields of this control for us to solve is reduced to the type of problem frequently met in engineering practice. The estimation of recovery by natural flow in fields of this control is given a reasonable foundation.

Where a field contains more than one productive formation on depth, it is clear that each provides a separate potential reservoir. They can, and in fact must, be handled independently of one another.

[18] The older wells will produce negatively. (See footnote 5, § 181, p. 536.)

187. *Restoration of gas pressure.*—Natural flow, we say, takes place on the basis of a circular drainage area. But one well is concerned with this flow, regardless of interception, and this well is located at one central point in the area. Each well possesses a volume cone that is only impaired at its edge in case of interception. To reduce the constant back pressure by means of a pump or lift merely provides the well with a larger volume cone. This alters the degree of interception, provided there is a well sufficiently close; otherwise the situation is unchanged geometrically. If the formation within the drainage area is laterally heterogeneous, we say that this area is not a perfect circle. It is a distorted, geometrically irregular, closed area. The cone exists none the less. Recovery from the isolated well is yet one-third of the mobile volume, since the volume of a cone is not dependent upon the shape of its base.[19] Such heterogeneous conditions may provide irregular areas that are not of equal extent for the various wells in the same field. In either case interception among the wells is not geometrically perfect and uniform throughout the field, even on the assumption that the wells are properly located at the corners of squares. All this we admit, but it is not the point at issue at the present moment. What is important for us is the fact that under no circumstances do we escape from cones with the wells at some unique center.

The well at this unique center produces oil, gas, or water, singly in the case of gas or in combination as between either or both liquids and gas, in virtue of gas pressure. Its volume cone, or volume differential cylinder, is impaired by neighboring wells on close drilling. Granted that all wells in a field are near or at the point of equilibrium for the given constant back pressure, there is yet mobile fluid within the formation, as reckoned at this constant back pressure. This fluid lies between the cut volume cones, or, if we prefer, it lies within the cut differential cylinders, in increasing amount as we measure outward from each and every individual well in the field.

In addition to this mobile volume there is all of the immobile volume, as reckoned at the constant back pressure, yet remaining within the reservoir. Efficient recovery first requires of the operator the reduction of this volume to an economic minimum.[20] In practice it is not possible to reduce this volume to zero, for to do so would require the application of a perfect vacuum at the well. Some fluid will then remain as immobile, and to this amount there must be added the accompanying portion of the mobile volume to account for all fluid yet within the reservoir. How can this retained volume be recovered?[21] It can be recovered either by the restoration of gas pressure or by the forced drive. Let us investigate the mechanics of the restoration of gas pressure.

[19] The base of the cylinder defining the mobile and immobile volumes will have identically the same shape and area as that of the irregular cone, and the volume of this cone will be one-third the volume of this cylinder. (See § 174.)

[20] This reduction is to be accomplished only by the reduction of the constant back pressure to its economic minimum. (See § 168, Case 3 in theoretic performance.)

[21] See footnote 13, § 174, page 506.

We recall the fact that our natural reservoirs in Hydraulic and Volumetric controls are of the open type. According to the discussion in section 60, if we pump gas into them through some of their wells, we contribute a small amount to their energy if we can cause a pocket of gas to form on the crest of a structure, such pocket forcing the free surface of liquid down the flanks of the structure. And if we are successful in accomplishing this, the gas which we have pumped into the reservoir must first be produced in order that the liquid may again come to the bottom of wells which are located within this gas pocket.[22] On the other hand, if we are unsuccessful in forcing the free surface of liquid down the flanks of a structure, such structure being locally absent, then we contribute nothing to their energy. Where gas is not the source of the energy, the wells produce regardless of the presence of the gas, and it is absurd to attempt to increase the energy by the addition of more gas.[23]

Quite the contrary, however, is the situation with respect to natural reservoirs of Capillary Control. These are, as we have learned, of the closed type. Where, as it is in these reservoirs, gas is the source of the energy, the addition of more gas is feasible and entirely proper. Each cubic foot of gas pumped into them increases their energy. As a consequence the pressure gauge attached to the wells through which the gas is pumped will show an increase, while such is not the case in Hydraulic and Volumetric controls.[24] *Only those natural reservoirs wherein cones and differential cylinders exist can have their energy restored by pumping gas into them—not those wherein cylinders of uniform density exist.*[25]

When gas is pumped into a well of this control a cone is built up within the reservoir. If the well is at equilibrium at the time pumping starts, the cone begins with zero dimensions, but if the well is at any other stage of production, the cone begins with the dimensions possessed by the active cone of the well at the time. In either event the base of the cone being built up,

[22] It is safe to assume that any successful action of gas pocket as herein described is absolutely worthless from the point of view of production.

[23] Our water wells in Hydraulic and Volumetric controls have produced water for years without gas. The fact that petroleum has a gaseous constituent at normal temperatures, while water has not, should not lead us astray from the principles of fluid mechanics. That we have been led astray is without doubt due to our contact with unrecognized principles of Capillary Control.

[24] The contrary situations are clear in the light of the mechanics of reservoirs. They have been heretofore corroborated in the field. The problem of the gas-lift must not be confused with the problem of the restoration of the gas pressure. These two problems must be solved separately. The gas-lift merely offers a problem in Case 3, theoretic performance. Having for its action a region exterior to the orifice of the natural reservoir (at the bottom of the well) it is equally and properly applicable to all wells, regardless of the control of the reservoir.

[25] Energy is repeatedly restored and withdrawn from the Zoar Storage Field. (See Appendix I.)

for ten equal intervals of pressure, is shown in Figure 183, a plan corresponding to the profile in Figure 181 (*b*). This cone we might appropriately call a "gas cone." It is taking the place of a former "oil cone" or "oil-water cone," depending upon the nature of the liquid previously present within the drainage area of the well.[26] Of course, we are not to overlook the fact that both globules and bubbles exist in all cones.

If pumping is continued until $r$, the radius of the gas cone, is equal to $R$, the original drainage radius of the well, the static pressure gradient plane is completely restored. It no longer contains a conical depression. The effects of this restoration can be seen in Figures 190 and 192. Let us say that $W$ is the well at which the gradient plane is restored. Clearly, then, the neutral plane at $M$ is rendered void. $L$'s volume cone is restored to the condition that would have existed if $W$ had not been drilled. Fluid within $B$ is restored to $L$. If $L$ has reached equilibrium at the time of restoration at $W$, it becomes active once more, and continues so until the quantity of fluid $B$ is produced. If $L$ has not reached equilibrium at the time, its activity is simply increased on account of the same quantity of fluid $B$. Where interception is less than fifty per cent there is sufficient mobile liquid within the formation to replenish $B$ for $L$, but where interception is more than this amount there may or may not be sufficient liquid for this purpose. In case there is not, a certain amount of gas that is pumped in at $W$ escapes at $L$ in place of so much liquid from $B$.

The same effects are to be seen in Figures 197, 198, and 199, where wells cover a field. Alternating wells in checker-board style, when restored in gas pressure, provide "repaired" volume cones for the alternating intervening wells.

There is no reason why the radius $r$ of the gas cone cannot exceed $R$, the original radius of the well. In fact, as shown in Figure 183, the final radius of the gas cone may have any value $R'$ greater than $R$, provided there are no abandoned and ignored wells such as $W_2$ remaining open, and provided further that no fault or fault zone, as at $F$, serves as a means of escape. We may, of course, be reasonably certain of no fault at $F$, if a well such as $W_3$ has produced a normal amount of fluid in the past.

Where the gas cone is given a radius $R'$, there exists a "conical hill" on the plane of the static pressure gradient. $L$'s volume cone—to return to our other figures—is not only repaired as to $B$, but it is enlarged to the extent of a "conical bulge" on the side facing $W$. $L$ simply performs for a greater radius $R$ in a sector facing that well. *The base of the bulge is elliptical.* This is inferred from the situation in connection with the forced drive, the subject to be discussed in the next chapter.

[26] We might include the original "water cone" and the original "gas cone," although there appears to be no economic reason for the restoration of gas pressure in reservoirs of this control producing water alone or gas alone for the purpose we now have in mind. In the storage of gas—for example, in the Zoar Storage Field—we do cause gas to take the place of a former gas cone.

The recovery $Q_{Pn}$, and therefore the recovery $Q_{Sn}$, is increased by the restoration of gas pressure. In case the static pressure is restored to its original amount at those wells where gas is pumped into the reservoir, the increase amounts to the quantity of fluid $B$ for each and every interception in the field. If the static pressure is partially or excessively restored, the increase is difficult to calculate in advance of actual production.

If the well at which gas is pumped into the reservoir were to be opened after such an operation and treated as a producer of gas, it is evident that its performance would not differ in the slightest degree from a normal gas well in this control. The gas cone that is built up by the pump becomes an ordinary active cone of production. It would make no difference whether the gas cone replaced an earlier oil cone, oil-water cone, or pre-existent gas cone. Thus an abandoned reservoir of any sort, provided it is by nature one in Capillary Control, may be used for the storage of gas. All gas pumped into it will of necessity be returned by the process of production.[27] *We may safely assume that a more secure container for gas does not exist.* Its worthiness as a container for stored gas is established with certainty in the fact that it had previously held fluid through geologic ages without loss in the vicinity of the well. Had it been an incompetent container, the well would not have been a producer at the time it was completed.

It would be unwise to attempt to store oil in an abandoned reservoir of Capillary Control. When oil is produced it loses its dissolved gas upon the release of pressure. To place it again under pressure in the presence of gas would effect the solution of the gas. The bubbles within the formation would thus become dissolved, if we pump gas-free oil into the reservoir, and upon subsequent production some of the dissolved gas must leave the reservoir with the oil. After repeating this combined process of storing and producing a few times the oil would remain in the container, since it would not possess the necessary energy for production.[28]

To return now to our subject. No particular mention has been made concerning the nature of the fluid with which the pressure is restored other than to speak of it as gas. Presumably we have had natural gas in mind. But physically we recognize air as a gas. Shall we restore pressure with natural gas or with air? Where interception is less than fifty per cent air has its advantages. It is to remain within the formation; none of it is to be produced at the rejuvenated wells; it is less valuable as a marketable commodity.[29] That any danger is involved because of the explosive mixture it is capable of form-

[27] This fact is clearly demonstrated in the operation of the Zoar Storage Field.

[28] This phenomenon is shown by reservoirs of the closed type, whether they be in Volumetric or in Capillary Control. It is easily verified with an "empty" inverted glass bottle, properly set up in the laboratory, and successively filled with water and allowed to produce.

[29] We are here assuming that the operation is confined to the restoration of gas pressure, and not extended to the forced drive.

ing with natural gas already present within the reservoir seems improbable. Where interception is more than fifty per cent, natural gas has its advantages. Not all of it is to remain within the formation, for some of it is to be produced at the rejuvenated wells; it is better than air as a mechanical conveyor of vapors. The latter circumstance is favorable toward an easy, and consequently an inexpensive, recovery of a valuable fractionated product. There is, of course, no explosive mixture to contend with.

To consider the restoration of pressure by pumping water into the reservoir seems unworthy. To do so, however, would not be absurd from the point of view of mechanics, for the pressure can be restored in this manner if desired.

The restoration of gas pressure is effectively a single process at one or more particular wells in the field. If the process is altered so that it becomes an intermittent one which is repeated a sufficient number of times, the results are effectively the same as those of the forced drive. Again, if the process is altered so that it becomes continuous, such a process actually constitutes the forced drive.

CHAPTER XXXIII

# Maxima of Production by Forced Drive

"Today we no longer beg of Nature; we command her, because we have discovered certain of her secrets and shall discover others each day. We command her in the name of the laws she cannot challenge, because they are hers; these laws we do not madly ask her to change, we are the first to submit to them. Nature can only be governed by obeying her."—HENRI POINCARÉ

**188.** *Conversion of control.*—We have learned that a natural reservoir of Hydraulic Control may become converted into one of Volumetric Control, and that one of Volumetric Control may become converted into one of either Hydraulic or Capillary Control. These conversions take place as a natural consequence of some factors that are within and other factors that are beyond our responsibility. Is it possible that a natural reservoir of Capillary Control might, as a consequence of factors beyond our responsibility, become converted into one of Volumetric Control? It is as likely to happen as a conversion from Volumetric Control to Capillary Control. Under given necessary conditions within the reservoir, namely, those with respect to the presence of both liquid and gas, and the porosity and texture of the formation, the failure of water to enter the formation from the surface will permit a conversion from Volumetric Control to Capillary Control, and its subsequent supply will permit a return to Volumetric Control.[1]

In section 150 the statement was made to the effect that $R$, the radius of action of $P$, does not extend from the orifice of the Jamin capillary tube to the other end unless $P$ is sufficiently great in its intensity. To convert the tube from Capillary Control to Volumetric Control it is only necessary to provide a pressure $P$ of sufficient intensity at the end of the tube to exceed the sum of all $f$'s within the tube, and thereby force all globules and bubbles out at the orifice.[2] So a subsequent supply of water from the surface, accord-

[1] I cannot cite a particular instance of conversion from Capillary to Volumetric Control in natural reservoirs. I only say that it is possible. If any constant supply of water enters the formation from the surface, obviously a hydrostatic head will be established and maintained within the reservoir, thus preventing the reservoir from performing in accordance with Capillary Control.

[2] This likely happened with Jamin in his experiments with oil and alcohol. (See § 147, Paragraph 7.) The sum of the $f$'s for a given number of globules of oil or alcohol is considerably less than that of the same number of globules of water in the same tube, because of their smaller values of surface tension.

ing to the preceding paragraph, might provide a pressure of sufficient intensity to exceed the sum of all the $f$'s within the formation and force all globules and bubbles out at the well, or wells.

If Nature fails to do this on her own account, why should we not do it? It is true that we can scarcely hope to cause a conversion on such a grand scale as she can; nevertheless, if we are content to confine our efforts to short distances between wells, we should be able to exceed the sum of all the $f$'s between such wells by applying a pressure of sufficient intensity at one of them. These is no question about our ability to do this, for we easily recognize this process as the *forced drive,* one that is already established in practice, and one that has already proved to be efficacious in fields of Capillary Control.

We have previously observed that where Nature has established, and continues to maintain, the forced drive, as she has in natural reservoirs of Hydraulic and Volumetric controls, there is little that we can do to help her. She drives with water, and if we are dissatisfied with the intensity of pressure at our wells, we can only seek an auxiliary supply of water at higher levels to be added to that already surrounding the pool. Lacking a copious and convenient supply we are helpless.[3]

There are differences—analytical differences—between a simple forced drive with the Jamin capillary tube and the more complicated one with the productive formation. The movement of fluid in the former is confined to one direction by walls of solid material. These walls are absent in the latter. We shall prefer to say that *the movement of fluid is here confined to a definite, localized area in the vicinity of the wells, these to be taken by pairs.* At one of the wells the driving fluid enters the formation, and at the other the driven fluid is produced. If we like, we can call them the "input" and "output" wells, respectively.

Let us investigate the geometrical shape of this localized area.

We have, we shall say, a well $W_1$ to use as an input well, and at a given distance $WS$ from it a well $W_2$ to use as an output well. If we imagine the latter to be closed while we fill the former with fluid under pressure, in agreement with section 187, a cone of the fluid selected for the purpose of creating the pressure is built up. This cone has a radius $r$ in the early stages of the process. It takes on a succession of values $r_1$, $r_2$, $r_3$, and so on, until it eventually attains a value equal to $WS$. Thereafter it exceeds this amount, reaching any prescribed value $R$, such as $R_1$, $R_2$, $R_3$, and so on. The fluid within the area defined by the base of the cone is under stress, whether the radius be $r$, $WS$, or $R$. None of it can flow from the formation, simply because there is no orifice from which it may escape. Inasmuch as there is a well at $W_2$ we might consider opening it from time to time during the process. As long as the radius has a value $r$, less than $WS$, the only fluid to be produced at $W_2$

[3] See § 142, last paragraph.

is that due to the restoration of pressure within the reservoir. We can hold $r$ constant by maintaining $p$ constant at $W_1$. This we do by closing $W_1$. Now we withdraw the fluid at $W_2$. When no more fluid comes to this well, we can increase $p$, say from $p_1$ to $p_2$, thereby increasing $r$ from $r_1$ to $r_2$, and withdraw the consequent additional fluid at $W_2$. On repeating this process a sufficient number of times we note a new feature at $W_1$. We close in the well at a pressure $p_3$ but it does not maintain this value. No matter what pressure we establish at $W_1$, and no matter how often we establish it on repetition, it invariably declines to a value $P'$. The radius $r$ is temporarily given a value $R$ greater than $WS$, since $p_3$ is a pressure $P$ greater than $P'$. The stressed condition cannot be maintained over so great an area with $W_1$ closed and $W_2$ opened. $P$ declines to $P'$, the latter corresponding to $WS$.

Instead of closing $W_1$ we might continue to add fluid under pressure. If we thus maintain a pressure $P_1$ at this well, we actually maintain a radius $R_1$, and this radius defines a circular area that is maintained under stress. No fluid can flow from the reservoir except that within the vicinity of $W_1$ and $W_2$. There is a restricted area here that will eventually be swept clean of the particular driven fluid, presumably oil.

The situation is depicted in Figure 204 (p. 570). $P_1$, as we say, maintains a radius $R_1$ in excess of $WS$. $R_1$ we know to be the radius of action of $P_1$ for the given formation with its fluid. Assuming $WS$ to be drawn to scale we may place pins at $W_1$ and $W_2$, and fasten to them a thread of length $R_1$ on the same scale. Now we can say that movement of the fluid within the formation is confined to all possible positions in which this thread may be imagined to extend between the two wells. In so far as $P_1$ alone is concerned, it can always move fluid over a path of length $R_1$. *It of course can move fluid over a shorter path, but certainly it cannot move fluid over a longer one.* But $W_2$ is the only orifice within the distance $R_1$ from $W_1$. Therefore $P_1$ can only move fluid a distance of $R_1$, as this might be made to extend between $W_1$ and $W_2$. If we take a pencil, and draw a closed area with it while it is held in the loop of the thread, we circumscribe the area to be swept out by $P_1$. $W_1cW_2$ represents one of an infinite number of positions for the pencil in the loop. It is clear that

$$W_1c + cW_2 = R_1$$

And this is true irrespective of the position of $c$ on the circumference of the closed area.

This closed area is the maximum area of influence of the pressure $P_1$. The manner in which we have drawn it, and its simple geometrical properties illustrated by the above equation, establishes the fact that it is an ellipse. The two wells are at its foci. *So long as we manipulate one well, either on filling or on producing, we are concerned with true circular areas with a unique center, but as soon as we manipulate wells in pairs, as we have in the present instance, we are concerned with "such circular areas which possess*

*two centers": namely, the ellipse.* The change from the circle to the ellipse is inevitable under the circumstances.[4]

We can cause a conversion of control to take place, but Nature restricts us to a localized area. During the intermittent process wherein $W_1$ is first

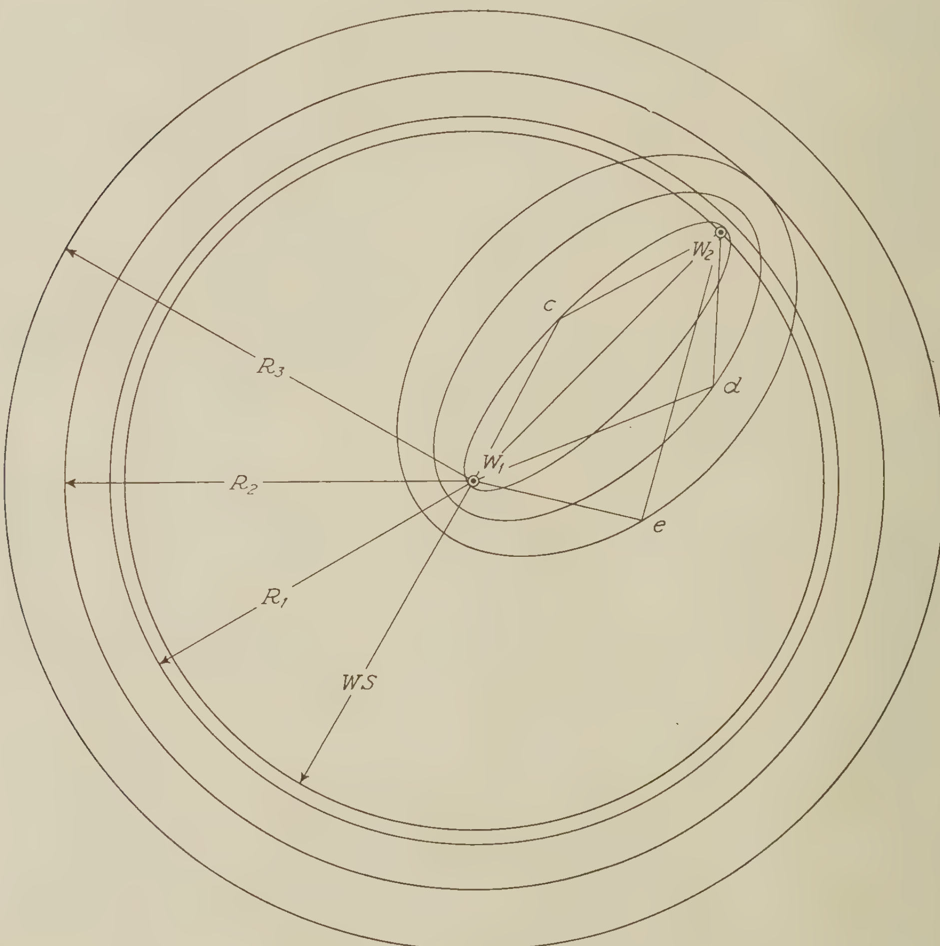



FIG. 204

opened for filling and then closed for flow at $W_2$, the control is Volumetric. Once that we have this control we can cause a further conversion to Hydraulic Control, as we have done in the continuous process just described.

$P'$ is a critical pressure. It is equal to the sum of all the $f$'s between $W_1$

[4] The principles of the forced drive, as herein described, are fundamental. They do not depend upon local features which are either imagined or known to exist in every field wherein the reservoir is in Capillary Control. Local features are not to be ignored in practice, for our procedure must be in accord with them. However, under no circumstances will these features upset or modify the general principles.

and $W_2$. The forced drive cannot exist unless $P$ exceeds this critical pressure. This means that the ellipse between the wells must have a minor axis greater than zero.[5]

If a pressure $P_2$, greater than $P_1$, is maintained at $W_1$, the ellipse is defined by

$$W_1d + dW_2 = R_2$$

where $R_2$ corresponds to $P_2$. Again, if a pressure $P_3$, greater than $P_2$, is maintained at $W_1$, the ellipse is defined by

$$W_1e + eW_2 = R_3$$

where $R_3$ corresponds to $P_3$. These ellipses are shown in the figure.

It is evident that the ellipses possess areas that depend upon the intensity of the pressure applied at the input well, so long as the constant back pressure at the output well is not changed during the process.[6] *Not one molecule of the driving fluid can pass the circumference of the ellipse, and not one molecule of fluid beyond the circumference of the ellipse can be driven to the output well.* In order that this might happen $R$ could not possess a value corresponding to $P$, and this condition, were it to exist, would be contrary to our abundantly verified conception of the resistant forces $f$ within reservoirs of Capillary Control. An ellipse, as we have just seen, may be enlarged by increasing the pressure at the input well, but if this is done, we merely repeat the statement with respect to molecules of driving and driven fluid as applied to the larger ellipse.[7]

The individual ellipses in Figure 204 are based upon the assumption that the formation is perfectly homogeneous in texture and fluid from top to

[5] If the minor axis is of zero length, the ellipse is simply the straight line between the wells, a line corresponding to a radius $r$ equal to, but not exceeding $WS$.

[6] All applied pressures $P$ are effective, or potential, pressures, in that their values, as measured at $W_1$, are reckoned in excess of a constant back pressure $C$ applied at $W_2$. In general, let $S$ represent the pressure applied at $W_1$, and let $C$ represent this constant back pressure at $W_2$. Then $P = S - C$.

[7] If, in Fig. 204, $W_1c$ is extended in a straight line until it intersects the circle of radius $R_1$, say at a point $q$ not shown, then $cq = cW_2$. The circle of $R_1$ is a "circular directrix" of the ellipse $W_1cW_2$. The same principle holds with respect to the circle of $R_2$ and the ellipse $W_1dW_2$, and likewise with respect to the circle of $R_3$ and the ellipse $W_1eW_2$. The points $c$, $d$, and $e$ are to be taken as representing any one of all points on their ellipses, and the point $q$ as its correspondent on the circular directrix of the ellipse. By means of this geometrical proposition we can easily visualize the dynamical situation at the points $c$, $d$, and $e$. Of all directions surrounding these points there is but one in which the movement of fluid can take place: namely, in that toward $W_2$. If the formation is not perfectly homogeneous in its lateral extent, the closed areas under stress are "irregular circles." They are then the irregular directrixes of irregular ellipses. The lines $cq$ and $cW_2$ are now so related that the sum of all the $f$'s along their paths are equal. Their relative lengths are in accord with this fact. (Compare the circular directrix of the ellipse with the straight line directrix of the parabola in Fig. $J_2$ of Appendix J.)

bottom. If it actually consists of strata varying in texture, or if, say, oil is underlain by water within the same stratum, more ellipses than one exist simultaneously between $W_1$ and $W_2$. Their separate focal radii $R$ will have dimensions in accordance with the factors which determine the radius $R$ on natural flow.

**189.** *Focal radial flow.*—Because the pressure at $W_1$ is exerted by a fluid, it must be exerted equally in all directions immediately surrounding this well. When $W_2$ is open, there is a release of pressure from its direction. All reservoirs in the process of flow have a kinetic pressure gradient. The elliptical area confines a reservoir in Volumetric or Hydraulic Control; there must exist within it a kinetic pressure gradient in accordance with either of these controls during movement of the fluid toward $W_2$. As shown in the lower half of Figure 205, there is an increase in the pressure drop along any and all focal radii existing between $W_1$ and $W_2$. The release of pressure at the latter is "reflected" at the circumference of the ellipse in the manner indicated. These focal radii intersect within the area. At any given point within this area there are two, and only two, intersecting focal radii. While they are alike with respect to their extreme points, they differ in the fact that one has a direct path to either well and the other has a reflected path. As the distances along these two paths must be different, representing, as they do, one side of a triangle against the other two sides, it is evident that at each point two pressure drops exist simultaneously. The two unite to form one; the value of this one is equal to the square of the mean square root of the two, in accordance with the usual method of averaging pressures that pertain to the motion of fluid. We can perhaps picture to ourselves the resulting kinetic pressure gradient surface in this area.[8] Motion of the fluid between the two wells is defined by this gradient surface.

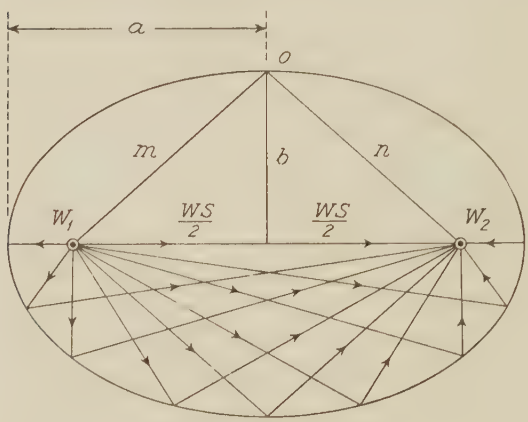


Fig. 205

In the upper half of Figure 205 we have the necessary dimensions for the determination of the relation between $P$, or $R$, and the area of the ellipse.

[8] It is feasible to construct a model of the gradient surface in accordance with given data on $P$, $P'$, and $WS$. This surface intersects the surface of the static gradient at the circumference of the ellipse.

As to the relation between $P$ and $R$, we already have the following equation[9] which we take from section 173 or section 176:

$$P = KR^2 \quad \ldots\ldots (610)$$

Thus if we determine the relation between $R$ and the area, it is a simple matter to determine the further relation between the pressure and the area.

Each half of an ellipse has one focal radius wherein its two sections are equal. This is obviously the case with $m$ and $n$ in the figure. As a matter of fact

$$m = n = a \quad \ldots\ldots (611)$$

where $a$ is one-half the major axis of the ellipse.[10] If we draw the perpendicular to the major axis from the point $o$, where $m$ and $n$ meet at the circumference, we have $b$, one-half the minor axis of the ellipse. The distance between the foot of the perpendicular and either well is one-half $WS$, the entire distance between the wells. The two right triangles are identical. Now

$$b^2 = m^2 - \left(\frac{WS}{2}\right)^2 \quad \ldots\ldots (612)$$

in accordance with the relation between the sides of a right triangle. In place of $m$ we might of course have $n$, or even $a$, in virtue of Equation 611. If we know the values of $a$ and $WS/2$, that of $b$ can be easily computed by means of Equation 612. By survey we know half the distance between the wells. How shall we determine $a$? In the preceding section we intermittently increased $p$ and $r$ until we reached a value $P'$ corresponding to $WS$. Then according to Equation 610 we may write

$$\frac{P}{P'} = \left(\frac{R}{WS}\right)^2 \quad \ldots\ldots (613)$$

or

$$R = WS\left(\frac{P}{P'}\right)^{1/2} \quad \ldots\ldots (614)$$

But we know that

$$R = 2a \quad \ldots\ldots (615)$$

consequently $a$ becomes known. Thus $b$ in turn becomes known, and the area of the ellipse is, as we know from geometry,

$$Ae = \pi ab \quad \ldots\ldots (616)$$

In this manner we determine the area to be swept out by the forced drive for any given pressure $P$ necessarily greater than $P'$.

[9] The relation between $P$ and $R$ is the same as it is in natural flow. We deal with a radial system in either case.

[10] Equation 611 is based upon a well-known geometrical property of the ellipse.

Incidentally it is important to note that Equations 613 and 614 furnish the means of determining the original radius $R$ for natural flow, for the given potential pressure $P$ at the beginning of production from a well in a given reservoir formation. I would propose, for practical purposes, that the first water wells encountered beyond the edge of the pool be allowed to reach equilibrium as soon as possible. Thereafter, by experimenting with these, a pressure $P''$ corresponding to $WS$, as of the formation containing water and gas, can be determined. From $P''$ we are to determine $P'$, knowing the surface tension of oil and water, and knowing the solubility of the gas in these liquids at a standard temperature and pressure. These results might be checked by further computations and experiments. The calculation of the dimensions of the volume cone should give results that agree fairly well, and the effects of interception will at least indicate their plausibility.[11] Later, when oil wells in equilibrium are available, the pressure $P'$ corresponding to $WS$ can be determined directly.

When the time is right for the installation of the forced drive, observations during the initial stages of the process will furnish the data necessary for the determination of the dimensions of the ellipse. The fact that there has been earlier production from the wells by natural flow does not hinder us in these observations, for the pre-existing cones are replaced before the actual drive begins.[12]

190. *The line drive.*—The movement of fluid during the forced drive takes place, then, from one focus of an ellipse to the other. If the pressure at the input well does not fluctuate, the movement of the fluid may be said to be "smooth." Under these conditions there is "zero banking" within the ellipse. When the pressure at the input well is increased, there is "positive banking," and when the pressure is decreased, there is "negative banking," within the ellipse. At the time the pressure is altered a high or low pressure cylinder originates at the input well; it is at first circular, and it thereafter assumes the contour of the ellipse.[13] The resulting positive or negative banking is no respecter of the contact between the input and output fluids; it is, however, a respecter of the contact between the fluid which has had its motion changed and the fluid which is to have its motion changed.[14] Zero banking

[11] See § 174, Equation 569; § 183, Fig. 196; and § 184, Fig. 200.

[12] See § 187. Allowances are to be made for the fact that a cone of different fluid is established before flow by the drive begins. Our observations are to be made independently of any earlier records of production by natural flow.

[13] See § 177. We here deal with cylinders in place of cones, because the drive, as herein described, is a process in Hydraulic Control. We have ideal performance when the pressure does not fluctuate, and theoretic performance when the pressure fluctuates.

[14] Being in the nature of a wave motion, it is clear that banking is independent of the motion of the fluids, and independent of the nature of the fluids within the ellipse.

—the complete absence of banking—is the usual accompaniment of actual operations in the forced drive.[15]

It is not customary to employ the forced drive between an individual pair of wells. Our purpose is, of course, to "sweep out" the formation over an area, and many pairs of wells are required for this. One arrangement of pairs is shown in Figure 206, where wells are located on lines across the field.

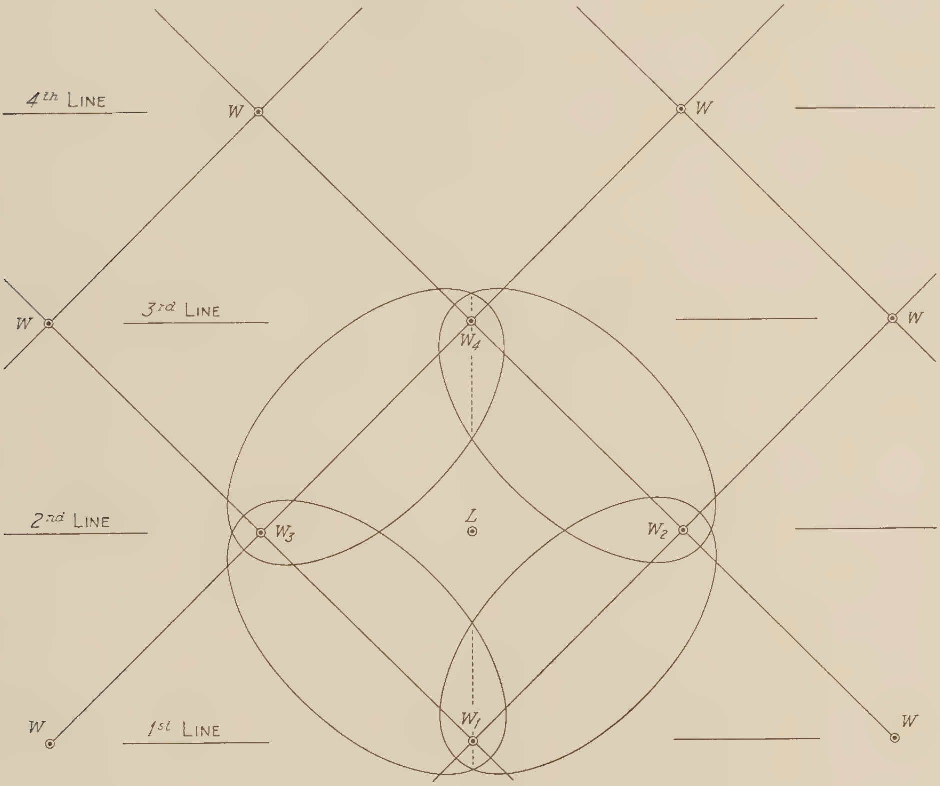



FIG. 206

These lines are parallel, and at equal distances apart. The wells on these lines are "staggered" in the manner shown. Four wells in position form a square; thus we have $W_1$, $W_2$, $W_3$, and $W_4$ forming one such square. $W_1$ and all other wells on the same line are input wells, while $W_2$ and $W_3$, and all other wells on their line, are at first output wells and subsequently input wells. Having presumably swept out the formation between the first and second lines we advance to the formation between the second and third lines, and so on, until the entire field is covered.

[15] If the pressure at the input well is fluctuating in its intensity, we of course have a series of high and low pressure cylinders successively emanating from the well. These give rise to banking which alternates in its sign.

In this arrangement it is obvious that the forced drive is in accord with the square pattern of locations. A glance at the figure reveals the situation with respect to the ellipses. Unless the pressure at the input wells is sufficiently great there are *star-shaped areas* remaining untouched at the center of the squares. A subsequent test well, as at $L$, will prove that there is no removal of fluid here by the drive.

To draw this figure it has been necessary to assume the value of the pressure $P$ in terms of $P'$ corresponding to $WS$. I have taken $P$ to be 137 per cent of $P'$. Had I taken $P$ equal to 200 per cent of $P'$, the ellipses would have intersected exactly at the center of the square. No star-shaped area would have remained. This is easily proved, for

$$W_1W_2 = WS, \text{ the distance between wells,}$$

and

$$W_1LW_2 = WS \text{ multiplied by the square root of 2,}$$

and the substitution of these values into Equation 613 (p. 573) gives

$$\frac{P}{P'} = \left(\frac{WS\sqrt{2}}{WS}\right)^2 = \frac{2}{1}$$

or 200 per cent.

In the process of driving, a neutral plane, as at $W_1$, is established at the input well. No driving fluid passes this plane to reach the other well, and no driven fluid passes this plane with the same purpose. If the ellipses between the first and second lines have been swept clean, there are areas, as at $W_2$ and $W_3$, that contain driving fluid which must be driven to the wells on the third line, before the ellipses between the second and third lines can be left clean. Within the first ellipses the driving and driven fluids always remain "radially distributed" with respect to the wells during the process.[16] Now we see fluid in subsequent ellipses that is not radially distributed with respect to the subsequent wells. Another neutral plane appears at the third line, as at $W_4$. Its function is analogous to that of the plane at the first line.

If star-shaped areas are untouched by the drive in practice, there are three methods by which they may be covered: namely, (*a*) increase the pressure $P$ to a value $2P'$, as we found above; (*b*) decrease $WS$ so that $P'$ has a value of $\frac{1}{2}P$; or (*c*) resort to subsequent diagonal forced drive, as from $W_1$ to $W_4$.

The last method is not a practical one when wells are located on the square pattern. The pressure corresponding to $P'$, as between $W_1$ and $W_4$, is exactly $2P'$, and this must be exceeded in order to form the ellipse. As a matter of fact a pressure $P$ equal to 240 per cent of $P'$ is required for cleaning out the particular area remaining in the figure. Why exceed 200 per cent of $P'$, and handle an excessive quantity of "non-radially distributed" driving fluid, when the application of this pressure in the beginning leaves no such area?

[16] By "radially distributed" I mean located within the ellipse according to focal radial flow from one well to the other.

As to the nature of the driving fluid, we might use water, air, or gas. Water breaks down the Jamin action within the elliptical areas by pushing the bubbles forward and dissolving their gas. The high viscosity effects of this action are therefore decreasing in their intensity during the process of driving between pairs of wells. The dimensions of the ellipses themselves undergo no increase as a result of this decrease in viscosity effects. Rather the driving and driven fluids, if the pressure is maintained at the same value, increase their lineal velocity of flow within the areas. Non-radially distributed water may prove to be expensive to handle at the output wells. It will at least affect the economic limit of profitable driving.

Air and gas provide for the maintenance of the Jamin action. They are less expensive to handle when non-radially distributed. In the case of air there is the usual hazard in an explosive mixture. Non-radially distributed gas will convey vapors to the output wells.

The formation is undoubtedly to remain filled with whatever fluid we may use for driving, since it must replace the driven fluid.

**191.** *The hexagonal drive.*—Another system of arranging the wells for the forced drive is illustrated in Figure 207. $W_1$ is the first input well, and

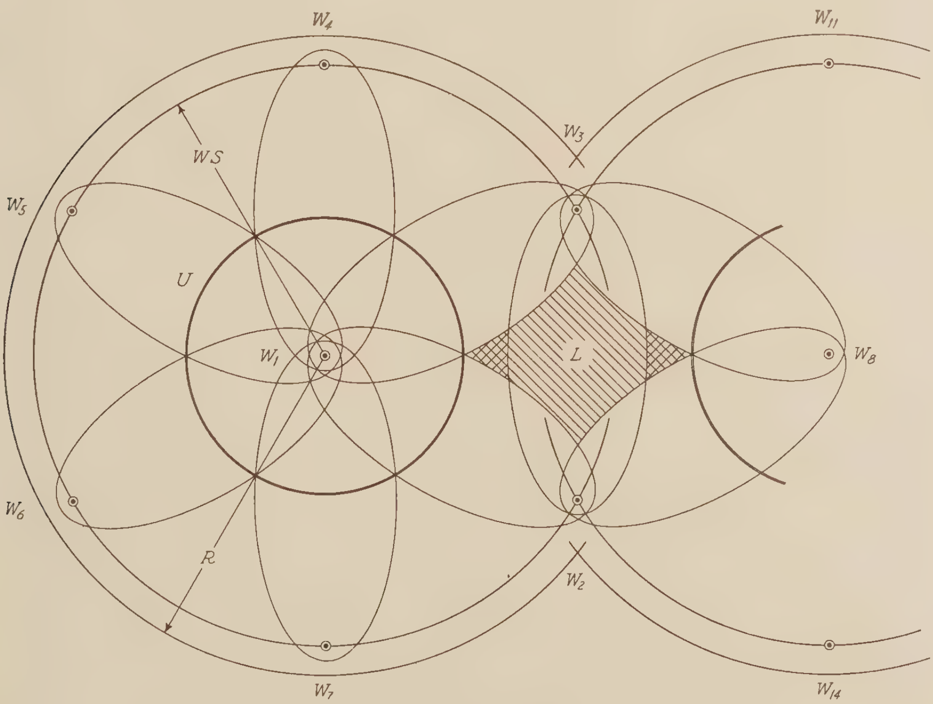



Fig. 207

$W_2$, $W_3$, $W_4$, and so on, are the first output wells. There are six of the latter in accordance with the hexagonal pattern of locations. $R$, as before, is the

radius of action of the pressure $P$ maintained at $W_1$, while $WS$ is the common distance between all wells. The drive is confined to elliptical areas in the same manner as before. We might speak of this as the circular, or center, drive, though we must not neglect the fact that there are areas remaining untouched within the circle described by $WS$ about $W_1$. Although I have completed the ellipses within the vicinity of the input well, these in reality represent no detectable feature in the formation during the driving process, for the drive is parted by six neutral planes through $W_1$. These planes extend obviously, to the circle $U$. The area within $U$ is effectively subjected to a complete sweeping.

On the completion of the process at $W_1$ the same operation is performed at $W_8$ with its six surrounding wells. Unless the pressure $P$ is sufficiently great, there are star-shaped areas remaining untouched in the manner indicated.

If we increase $P$, or decrease $WS$, in order that no star-shaped area will remain, the ellipses must overlap at the center, for the area defined by $W_1W_2W_8W_3$ is not square. This means an excess of non-radially distributed driving fluid for the operation between $W_8$ and $W_2$, and between $W_8$ and $W_3$.

Subsequent diagonal driving here has its advantages. All wells are equally distant apart; consequently we have the same dimensions for our ellipse between $W_2$ and $W_3$ as between $W_1$ and all immediately surrounding wells, on the assumption that the pressure $P$ is the same amount throughout.[17] It is true that there is an excess of non-radially distributed driving fluid to be handled with this diagonal drive, but here we encounter an interesting point. The diagonal drive promises to be unsuccessful if water is used as the driving fluid. The globules and bubbles remain intact within the star-shaped area, whereas water with little, if any, gas completely surrounds it. The high viscosity effects within the area will deflect the water, for it will more readily pass around than through the area. Air or gas, on the other hand, promises success. Bubbles exist everywhere in the formation; in fact there will tend to be more of them outside than inside the star-shaped area, and as a result there will be no tendency for the driving fluid on the diagonal drive to pass by a circuitous route between the wells. With air or gas, as we have already noted, the expense of handling non-radially distributed fluid is not excessive.

192. *Recovery by forced drive.*—If the oil that remains within the formation after the wells have reached equilibrium by natural flow is to be recovered, it must be driven out by force. *It is not because the pressure within the formation has declined to a low value that force must be used, but it is because this oil is securely locked in place by bubbles of gas.* Force is required to overpower these bubbles. When these are overpowered, all the oil—the static

[17] The applied pressure at $W_1$ is common to all ellipses having $W_1$ as a focus, and unless the constant back pressure $C$ at the surrounding wells is of the same value throughout, the effective pressure $P$ will be different for some ellipses than for others.

or absolute volume of oil—within the elliptical area is swept out. We are now to investigate recovery by the forced drive. Our procedure must necessarily differ somewhat from that in the investigation of recovery by natural flow.

We shall confine our attention to the square pattern, knowing that the hexagonal pattern may be treated in the same way. The problem is first evident in Figure 206. The reservoir is perfectly homogeneous in all respects, and the driving operations are carried out under exactly identical conditions at all wells. For the present investigation we shall make an unusual assumption: namely, that no fluid has been, or is to be permitted to leave the formation by natural flow before the drive. With general principles thus established the necessary modifications on account of earlier natural flow will become clear.

The problem is simply this: What percentage of the square area defined by the four wells is covered by portions of four ellipses for any and all values of the pressure $P$ between 100 and 200 per cent of the value of the pressure $P'$ corresponding to $WS$, the length of the side of the square?[18] The solution is not a difficult one. In Figure 208 (p. 580) I have set up the square and ellipses with their geometrical features in display. $X$ and $Y$ axes are chosen in the manner shown; consequently the ellipses which extend into the first quadrant have the equations attached to them in the drawing. As to the symbols, we have the following:

$s$ = one-half $WS$

$a$ = one-half the major axis of the ellipses

$b$ = one-half the minor axis of the ellipses

$L$ = the entire area of the star

One-fourth of $L$ lies in the first quadrant. We are to compute this area and subtract the amount from $s^2$, one-fourth of the entire area of the square, thus determining the portion covered by the ellipses. For convenience we shall give $s$ the value one, and thereafter assume eleven values of $b$ in terms of it. The quantity $b$ is then to have the successive values of 0, 0.1, 0.2, 0.3, . . . . . . . 1.0. Corresponding values of $a$ follow from the equation

$$a^2 = b^2 + 1 \qquad (617)$$

The points at which the eleven sets of ellipses intersect must be computed. These are the points $x'y'$, as indicated in the drawing. To do this we note the fact that the 45-degree line, $x = y$, intersects the ellipses at the same point;

[18] As we now know, with $P$ at 100 per cent the well at the distance $WS$ is just reached, while with $P$ at 200 per cent, there is no star-shaped area at the center of the square, and neither do the ellipses overlap immediately at the center.

therefore we need only let $x$ be equal to $y$ in either of the equations for the ellipses and solve, with the result that

$$x' = y' = \frac{a^2 - ab^2\sqrt{2}}{a^2 + b^2} \qquad (618)$$

From these points we determine one-quarter of the area $U$ inclosing one-quarter of the area $L$.[19] From them we also determine the area of the strip cut

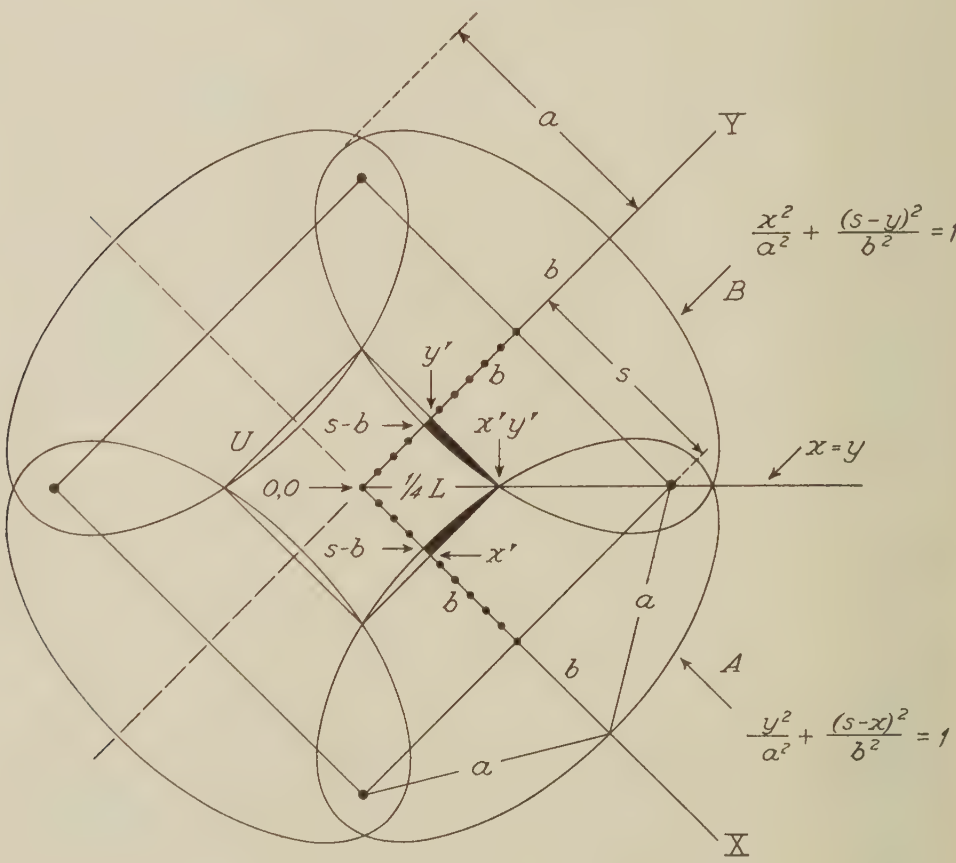



FIG. 208

from the ellipses by perpendicular lines at $x'$ and $y'$. It is clear that if we subtract the area of the strip from one-quarter of $U$ we have one-quarter of $L$.

To calculate the area of the strip it is necessary to employ the method of calculus. Solve the equation of one of the ellipses for the value of the variable

[19] There is another set of values for $x'$ and $y'$ by the same solution, since the fraction can also appear with a plus sign in the numerator. These values designate the points of intersection exterior to the square, points in which we are not interested.

we choose to adopt as the dependent one. Say for the ellipse $A$ we solve for $y$. Thus

$$y = \pm \frac{a}{b} \sqrt{b^2 - z^2} \quad \ldots\ldots\ldots\ldots (619)$$

where, for convenience, $z$ replaces $(s - x)$, $x$ now having the value $x'$, in accordance with Equation 618. The entire strip cut from the ellipse $A$ requires both plus and minus signs. But half of this strip is associated with one-quarter $L$. Nevertheless, there is another half-strip associated with one-quarter $L$ in the ellipse $B$. We can either employ both plus and minus signs, knowing that the two half-strips are equivalent to one entire strip, or simply substitute the constant 2 in the equation in place of the signs. The latter is to be preferred.

Now we are ready to set up our differential equation, as follows:

$$dy'' = 2\,\frac{a}{b}\sqrt{b^2 - z^2}\,dz \quad \ldots\ldots\ldots\ldots (620)$$

where $dy''$ represents a differential area within the ellipse. By integration this becomes[20]

$$y'' = \frac{a}{b}\left( z\sqrt{b^2 - z^2} + b^2 \operatorname{arcsine} \frac{z}{b} \right) \ldots\ldots\ldots (621)$$

For the strip we have this to evaluate between the limits

$$z_1 = 1 - x'$$

and

$$z_2 = b$$

By substituting these limits into the equation we obtain

$$As = A - B - C \quad \ldots\ldots\ldots\ldots (622)$$

wherein[21]

$As =$ the area of the strip

$$A = \frac{a}{b}\left( b^2 \frac{\pi}{2} \right)$$

$$B = \frac{a}{b}\left( [1 - x'] \sqrt{b^2 - [1 - x']^2} \right)$$

and

$$C = \frac{a}{b}\left( b^2 \operatorname{arcsine} \frac{1 - x'}{b} \right)$$

Now the various values of $b$, with the corresponding values of $a$ and $x'$,

[20] The constant of integration is omitted in the equation in view of the fact that it disappears upon substituting the limits of $z$.

[21] In order to retain their symmetry I have not reduced the terms in the following equations to their simplest form.

can be substituted into this equation. Let $Ae'$ represent the portion of the square in the first quadrant covered by ellipses; then

$$Ae' = s^2 - (x'^2 - As) \quad \ldots\ldots\ldots\ldots\ldots\ldots (623)$$

Express $Ae'$ in percentage values of $s^2$, the latter taken as 100 per cent, since $s$ is unity.

From Equations 613 and 615 (p. 573) we have

$$P = P' \left(\frac{R}{WS}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (624)$$

and

$$R = 2a \quad \ldots\ldots\ldots\ldots\ldots\ldots (625)$$

Substitute this value of $R$ into Equation 624, and replace $WS$ by $2s$. The result is

$$P = P' \frac{a^2}{s^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (626)$$

Take $P'$ as 100 per cent, and express $a^2$ in terms of $s^2$. As a percentage equation we then have

$$P = a^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (627)$$

since $s$, as usual, is unity. $P$ is evidently greater than 100 per cent, for $a$ is always greater than $s$. Inasmuch as we are to take $P'$ as 100 per cent, we can conveniently subtract $P'$ from $P$ to obtain the excess of the latter over the former.

Now finally we have the following set of values to plot as in Figure 209:

| $P-P'$ | $Ae'$ |
|---|---|
| 0.00 | 00.00 |
| 1.00 | 15.75 |
| 4.00 | 31.51 |
| 9.00 | 46.76 |
| 16.00 | 60.76 |
| 25.00 | 73.99 |
| 36.00 | 82.86 |
| 49.00 | 90.22 |
| 64.00 | 95.56 |
| 81.00 | 98.95 |
| 100.00 | 100.00 |

These are abscissas and ordinates, respectively, of the curve $AB$.[22] The four quarters of the square defined by the wells are identical; consequently the percentage values which apply to one quarter apply as well to the four taken as a whole.

[22] The solution to the problem can be checked graphically by constructing successive sets of ellipses on a square, and measuring $2a$ and $L$ for each set by a scale and a planimeter, respectively.

$Ae'$ represents, as we recall, the portion of the square covered by ellipses. It actually represents, then, the percentage recovery from the square by the forced drive, on the assumption that no fluid is permitted to leave the formation by natural flow before the drive. The driven fluid lies in the form of an elliptical cylinder of uniform density. Its lateral dimensions are determined by such factors as $P$, $P'$, $R$, and $WS$. What is its vertical dimension?

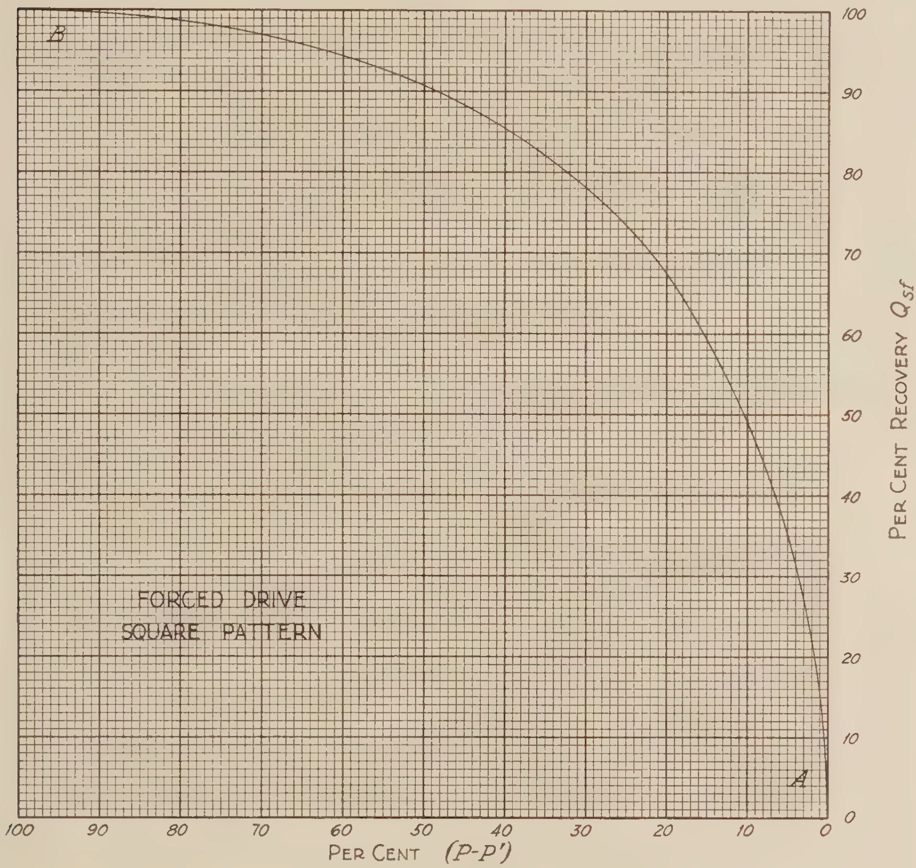



FIG. 209

If we were to represent the cylinder by a space diagram, after Figure 66, we would give it the altitude represented by the distance between planes at $K$ and $I$. In other words, the forced drive is concerned with a *static volume cylinder* with an elliptical base. It will be convenient to designate the present recovery as $Q_{sf}$, indicating the fact that it is on the same basis as $Q_{sn}$ for natural flow.

Given any value of $P - P'$ in a field subjected to the forced drive, where wells are located according to a square pattern, and where no fluid is permitted to leave the formation by natural flow, the corresponding ordinate in Figure 209 represents in a mathematical sense the maximum of recovery for

the forced drive under the conditions. *The series of points along the curve represents the locus of all possible maxima by the forced drive.*[23]

If the recovery from the field at large is $Q_{Sn}$ by natural flow before the drive is instituted, then

$$100\% - Q_{Sn}\% = Q_{Sv}\% \quad \text{.................(628)}$$

still remains within the formation. The drive, instead of operating upon 100 per cent of the static volume, in fact operates upon $Q_{Sv}$ per cent. If this volume lay as a flat prism between two horizontal planes over the field, we might say that the actual recovery by the forced drive is

$$Q_{Sf}\% \times Q_{Sv}\% = Q_{Sd}\% \quad \text{.................(629)}$$

And this, of course, would be exact under the conditions. But we know that $Q_{Sv}$ really lies as a flat prism between two such planes, the prism having conical depressions in it on account of production by natural flow. The ordinates of Figure 209, treated in accordance with Equation 629, represent exact conditions in practice only at 0 and 100 per cent; for all positions between these extremes they represent approximations. *As we make the squares for the drive smaller in comparison with the squares for natural flow, the approximations approach the exact values shown by the curve* AB.[24]

For our final step we shall assume that $Q_{Sd}$ is a close approximation to the truth. Then we may write

$$Q_{Sn}\% + Q_{Sd}\% = Q_{Su}\% \quad \text{.................(630)}$$

where $Q_{Su}$ per cent represents the total, or ultimate, recovery from the field at large. *If there is a remainder,*

$$100\% - Q_{Su}\% = Q_{Sa}\% \quad \text{.................(631)}$$

*it represents the fluid that is yet within the formation, securely locked in place by Jamin action.*[25] This we abandon.

[23] It is of interest to note that at $P - P'$ equal to 50 per cent, that is, at $P$ equal to 150 per cent of $P'$, the recovery exceeds 90 per cent.

[24] In application the local features resulting from earlier operations by natural flow must now be taken into account. (See footnote 4, § 188, p. 570.) If we deal with the ratio between the area swept out and the area defined by the square for the drive, without regard to the lay of the fluid, then the ordinates in Fig. 209 may be changed to read, "Per Cent Area Swept Out," and the curve is then exact for all positions between 0 and 100 per cent, irrespective of the ratio between the size of the square for the drive and that of the square for natural flow. It is, in other words, independent of earlier production by natural flow.

[25] See footnote 14, § 186, page 559, for the description of symbols. In addition now, $f$ and $d$ stand for forced drive, $v$ stands for retained volume, $u$ stands for ultimate recovery, and $a$ stands for abandoned fluid.

*In the physical sciences, "possible irregularities in geometrical shape are neglected and, merely for the sake of simplification, a far simpler behavior is assumed than that which actually occurs. If theoretical mechanics and physics are to proceed deductively at all, any other treatment of reality is impossible. For here the empirical phenomena are so complicated that, by excluding subsidiary factors which are, however, always at work, abstract relationships are made the basis of the method, and the behavior of phenomena is then treated as if it were dependent only upon these abstract factors and the others did not come into consideration at all."*—Hans Vaihinger

*"By this way of analysis we may proceed from compounds to ingredients, and from motions to the forces producing them; and in general, from effects to their causes, and from particular causes to more general ones, till the argument end in the most general. This is the method of analysis; and the synthesis consists in assuming the causes discovered, and established as principles, and by them explaining the phenomena proceeding from them, and proving the explanations."*—Sir Isaac Newton

Summary. Analytical Principles of Production
Their Value to the Oil and Gas Industry

SUMMARY

# Analytical Principles of Production Their Value to the Oil and Gas Industry[1]

[Herein Mr. Parks presents a summary of the theory set forth in the foregoing treatise, emphasizing its relation to the problems of oil and gas field development, operation, and rejuvenation. His thorough study of the theory itself and his knowledge of oil and gas field practice undoubtedly place him in a position to select and explain in not too technical language those items of greatest concern to the executive or operator. It is assumed that this executive or operator desires information concerning the nature of the subject treated, the author's method of attack, and the value of the work to the industry, without entering upon a detailed study of the contents of the book.—S. C. H.]

A well, properly fitted with its casing, its tubing, its head equipment, and any other accessory apparatus, such as a pump or a gas-lift, is, after all, merely the orifice of a natural reservoir that is capable of producing oil and gas. It is the hole through which these fluids are brought to the surface. To serve efficiently it must of course be suitably equipped and properly maintained in condition during operation. By means of this hole we obtain not only production itself, but, since it is our only point of access to the productive formation lying below, we obtain also the data of such production. From these data we know the behavior of the reservoir; consequently we know how best to treat the reservoir and what to expect of it in return. It is the reservoir which is of greater concern to us. Without it the well would have no value.

The industry asks, "What do we know about the physical laws governing the habits of oil and gas deposits, and the laws which govern and influence the extraction of oil and gas from those deposits?"[2] We may assume that the answer is indeed given in the foregoing treatise.

Seven so-called primary functions of performance—pressure, velocity, volume, acceleration, energy, power, and time—give us the key to the entire situation within the reservoir. We must know about *pressure,* because this determines whether or not the reservoir will produce; it determines the rate of production; and it is necessary to the calculation of future production. *Velocity* is merely a convenient term for the rate of production. *Volume* is the total amount of fluid to be produced. *Acceleration,* or the change in the

[1] By Ernest K. Parks, Petroleum Engineer, Standard Oil Company of California.

[2] This question was framed by M. E. Lombardi, Standard Oil Company of California, and presented to the Pacific Coast Technical Sub-Committee of the American Petroleum Institute's Gas Conservation Committee, San Francisco, September, 1927.

rate of production, is an essential feature of our studies in decline curves. *Energy* is utilized in the work of production, and it is related to the amount of gas accompanying oil within the reservoir. *Power* is the rate at which energy leaves the reservoir, and it is also the rate at which gas is produced with oil. Obviously it is a feature of direct application in the problem of gas-oil ratios. *Time,* of course, is vital to all our calculations.

In contrast to these primary functions which deal with the behavior of the reservoir, as observed at the well, there are the secondary functions. These deal with the events and conditions which take place or exist within the reservoir itself. They pertain to such matters as the source of energy within the reservoir, the conservation of this energy, the drainage radius and the drainage area of wells producing from the reservoir, the encroachment of water upon the pool of oil and upon the well itself, the fitness of the reservoir formation for the storage of gas and for reserves of oil, the percentage recovery from the formation by natural flow, the fitness of the reservoir formation for the restoration of gas pressure, and for the forced drive with water, air, or gas. These features are called secondary because they are to be identified by means of the primary functions, and not because they are of less importance.

It is evident, then, that we have sufficient reasons for understanding the nature of the primary functions, their characteristics, their interrelations, and their variations during production. By means of them we learn to regulate the performance of the well to the best advantage, in that we know the proper back pressure to be applied, the most desirable rate of production, the most economical gas-oil ratio to be attained, and the most desirable rate for pumping or lifting the oil from the bottom of the well. Upon them we base our forecasting of the future performance of the well. For this they prescribe the necessary data to be collected in the field, and they indicate the manner in which these data should be used. The collection of irrelevant data is to be avoided, and the loss of relevant data by neglecting the proper observations is to be prevented. Finally, as we have just noted, they indicate to us the events and conditions within the reservoir. By means of them we learn the proper location and spacing of wells in developing the field, we know the necessity or lack of necessity in purchasing protective acreage, and we know the amount and security of our reserves.

The primary functions are related differently under various circumstances, and, as we might expect, the secondary functions are different in correspondence with them. These differences are not to be confused with minor features that are due simply to local circumstances within the vicinity of the single well or group of wells in the field. It is true that such local variations in structure and texture, as are known to exist in our fields, combined with a greater or less proximity of water, tend to make each well distinguishable from every other well in the same field, and each field distinguishable from every other field in existence. In spite of these features all natural reservoirs possess

properties which permit us to classify them into three distinct groups. These groups, called "controls," are clearly defined, and they are therefore easily recognized in the field. Each control has its distinct set of performance curves in accordance with different relations between the primary functions. Every natural reservoir must belong to one of the three.

The three controls are as follows:

1. *Hydraulic Control.*—Pressure and rate of production remain constant during the course of production from the reservoir. The situation is analogous to that of a water tank delivering its water with a head, or depth of water, maintained at a constant level within it by the necessary inflow of more water.

2. *Volumetric Control.*—Pressure and rate of production decline and approach zero during the course of production from the reservoir. The situation is now analogous to a water tank delivering its water with a head declining in accordance with production from it.

3. *Capillary Control.*—Pressure and rate of production also decline and approach zero during the course of production. The decline here takes place in accordance with the characteristics of the "Jamin capillary tube," a tube of capillary bore, filled with alternating globules of liquid and bubbles of gas.

Natural reservoirs in Hydraulic and Volumetric controls perform according to the well-known laws of hydraulics. As we see, the only difference between them lies in the fact that the pressure head remains constant in the first one, while it declines in the second one. The fact that the reservoir is associated with a porous formation does not affect the laws of production, as these are displayed by the pressure and rate of production curves. Only the numerical values are different in the presence of this formation. The column of liquid bearing its weight upon the bottom of the well is the source of the energy; consequently the water surrounding the pool of oil must be conserved. The encroachment of this water upon the pool and upon any well in the pool is inevitable in time. The drainage radius and drainage area of each well in the pool is coextensive with the formation itself. This means that all wells are producing from the same space. These reservoirs are unsuitable for the storage of gas and for oil reservoirs, if wells under other ownership are producing from the same formation. On the assumption that there is at least one well to each minor feature in the structure, recovery is one hundred per cent of the mobile oil in Hydraulic Control, and the same in Volumetric Control, depending upon whether or not the pool is depleted before the pressure and rate of production curves reach zero in the course of production. It is useless to attempt the restoration of gas pressure within these reservoirs, for the gas is not the source of the energy. The forced drive with water, air, or gas is unnecessary on our part, for Nature is already driving with water —with that water which surrounds the pool and extends up the general dip of the formation.

The Kern River Field of California is a good example of production in

Hydraulic Control, and the Coalinga Field of the same state is a good example of production in Volumetric Control.

The discovery of Capillary Control in our producing fields and the complete analysis of its important economic features constitute the outstanding contribution of Mr. Herold's work. While "Jamin action," as illustrated by the behavior of the Jamin capillary tube, plays a rôle in all natural reservoirs in porous formations when both liquid and gas are present, its rôle is a subordinate one in Hydraulic and Volumetric controls, but a predominant one in Capillary Control. The situation as explained by Capillary Control, with respect to both primary and secondary functions, accounts for the supposed erratic behavior of many of our oil and gas wells.

Jamin action is due to alternating globules of liquid and bubbles of gas within channels sufficiently small to permit capillary forces to be an important feature. The globules, on the release of pressure at one end of the channel, become distorted in shape, and in this condition they offer a resistance to the pressure tending to drive both the liquid and the gas toward this end. Consequently the amount of liquid and gas to issue from the channel depends upon the intensity of the pressure bearing upon the fluids within it.

The action of these globules and bubbles is easily observed in the Jamin tube having glass walls. A porous formation may be taken as a bundle of such tubes radiating in all directions from any well which penetrates it. The hydrostatic pressure bearing upon a pool of oil in the vicinity of the well may be sufficiently intense to overpower the resistance of the globules. In this case the well produces either in Hydraulic or in Volumetric Control, depending upon whether or not this hydrostatic pressure is constant during production. But if this hydrostatic pressure is absent, or if it is not sufficiently intense to overpower the resistance, the well produces in Capillary Control.

The source of energy in reservoirs of Capillary Control is due to gas pressure; consequently the conservation of this energy is only to be attained by the conservation of the gas within the reservoir. The drainage radius of a well in such a reservoir is of limited length, depending upon the intensity of the pressure tending to move the oil toward the well, and the intensity of the resistance offered by the globules in the path of movement. The drainage area of a well is circular—perhaps an imperfectly circular one because the texture of the formation is not the same in all directions from the well; nevertheless the area is a closed one, and it does not cover the extent of the formation. Water surrounding the pool of oil cannot encroach upon it. If the well happens to be located near the edge, water of course can encroach upon the well, for this water lies within the drainage area of the well. The formations having such characteristics are eminently suited to the storage of gas and to reserves of oil. Neighboring wells drain from a property only within a limited radius, and an offset well prevents even this. The percentage recovery from the formation by natural flow varies considerably in different fields of this

control, for the recovery depends entirely upon the radius of drainage of the wells and the spacing between them. In any given field we can actually determine the recovery. The restoration of gas pressure is entirely proper, for the gas, as just stated, is the source of all energy within the reservoir. Complete recovery is only to be attained by the forced drive with water, air, or gas. We must apply a pressure at one well of sufficient intensity to overpower the resistance offered by the globules between it and another well. Nature is not flooding with water in these reservoirs, since the hydrostatic pressure is either absent or not of sufficient intensity; consequently we must flood the formation ourselves, if we want the oil which remains after natural flow and pumping have reduced production to the low rate that is said to be the economic limit of operation. Nature permits us to imitate her in flooding, but in this she confines us to a local area within the vicinity of the input and output wells. The area proves to be elliptical in shape, and its dimensions can be determined in any case.

There are, as we have seen, two controls—Volumetric and Capillary controls—in which pressure and rate of production decline in the course of production. The secondary functions—the events and conditions within the reservoirs—are very different in the two. We cannot make personal visits to the formations. How shall we decide which of the two controls we have in any field of interest to us? Easily and efficaciously by means of the primary functions. As stated above, they indicate to us the events and conditions within the reservoir.

The pressure curves are the same in both declining controls; consequently we cannot tell from pressure curves alone the control of the field. The rate of production curves, however, are different. To be absolutely certain of the control we should have both pressure and rate of production curves. We have used both in gas field practice for several years, and there seems to be no reason why we should not use both in oil field practice. Where we have them we can plot the values of pressure and rate of production in percentages of the initial values against time. If the rate curve lies above the pressure curve we have Volumetric Control, but if it lies below the pressure curve we have Capillary Control. We see how easy this is. As a matter of fact the gas engineers in the fields of Ohio, Pennsylvania, New York, and the Canadian provinces adjoining, have long ago recognized the fact that the rate of production curve underlies the pressure curve. (They have been producing from reservoirs in Capillary Control without being aware of the significance of it.) They found that the well-known laws of hydraulics are not applicable to their gas wells. They were right.

Another way by which the distinction between the two controls can be made is by plotting the rate of production against the pressure on logarithmic paper. If the points do not lie on a straight line they can be made to do so approximately by shifting them to the right or to the left in the usual manner. If this line has a slope of one to two we have Volumetric Control, but if it

has a slope of three to two we have Capillary Control. We also see how easy this is. As a matter of fact gas engineers in Kansas and Oklahoma have long ago recognized the three-to-two slope. (They likewise have been producing from reservoirs in Capillary Control without being aware of the significance of it.)

Oil wells in the regions mentioned above also produce from reservoirs in Capillary Control. Until recently the oil engineers have not had pressure data on their wells; so the situations with respect to the two curves are not very well known among them. Nevertheless they have frequently noted that the rate of production curves have a slope of three to one on logarithmic paper. This is the slope for Capillary Control. If the reservoirs were in Volumetric Control these curves would have a slope of one to one. All these circumstances are in agreement with the fact that there are two declining controls.

The Cushing Field in Oklahoma is an excellent example of production in Capillary Control. It has been particularly noted there that edge water does not encroach upon the pool, as it does in the Coalinga Field.

If we say that in the Kern River and Coalinga fields every barrel of oil produced at one well means one barrel less for some other well or group of wells, and that such is not the case in the Cushing Field, we are basing our statement upon a knowledge of the secondary functions of producing reservoirs in the three controls. But how can we have this knowledge? The answer is: by mathematical analysis. This simply means that from the known properties of fluids, such as viscosity, surface tension, and capillarity, and from the known laws of physics, such as Boyle's Law, Henry's Law, and the laws of fluid mechanics, we can proceed with mathematical operations that are mostly algebraic and geometrical in their nature, and derive results which can be interpreted in terms of the producing reservoir. Of course we know in advance what we wish to determine; consequently we continually steer our mathematical course in that one direction until we arrive at our destination. Upon arrival we cannot be sure of ourselves until we verify the situation. Now this verification is a simple matter in the case of oil and gas production, for we have been noting the performance of wells for many years. The experience we have already had with producing properties is generally sufficient for us to decide whether or not our results are correct. We may expect different results in the three controls; therefore we must be careful in judging them from our experience. If our experience should be confined to reservoirs of one control, it is almost certain that we will question results which pertain to a reservoir of a different control. Where question arises, perhaps the best we can do is to ask someone who has had experience in the other control if he can say that the result is correct.

By mathematical analysis Mr. Herold leads us to the analytical principles of production. But what are analytical principles of production? They are simply laws pertaining to production. We have had some of them for many

years. Take the *law of equal expectation* as given by Lewis and Beal:[3] "If two wells under similar conditions produce equal amounts during any given year, the amounts they will produce thereafter, on the average, will be approximately equal, regardless of their relative ages." This is one analytical principle of production. Take another, the *equal production per pound decline law* of unknown authorship: "For equal amounts of decline in the pressure at a well equal amounts of oil are produced." By mathematical analysis we find that the first of these laws is a correct one, regardless of the control, provided the two wells being compared are not in different controls. Likewise by mathematical analysis we find that the second of these laws is correct for wells which produce from reservoirs in Volumetric Control, but not correct for those which produce from reservoirs in Capillary Control.

These two laws, as stated, illustrate to us a very important matter. Laws, or analytical principles—whichever we wish to call them—can be determined by carefully observing wells during production; they can be determined without mathematical analysis. Then why resort to mathematical analysis at all? Not all of us are fluent in mathematics; some of us cannot see anything about a producing well in an equation. Why not study the well itself, and leave the mathematics to those who like it? But if it were not for mathematics we would not know that these very two laws have restrictions in their application. Furthermore, it is quite likely that there are other laws of easy application which it might profit us to know about. Instead of attempting to find them by months of observation at the wells, why not spend a few minutes with mathematics? If we like them when we get them we can apply them; if not, we can discard them and look for others. Then there are all those questions which the industry asks of us concerning all those events and conditions within the producing reservoir. There are analytical principles which pertain to them. Years spent in observations at the wells have not given, and cannot give, the information that we feel we must have in order to operate our properties to the greatest advantage. A few hours spent with mathematics will give us that information.

Many new analytical principles of production are brought to light in the present treatise. So long as they are new we must have their proofs; otherwise we can hardly afford to accept them. For this reason the mathematical proofs are included in the text. When we are more familiar with these principles, we can probably get along without proofs. Until then it will be well at least to have them available. Proofs of necessity appear in the form of equations. Verifications in the field are often only of an incidental character. They cannot always be relied upon, unless we know well the mathematical relation between the verifying observations and the principles themselves.

Mathematical analysis is not dependent upon so-called unknown factors

[3] J. O. Lewis and Carl H. Beal, "Some New Methods for Estimating the Future Production of Oil Wells," *Transactions of the American Institute of Mining Engineers*, Vol. LIX, February, 1918, pp. 492–520.

of production. But how can analysis proceed without evaluating the effects of such properties as viscosity, surface tension, and capillarity? It is of first importance to know *how* the factors affect the pressure, the rate of production, the volume to be produced, and so on, rather than to know *how much* they affect these things. The "how" really determines the form of the equation, while the "how much" merely determines the value of some constant which appears in the equation. We can call this constant "$K$," and be assured that it represents the unknown factors of production. In the statement of any law, such as the two given above, the situation is so worded that undetermined constants do not appear in the expression. The author is therefore justified in handling $K$ as an unevaluated constant throughout his work. This simplifies the study of production considerably. While the unknown factors are acknowledged as existing, their values remain unevaluated, and the result is the same as if they had been evaluated.

It is now clear how this treatise simplifies the solution of problems encountered in oil and gas field practice by:

1. Resolving the principal features of production—features observed at the well—into seven primary functions the relations of which to one another are systematically developed.
2. Classifying oil and gas reservoirs into three controls.
3. Establishing the relations between the primary and secondary functions of reservoirs.
4. Describing the secondary functions of reservoirs.
5. Removing from consideration all unknown factors of production.

A further simplification is provided by the adoption of the method of measuring pressure and volume long ago established by the hydraulic engineer. Pressure is expressed as "effective pressure" and volume is expressed as "available volume" under the particular conditions regarding back pressure at the well. From this it follows that the rate of production should be expressed as the "actual rate" under the particular conditions regarding back pressure at the well, and not as an "open-flow rate" so frequently dealt with in gas field practice. By this method the calculations concerning liquids, gases, and vapors are made in the same manner. The fact that these fluids behave differently when subjected to changes in pressure does not influence the method of calculation. Instead of such words as "effective," "available," and "actual," required to suit the various functions, the one word "potential" is applied to all of them. Thus the potential pressure is the effective pressure, the potential volume is the available volume under the given conditions, and so on. No change is made in naming the units of the functions; pressure appears in pounds per square inch or feet of liquid, volume appears in barrels or cubic feet at atmospheric pressure, and rate of production appears in barrels or cubic feet per day, per week, per month, and so on. Undoubtedly this method of placing calculations on a single basis for all fluids will be appreciated by the oil and gas engineer.

The classification of reservoirs, the consequent classification of problems concerning production from them, and the simplification of the solution of these problems all tend toward more efficient operations in our oil and gas fields.

Throughout the treatise much emphasis is placed upon the graphical representation of well performance. Curves for pressure decline, rate of production, cumulative production, and so on, are explained for each control under varying conditions of production. Curves plotted upon logarithmic paper are also explained. Logarithmic paper is a valuable tool in the hands of the oil and gas engineer; it has been in use for some years, yet no book has heretofore discussed the properties of this paper with respect to production from wells. A thorough understanding of all graphical methods will assist the engineer in his calculations. Short cuts will suggest themselves, and greater precision on the whole will result from this study.

Mr. Herold's work is comprehensive in the theory of production. He does not deal with hypotheses of production. The principles set forth are already abundantly verified in oil and gas field practice. These principles are fundamental; they do not depend upon local features within the vicinity of the well or group of wells. They may be compared with the law of gravity in that they hold regardless of local conditions. The application of the principles will reveal the local features. These features need not be known in advance of application. The principles are offered to us in lieu of costly and indecisive experiments in our laboratories and in our fields. Misconceived notions regarding the conservation of gas pressure, the restoration of gas pressure, the percentage recovery from the formations, the application of back pressures, and the drainage of properties by the adjoining wells of others are avoided by understanding these fundamental principles. These things are only misconceived when applied to the wrong control; otherwise they are properly conceived. Thus it becomes clear that we must know the control of our fields from the time of its discovery, in order that we may properly judge the wells in their behavior, properly treat the wells during their production, and properly develop the field at large.

One of the causes of overproduction is the fear that neighboring wells will drain the property if it is not developed rapidly. This fear is well founded in reservoirs of Hydraulic and of Volumetric Control. In these we need co-operation among the various operators. On the other hand, this fear is absolutely unfounded in reservoirs of Capillary Control. Co-operation is unnecessary; single offsets to the adjoining wells are entirely sufficient to prevent drainage; the remaining oil within the interior of the property will rest untouched until the end of time. By means of the principles brought forth in this treatise we shall see the stabilization of the industry. We shall obtain the greatest amount of oil at the least cost only so rapidly as we need it for the market.

San Francisco, California
June, 1928

# Appendixes

## APPENDIX A

### Tabulated Summary of Fundamental Primary Function Relations

| Relations | Hydraulic Control (Time Elapsed) | Volumetric Control (Time Remaining) | Capillary Control (Time Remaining) |
|---|---|---|---|
| Pressure-Time ........... | $P = K$ | $P = KT^2$ | $P = KT^2$ |
| Volume-Time ............ | $Vo = KT$ | $Vo = KT^2$ | $Vo = KT^4$ |
| Velocity-Time ........... | $Ve = K$ | $Ve = KT$ | $Ve = KT^3$ |
| Acceleration-Time ........ | $Ac =$ zero | $Ac = K$ | $Ac = KT^2$ |
| Energy-Time ............ | $E = KT$ | $E = KT^4$ | $E = KT^6$ |
| Power-Time ............. | $Po = K$ | $Po = KT^3$ | $Po = KT^5$ |

## APPENDIX B

### Tabulated Summary of Derived Primary Function Relations

| Relations | Hydraulic Control | Volumetric Control | Capillary Control |
|---|---|---|---|
| Pressure-Volume ......... | $P = K$ | $P = KVo$ | $P = KVo^{1/2}$ |
| Velocity-Pressure ........ | ....... | $Ve = KP^{1/2}$ | $Ve = KP^{3/2}$ |
| Acceleration-Pressure ..... | ....... | $Ac = K$ | $Ac = KP$ |
| Pressure-Energy ......... | $P = K$ | $P = KE^{1/2}$ | $P = KE^{1/3}$ |
| Pressure-Power .......... | ....... | $P = KPo^{2/3}$ | $P = KPo^{2/5}$ |
| Velocity-Volume ......... | $Ve = K$ | $Ve = KVo^{1/2}$ | $Ve = KVo^{3/4}$ |
| Acceleration-Volume ...... | $Ac =$ zero | $Ac = K$ | $Ac = KVo^{1/2}$ |
| Volume-Energy .......... | $Vo = KE$ | $Vo = KE^{1/2}$ | $Vo = KE^{2/3}$ |
| Power-Volume ........... | $Po = K$ | $Po = KVo^{3/2}$ | $Po = KVo^{5/4}$ |
| Acceleration-Velocity ..... | ....... | $Ac = K$ | $Ac = KVe^{2/3}$ |
| Velocity-Energy .......... | $Ve = K$ | $Ve = KE^{1/4}$ | $Ve = KE^{1/2}$ |
| Velocity-Power .......... | ....... | $Ve = KPo^{1/3}$ | $Ve = KPo^{3/5}$ |
| Acceleration-Energy ...... | $Ac =$ zero | $Ac = K$ | $Ac = KE^{1/3}$ |
| Acceleration-Power ....... | ....... | $Ac = K$ | $Ac = KPo^{2/5}$ |
| Power-Energy ........... | $Po = K$ | $Po = KE^{3/4}$ | $Po = KE^{5/6}$ |

# APPENDIX C

## Tabulated Summary of Secondary Functions (Natural Reservoirs)

| Function | Hydraulic Control | Volumetric Control | Capillary Control |
|---|---|---|---|
| Fluids produced | *a*) Liquid only<br>*b*) Gas only<br>*c*) Liquid and gas | *a*) Liquid only<br>*b*) Gas only<br>*c*) Liquid and gas | *a*) . . . . . . . . . .<br>*b*) Gas only<br>*c*) Liquid and gas |
| Source of energy | Hydrostatic head | Hydrostatic head | Gas pressure head |
| Conservation desired | Water pressure | Water pressure | Gas pressure |
| Drainage radius and drainage area | Coextensive with the limits of the container and its fluid or fluids | Coextensive with the limits of the container and its fluid or fluids | Limited to a circular area with the well as a center |
| Water encroachment upon the pool | Inevitable | Inevitable | Impossible |
| Storage and reserves | Unsafe from wells on the same structure | Unsafe from wells on the same structure | Safe from wells on the same structure |
| Recovery by natural flow | 100 per cent, assuming proper location of wells on structure | 100 per cent, assuming proper location of wells on structure, and provided equilibrium is not attained before this recovery | Less than 100 per cent, depending upon drainage radius and well-spacing |
| Possible conversion of control | To Volumetric Control | To Hydraulic or to Capillary Control | To Volumetric Control |
| Restoration of gas pressure | Inappropriate | Inappropriate | Appropriate |
| Forced drive | Inappropriate | Inappropriate | Appropriate |

APPENDIX D

## A Selected List of Books for Reference

### I. Mathematics


Slichter, *Elementary Mathematical Analysis,* McGraw-Hill Book Company, Inc.

Karpinski, Benedict, and Calhoun, *Unified Mathematics,* D. C. Heath & Company

Siceloff, Wentworth, and Smith, *Analytic Geometry,* Ginn & Company

Woods and Bailey, *Analytic Geometry and Calculus,* Ginn & Company

Woods and Bailey, *A Course in Mathematics,* Ginn & Company

Thompson, *Calculus Made Easy,* The Macmillan Company

Palmer, *Practical Calculus for Home Study,* McGraw-Hill Book Company, Inc.

Granville, *Differential and Integral Calculus,* Ginn & Company

March and Wolff, *Calculus,* McGraw-Hill Book Company, Inc.

Franklin, MacNutt, and Charles, *Calculus,* Franklin & Charles

Woods, *Advanced Calculus,* Ginn & Company

Steinmetz, *Engineering Mathematics,* McGraw-Hill Book Company, Inc.

Mellor, *Higher Mathematics for Students of Chemistry and Physics,* Longmans, Green & Company

Lipka, *Graphical and Mechanical Computation,* John Wiley & Sons, Inc.

Murray, *Introductory Course in Differential Equations,* Longmans, Green & Company

Piaggio, *An Elementary Treatise on Differential Equations and Their Applications,* G. Bell & Sons, Ltd.

Frost, *An Elementary Treatise on Curve Tracing,* The Macmillan Company

Bridgman, *Dimensional Analysis,* Yale University Press

Von Sanden, *Practical Mathematical Analysis,* translation by Levy, E. P. Dutton & Company

Whittaker and Robinson, *Calculus of Observations,* D. Van Nostrand Company, Inc.


### II. Physics

#### properties of matter


Edser, *General Physics for Students,* The Macmillan Company

Poynting and Thomson, *Properties of Matter,* Charles Griffin & Company, Ltd.

McEwen, *Properties of Matter,* Longmans, Green & Company

Loeb, *Kinetic Theory of Gases,* McGraw-Hill Book Company, Inc.

Fowle, *Smithsonian Physical Tables,* Smithsonian Institution

### THEORETICAL MECHANICS

POORMAN, *Applied Mechanics,* McGraw-Hill Book Company, Inc.

SEELY and ENSIGN, *Analytical Mechanics for Engineers,* John Wiley & Sons, Inc.

FULLER and JOHNSTON, *Applied Mechanics,* John Wiley & Sons, Inc.

FRANKLIN and MACNUTT, *Mechanics,* Franklin & Charles

JEANS, *Theoretical Mechanics,* Ginn & Company

### HYDRAULICS AND HYDRODYNAMICS

RUSSELL, *Textbook on Hydraulics,* Henry Holt & Company

HOSKINS, *A Textbook on Hydraulics,* Henry Holt & Company

MERRIMAN, *Treatise of Hydraulics,* John Wiley & Sons, Inc.

DAUGHERTY, *Hydraulics,* McGraw-Hill Book Company, Inc.

BOND, *An Introduction to Fluid Motion,* Edward Arnold & Company

PANNELL, *The Measurement of Fluid Velocity and Pressure,* Edward Arnold & Company

EASON, *The Flow and Measurement of Air and Gases,* Charles Griffin & Company

DRYSDALE and OTHERS, *The Mechanical Properties of Fluid,* D. Van Nostrand Company, Inc.

### HEAT AND THERMODYNAMICS

EDSER, *Heat,* The Macmillan Company

POYNTING and THOMSON, *Heat,* Charles Griffin & Company, Ltd.

FRANKLIN and MACNUTT, *Heat,* Franklin & Charles

PRESTON, *The Theory of Heat,* The Macmillan Company

INGERSOLL and ZOBEL, *An Introduction to the Mechanical Theory of Heat Conduction,* Ginn & Company

GOODENOUGH, *Principles of Thermodynamics,* Henry Holt & Company

PLANCK, *A Treatise on Thermodynamics,* translation by Ogg, Longmans, Green & Company

### PHYSICAL CHEMISTRY

EUKEN, JETTE, and LAMER, *Fundamentals of Physical Chemistry,* McGraw-Hill Book Company, Inc.

WASHBURN, *Principles of Physical Chemistry,* McGraw-Hill Book Company, Inc.

NERNST, *Theoretical Chemistry,* translation by Codd, The Macmillan Company

BANCROFT, *Applied Colloid Chemistry,* McGraw-Hill Book Company, Inc.

BINGHAM, *Fluidity and Plasticity,* McGraw-Hill Book Company, Inc.

BOGUE, *Theory and Application of Colloidal Behavior,* McGraw-Hill Book Company, Inc.

Freundlich, *Colloid and Capillary Chemistry,* translation by Hatfield, E. P. Dutton & Company

Lewis and Randall, *Thermodynamics and the Free Energy of Chemical Substances,* McGraw-Hill Book Company, Inc.

Taylor, *A Treatise on Physical Chemistry,* D. Van Nostrand Company, Inc.

Lewis, *A System of Physical Chemistry,* Longmans, Green & Company

### III. Foundations of Mathematical and Physical Sciences

Young, *Fundamental Concepts of Algebra and Geometry,* The Macmillan Company

Mach, *The Science of Mechanics,* translation by McCormack, The Open Court Publishing Company

Burtt, *The Metaphysical Foundations of Modern Physical Science,* Harcourt, Brace & Company, Inc.

Poincaré, *The Foundations of Science,* translation by Halstead, Science Press

Vaihinger, *The Philosophy of "As If,"* translation by Ogden, Harcourt, Brace & Company, Inc.

Lewis, *Anatomy of Science,* Yale University Press

Bridgman, *Logic of Modern Physics,* The Macmillan Company

APPENDIX E

## A Table of Natural Logarithms and Arcsines in Radians for Values between Zero and One by Hundredths

| $u$ | $\log_e u$ | arcsine $u$ |
|---|---|---|
| 0.00 | $-\infty$ | 0.000000 |
| 0.01 | —4.605170 | 0.010000 |
| 0.02 | —3.912023 | 0.020001 |
| 0.03 | —3.506558 | 0.030004 |
| 0.04 | —3.218876 | 0.040011 |
| 0.05 | —2.995732 | 0.050021 |
| 0.06 | —2.813411 | 0.060036 |
| 0.07 | —2.659260 | 0.070057 |
| 0.08 | —2.525729 | 0.080086 |
| 0.09 | —2.407946 | 0.090122 |
| 0.10 | —2.302585 | 0.100167 |
| 0.11 | —2.207275 | 0.110223 |
| 0.12 | —2.120264 | 0.120290 |
| 0.13 | —2.040221 | 0.130369 |
| 0.14 | —1.966113 | 0.140461 |
| 0.15 | —1.897120 | 0.150568 |
| 0.16 | —1.832581 | 0.160691 |
| 0.17 | —1.771957 | 0.170830 |
| 0.18 | —1.714798 | 0.180986 |
| 0.19 | —1.660731 | 0.191162 |
| 0.20 | —1.609438 | 0.201358 |

A Table of Logarithms and Arcsines (*Continued*)

| $u$ | $\log_e u$ | arcsine $u$ |
|---|---|---|
| 0.20 | —1.609438 | 0.201358 |
| 0.21 | —1.560648 | 0.211575 |
| 0.22 | —1.514128 | 0.221814 |
| 0.23 | —1.469676 | 0.232078 |
| 0.24 | —1.427116 | 0.242366 |
| 0.25 | —1.386294 | 0.252680 |
| 0.26 | —1.347074 | 0.263022 |
| 0.27 | —1.309333 | 0.273393 |
| 0.28 | —1.272966 | 0.283794 |
| 0.29 | —1.237874 | 0.294227 |
| 0.30 | —1.203973 | 0.304693 |
| 0.31 | —1.171183 | 0.315193 |
| 0.32 | —1.139434 | 0.325729 |
| 0.33 | —1.108663 | 0.336304 |
| 0.34 | —1.078810 | 0.346917 |
| 0.35 | —1.049822 | 0.357571 |
| 0.36 | —1.021651 | 0.368268 |
| 0.37 | —0.994252 | 0.379009 |
| 0.38 | —0.967584 | 0.389796 |
| 0.39 | —0.941609 | 0.400632 |
| 0.40 | —0.916291 | 0.411517 |

A Table of Logarithms and Arcsines (*Continued*)

| $u$ | $\log_e u$ | arcsine $u$ |
|---|---|---|
| 0.40 | —0.916291 | 0.411517 |
| 0.41 | —0.891598 | 0.422454 |
| 0.42 | —0.867501 | 0.433445 |
| 0.43 | —0.843970 | 0.444493 |
| 0.44 | —0.820981 | 0.455599 |
| 0.45 | —0.798508 | 0.466765 |
| 0.46 | —0.776529 | 0.477995 |
| 0.47 | —0.755023 | 0.489291 |
| 0.48 | —0.733969 | 0.500655 |
| 0.49 | —0.713350 | 0.512090 |
| 0.50 | —0.693147 | 0.523599 |
| 0.51 | —0.673345 | 0.535185 |
| 0.52 | —0.653926 | 0.546851 |
| 0.53 | —0.634878 | 0.558601 |
| 0.54 | —0.616186 | 0.570437 |
| 0.55 | —0.597837 | 0.582364 |
| 0.56 | —0.579818 | 0.594386 |
| 0.57 | —0.562119 | 0.606506 |
| 0.58 | —0.544727 | 0.618729 |
| 0.59 | —0.527633 | 0.631059 |
| 0.60 | —0.510826 | 0.643501 |

**A Table of Logarithms and Arcsines (*Continued*)**

| $u$ | $\log_e u$ | arcsine $u$ |
|---|---|---|
| 0.60 | —0.510826 | 0.643501 |
| 0.61 | —0.494296 | 0.656060 |
| 0.62 | —0.478036 | 0.668743 |
| 0.63 | —0.462035 | 0.681553 |
| 0.64 | —0.446287 | 0.694498 |
| 0.65 | —0.430783 | 0.707585 |
| 0.66 | —0.415515 | 0.720819 |
| 0.67 | —0.400478 | 0.734209 |
| 0.68 | —0.385662 | 0.747763 |
| 0.69 | —0.371064 | 0.761489 |
| 0.70 | —0.356675 | 0.775397 |
| 0.71 | —0.342490 | 0.789498 |
| 0.72 | —0.328504 | 0.803802 |
| 0.73 | —0.314711 | 0.818322 |
| 0.74 | —0.301105 | 0.833070 |
| 0.75 | —0.287682 | 0.848062 |
| 0.76 | —0.274437 | 0.863313 |
| 0.77 | —0.261365 | 0.878841 |
| 0.78 | —0.248461 | 0.894666 |
| 0.79 | —0.235722 | 0.910809 |
| 0.80 | —0.223144 | 0.927295 |

A Table of Logarithms and Arcsines (*Concluded*)

| $u$ | $\log_e u$ | arcsine $u$ |
|---|---|---|
| 0.80 | —0.223144 | 0.927295 |
| 0.81 | —0.210721 | 0.944152 |
| 0.82 | —0.198451 | 0.961411 |
| 0.83 | —0.186330 | 0.979108 |
| 0.84 | —0.174353 | 0.997283 |
| 0.85 | —0.162519 | 1.015985 |
| 0.86 | —0.150823 | 1.035270 |
| 0.87 | —0.139262 | 1.055203 |
| 0.88 | —0.127833 | 1.075862 |
| 0.89 | —0.116534 | 1.097345 |
| 0.90 | —0.105361 | 1.119769 |
| 0.91 | —0.094311 | 1.143284 |
| 0.92 | —0.083382 | 1.168080 |
| 0.93 | —0.072571 | 1.194413 |
| 0.94 | —0.061875 | 1.222630 |
| 0.95 | —0.051293 | 1.253236 |
| 0.96 | —0.040822 | 1.287002 |
| 0.97 | —0.030459 | 1.325230 |
| 0.98 | —0.020203 | 1.370462 |
| 0.99 | —0.010050 | 1.429257 |
| 1.00 | 0 | 1.570796 |

APPENDIX F

## The One-Zero Differential Sphere

Let us imagine a reservoir in Capillary Control to be of a lateral extent greater than $R$ in all directions from an orifice, and of a vertical thickness greater than $2R$. The orifice will be supposed to be at the bottom of an unperforated tube which extends downward one-half the thickness of the porous medium. By the action of the globules and bubbles such an orifice produces fluid from a spherical space of radius $R$ surrounding it. One hundred per cent of the mobile volume immediately at the orifice is produced, while zero per cent of this volume at the distance $R$ from the orifice is produced. The potential volume of the reservoir thus equipped lies as a differential sphere.

If the tube extended through the medium, and if it were perforated from the top to the bottom of the medium, we would, of course, have the usual differential cylinder surrounding the tube, regardless of the great thickness of the medium. The differential cylinder was found to be equivalent to a cone of uniform density. In fact we preferred the cone to the cylinder in mathematical operations. Now for the differential sphere we have no equivalent geometrical body of uniform density; consequently we must deal with this sphere directly, as we might have done with the differential cylinder. In Figure $F_1$ we have the sphere with its mathematical setting.

If we say that this sphere has a volume, or preferably a weight, of one hundred per cent in its entirety, we shall wish to know how this weight compares with that of a one-one sphere of the same radius, and how it compares with the weight of a circumscribed one-one cylinder. The first shows us the ratio in percentages between the potential and mobile volumes described by the radius $R$, and the second shows us the corresponding ratio between the potential volume and that mobile volume underlying the surface, of an area described by $R$ in all lateral directions, on the assumption that the medium has the specified thickness of $2R$. Furthermore, we shall wish to know the percentage of weight remaining to the differential sphere after interception by a plane to any degree between zero and one hundred per cent of its diameter.

Let us say that we have a natural reservoir in this control, wherein the productive formation is exceedingly thick. The well merely penetrates the top of this formation. Clearly we have to deal with the bottom half of our differential sphere. Perhaps it is unreasonable to expect a formation to be of a thickness $R$, as required by these conditions. There is no doubt about its having a thickness less than $R$ in many fields; therefore in these cases our differential sphere is intercepted by a horizontal plane denoting the bottom of the porous formation and the top of the underlying impervious stratum. In such artificial reservoirs as we have seen in Figures 111 and 112 the thickness of the porous medium, as measured from the orifice, is frequently greater than $2R$. Here, then, there is no interception by a plane. We have a differential hemisphere that is complete, provided the container itself is at least of radius $R$ on the plane of the orifice. Otherwise there is prismatic or cylindrical interception in accordance with the shape of the container.

In Figure $F_1$ the origin of co-ordinates is taken at the center of the sphere. First, there is a horizontal slice of thickness $dy$, located at a distance $y$ from the $X$ axis. Secondly, within this slice there is a ring $BB$, having a radius $x$ about the $Y$

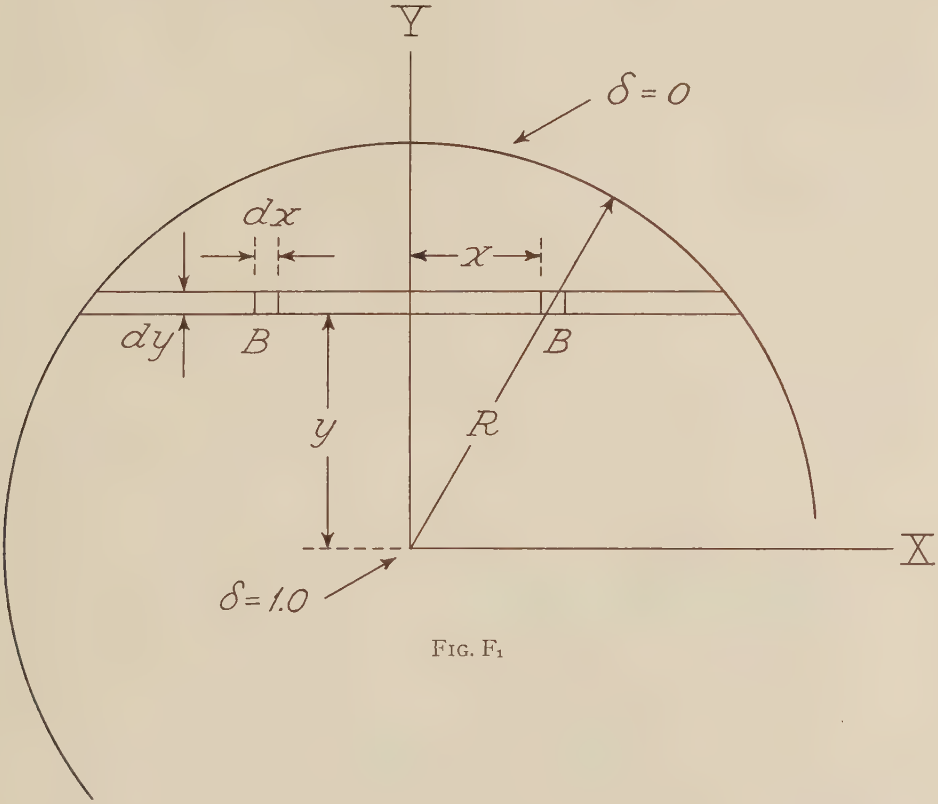



FIG. $F_1$

axis, and a thickness $dx$. We are to determine the weight of this ring, then the weight of the slice, and lastly the weight of all slices lying between designated limits, the latter being defined by horizontal planes.

The volume of the ring is

$$V = 2\pi x \, dx \, dy \quad \text{.......} (1)$$

and its density is

$$D = \frac{R - \sqrt{y^2 + x^2}}{R} \quad \text{.......} (2)$$

The weight of the ring is equal to the product of these quantities, since density is defined as volume divided by weight. But this product is actually the differential weight of the entire slice; that is,

$$dW_s = 2\pi x dx dy \left( \frac{R - \sqrt{y^2 + x^2}}{R} \right) \quad \text{.......} (3)$$

It then follows that the weight of the entire slice is

$$W_s = \frac{2\pi dy}{R} \int_0^{\sqrt{R^2 - y^2}} \left( R - \sqrt{y^2 + x^2} \right) x dx \quad \text{.......} (4)$$

By integration with respect to $x$, holding $y$ constant, and thereafter substituting the limiting values of $x$ into the result, we have

$$W_s = \frac{2\pi dy}{R}\left(\frac{R^3}{6} - \frac{Ry^2}{2} + \frac{y^3}{3}\right) \qquad (5)$$

This is the weight of the slice. It is also the differential weight of the sphere We may therefore write

$$dW_s = \frac{2\pi}{R}\left(\frac{R^3}{6} - \frac{Ry^2}{2} + \frac{y^3}{3}\right)dy \qquad (6)$$

The weight of the sphere between the limits $y =$ zero and $y = nR$, where $n$ is a decimal fraction with the limits zero and unity, is

$$W_s = \frac{2\pi}{R}\int_0^{nR}\left(\frac{R^3}{6} - \frac{Ry^2}{2} + \frac{y^3}{3}\right)dy \qquad (7)$$

By integration with respect to $y$ this becomes

$$W_s = \frac{\pi}{R}\left[\frac{R^3y}{3} - \frac{Ry^3}{3} + \frac{y^4}{6}\right]_0^{nR} \qquad (8)$$

Now a series of substitutions for $n$ can be carried out.

By substituting $n = 0$ and $n = 1$ we have the weight of half the entire sphere. It is

$$\frac{1}{6}\pi R^3$$

This is one-quarter of the weight of a one-one hemisphere having the same radius $R$. Of course the whole one-zero sphere is likewise one-quarter of the whole one-one sphere. *The potential volume of fluid to be delivered from the hemispherical or spherical space within the porous medium is equal to one-fourth of the mobile volume of fluid within the respective space.*

The weight of the one-zero hemisphere is one-sixth of the weight of the circumscribing one-one cylinder. The weight of the one-zero sphere is likewise one-sixth of the weight of the circumscribing one-one cylinder, this cylinder being of twice the weight of the previous one. *The potential volume of fluid to be delivered from a porous medium of a surface area defined by* R *is one-sixth of the mobile volume of fluid underlying such area, on the assumption that the medium has the thickness* R *for the hemisphere or* 2R *for the sphere.*

In case the porous medium is of greater thickness than the specified amounts, the potential volume is less than one-sixth. We need not give detailed consideration to this situation, since it is unlikely to be met in natural reservoirs. But in case the porous medium is of less thickness, the potential volume is greater than one-sixth. It cannot become greater than one-third, as we shall presently see.

To determine the percentage of weight remaining to the differential sphere after interception by a plane we simply give $n$ a series of values in Equation 8, subtract the values from the weight of the hemisphere to determine the weight of the intercepted portion, and divide these in turn by the weight of the entire sphere. The difference between these values and the weight of the entire sphere as unity

or one hundred per cent represents the series of quantities sought. We have the following table of such values:

| Per Cent Diameter Intercepted | Per Cent Volume Intercepted |
|---|---|
| 0.0 | 0.000 |
| 5.0 | 0.195 |
| 10.0 | 0.720 |
| 15.0 | 2.295 |
| 20.0 | 5.120 |
| 25.0 | 9.375 |
| 30.0 | 15.120 |
| 35.0 | 22.295 |
| 40.0 | 30.720 |
| 45.0 | 40.095 |
| 50.0 | 50.000 |

These points when plotted give half the curve. The other half may be drawn by revolving the plat 180 degrees in its own plane, and repeating the points as of the same position. The curve is shown in Figure $F_2$. For the purpose of compari-

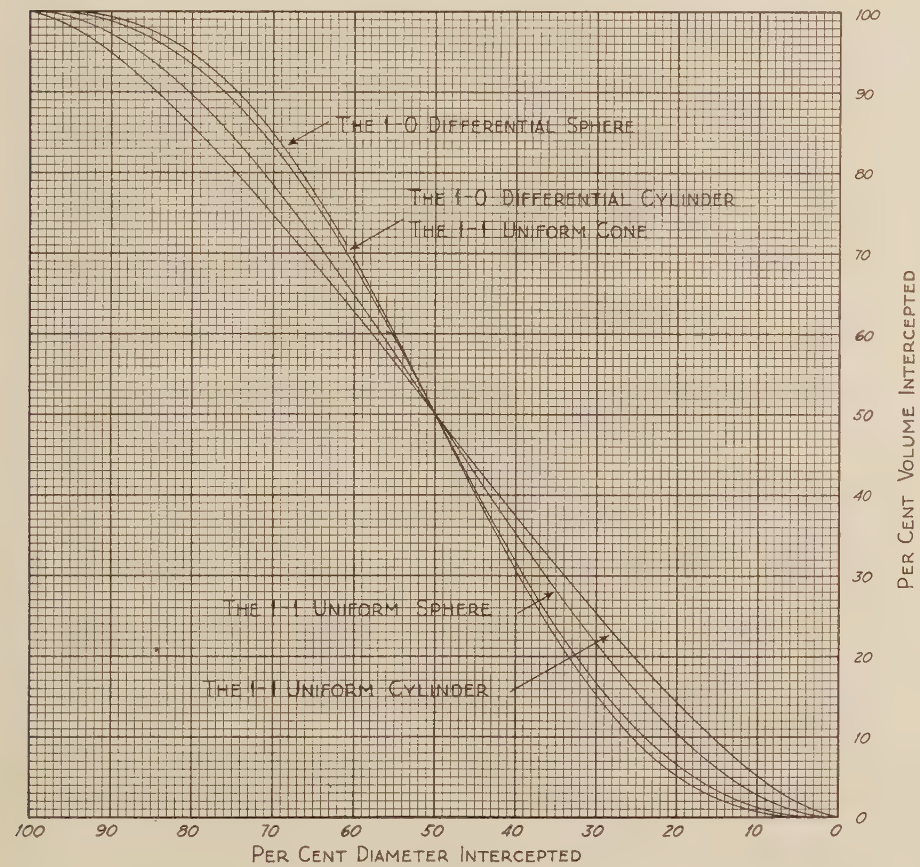



Fig. $F_2$

son I have added curves which were found by the same process within the text. These are as follows:

*a*) Interception between a plane and a one-one cylinder. (Section 137.)
*b*) Interception between a plane and a one-one sphere. (Section 138.)
*c*) Interception between a plane and a one-one cone or a one-zero cylinder. (Section 183.)

If the present curve is divided into two at its center, and these halves are stretched so as to reach from corner to corner on the plat, the resulting curves pertain to the two hemispheres to be obtained by cutting the differential sphere horizontally by a plane through the center, each respective hemisphere being taken as unity.

In Figure $F_3$ a well is shown to have perforated the top of a productive formation of thickness $h$ between $H_0$ and $H_2$. This well does not penetrate the formation to any appreciable extent. The formation itself is perfectly homogeneous in all directions, and $R$ is the drainage radius corresponding to the initial value of the potential pressure $P$. $R$ defines the differential hemisphere and its circumscribing cylinder. We have just found that the recovery from the formation underlying the circular area described by $R$ is one-sixth of the mobile volume of fluid present.

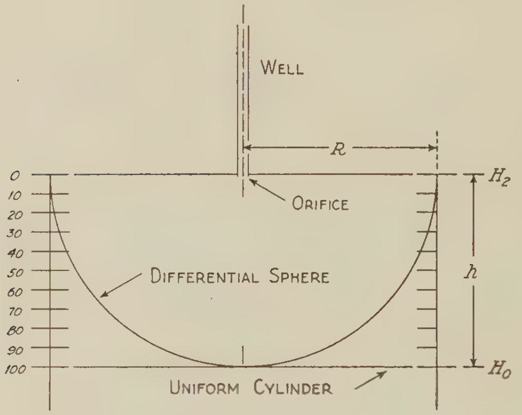



Fig. $F_3$

As $h$ is caused to decrease by moving $H_0$ upward toward $H_2$, both the hemisphere and the cylinder are intercepted by the plane defined by $H_0$. We can select eleven values of $h$ in percentage ratios of $R$, as indicated in the figure, and compute the corresponding percentage values of recovery by the well. The latter values are merely the ratios between the weight of the intercepted differential hemisphere and the weight of the uniform cylinder inclosing it, both possessing, of course, the same altitude. The first weight is determined from the stretched lower half of the curve in Figure $F_2$, and the second is determined directly by geometry. The table of values follows:

| Per Cent Ratio between $h$ and $R$ | Per Cent Recovery $Q_{Pn}$ |
|---|---|
| 0.0 | 33.33 |
| 10.0 | 33.02 |
| 20.0 | 32.13 |
| 30.0 | 30.78 |
| 40.0 | 29.07 |
| 50.0 | 27.08 |
| 60.0 | 24.93 |
| 70.0 | 22.72 |
| 80.0 | 20.53 |
| 90.0 | 18.48 |
| 100.0 | 16.67 |

Recovery is designated as $Q_{Pn}$ in conformity with the symbols used in chapters xxxii and xxxiii. It pertains to recovery in terms of a mobile volume of fluid that is determined by the initial value of the potential pressure. The recovery $Q_{Sn}$ can be determined in the usual way.

The curve for the above values is shown in Figure $F_4$. We here see the effects

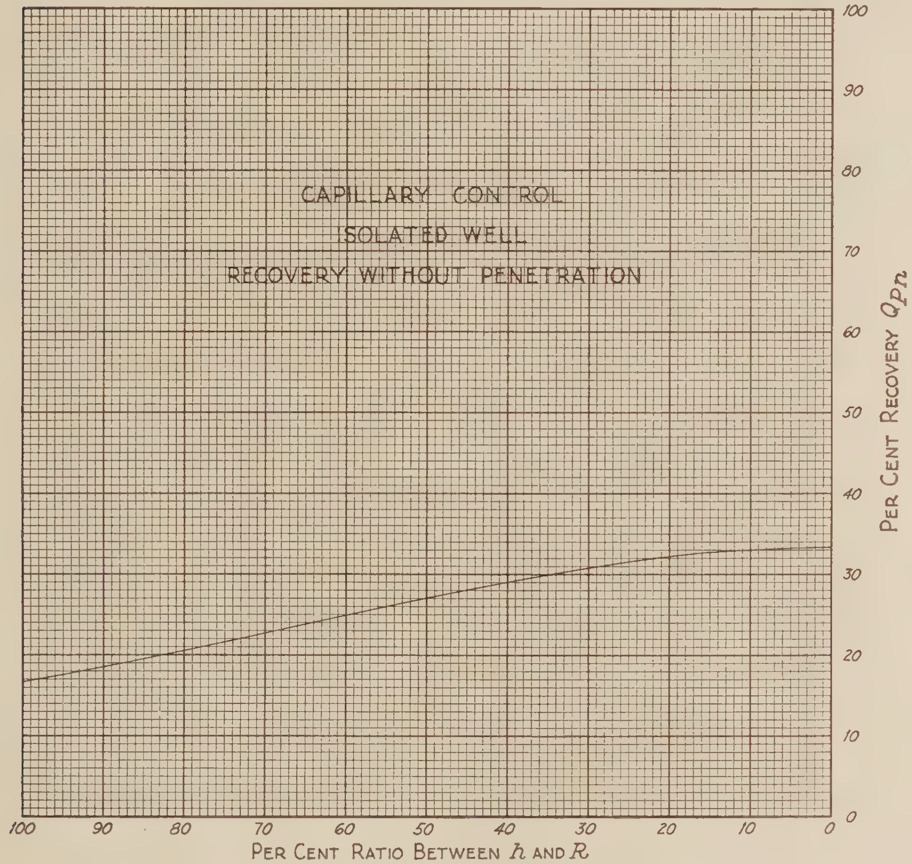



FIG. $F_4$

of failing to penetrate the formation with the drill. In the limit, as $h$ approaches zero, the differential hemisphere becomes equivalent to a differential cylinder, as of full penetration. This is the maximum of recovery $Q_{Pn}$ for the isolated well, under the specified conditions of drilling.

## APPENDIX G

### The Logarithmic Slide-Rule[1]

The logarithmic slide-rule is provided with the so-called logarithmic scale, the same scale we considered in connection with the logarithmic plat for the curves of production, as described in chapter xix. While the scale on the rule might properly extend in either direction, that from left to right is customary, perhaps to conform to our usual manner of writing down increasing numbers side by side. A unit length $s$ of a given number of inches is selected for the scale, and this length is divided according to the logarithms of numbers between successive powers of ten. The left-hand end of the scale denotes $10^n$, while the right-hand end denotes $10^{n+1}$, where $n$ is an integral number, either positive or negative. These divisions of $s$ clearly comply with the equation

$$y = \log x$$

in which $x$ is any number between $10^n$ and $10^{n+1}$, integral or fractional, and $y$ is a corresponding distance for $x$ to be measured off from the left end of $s$. The index numbers ascribed to marks in accordance with such measurements express $x$ directly, instead of the logarithm of $x$. In virtue of this feature we need not consciously concern ourselves with logarithms in the operations of multiplication and division.

For the unit length $s$ we have numbers in the same form of periodicity as we have in the case of the logarithmic plat; that is to say, we have for $s$ the numbers

$$x = 0.1,\ 0.2,\ 0.3,\ \ldots\ldots\ 1.0$$
$$x = 1.0,\ 2.0,\ 3.0,\ \ldots\ldots\ 10.0$$
$$x = 10.0,\ 20.0,\ 30.0,\ \ldots\ldots\ 100.0$$

each set including all intermediate numbers. These are but three of a possible infinite number of sets, all others being of a magnitude greater or less than these. Although the scale is actually of the length $s$, it may be regarded as one of infinite length because of this periodicity.

For any length $s$ there is, of course, a smallest value $ds$ which can be appreciated with accuracy; consequently there is a smallest value $dy$ in $y$, and, in turn, a smallest value $dx$ in $x$, which can be likewise appreciated with accuracy. The last is the error in $x$ which must attend a given operation in multiplication or division. The absolute value of $dx$ depends upon the magnitude of the set of numbers ascribed to $s$, while the relative value of $dx$ in terms of $x$ is always the same, regardless of the magnitude of the set of numbers. In order to diminish the value of $dx$, either as an absolute or a relative quantity, it is necessary and sufficient to take $s$, as measured in inches, greater.

[1] The following description is based largely upon the contents of the pamphlet furnished by Keuffel & Esser Company.

The rule for multiplication and division may be deduced immediately from the equation

$$\log x \pm \log y = \log z$$

For multiplication it is merely necessary to add the logarithmic lengths representing $x$ and $y$, and for division to subtract them. Thus the operations implied by

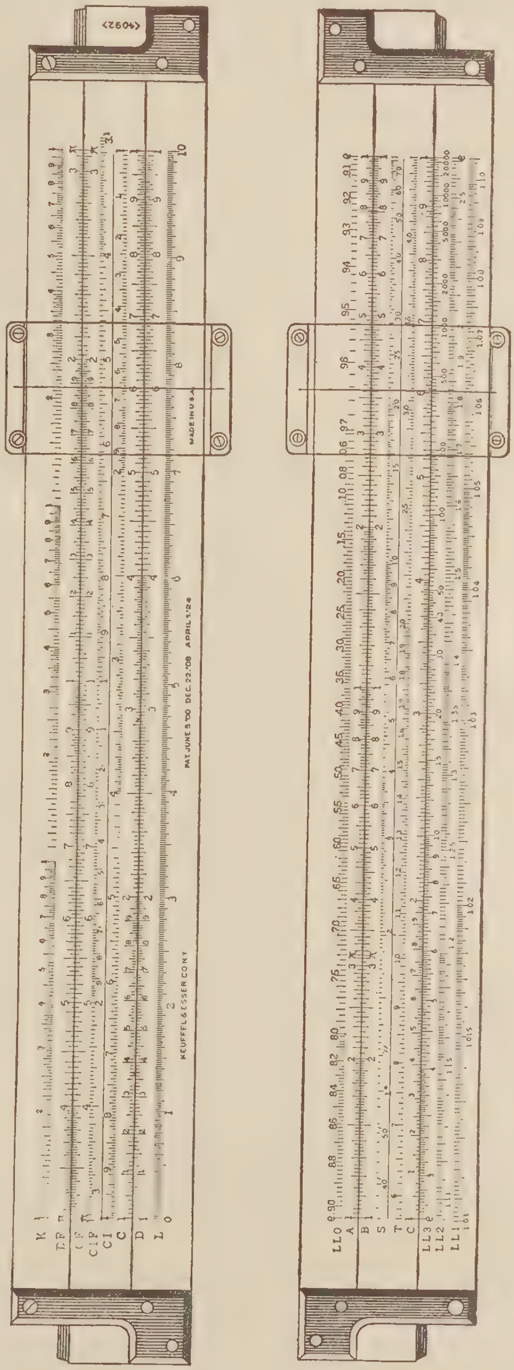
$$xy = z \text{ and } x/y = z$$

are transformed into the simpler operations

$$x + y = z \text{ and } x - y = z$$

respectively, by means of the slide-rule. The scale has the property of canceling the term "log" in the preceding logarithmic equation.

As we see in Figure $G_1$ the unit of length for the scales $A$ and $B$ is half that for the scales $C$ and $D$. The error encountered in the use of the former is twice that with the latter. Between these two sets of scales we have squares and square roots of numbers. In virtue of the fact that $A$ and $B$ actually contain two unit scales end on end, a desired product or quotient is always to be found for the particular setting of the slide. With $C$ and $D$ it frequently happens that we do not discover a wrong setting of the slide until we find the desired number to be beyond the range of the scale. We must then reset the slide. This resetting is avoided, however, by the scales $CF$ and $DF$, which have the same unit length as $C$ and $D$, but are divided conveniently at $\pi$ in place of powers of ten. Between these two sets the desired number is always within range. $CF$ and $DF$ make our linear rule equivalent to a circular one wherein scales are joined end to end.



*Courtesy of Keuffel & Esser Company*

FIG. $G_1$

The inverted scales *CI* and *CIF* permit us to change an operation of division to one of multiplication, and vice versa, without any intermediate operation. Like the preceding scales these two "complete a circle."

The unit of length for the scale *K* is one-third that for *C* and *D;* consequently between these we have cubes and cube roots of numbers. Between *K* and *A* or *B* we have squares of cube roots and cubes of square roots of numbers; that is, numbers raised to the 2/3 and 3/2 powers.

The scale *S* gives the angles from 35 minutes to 90 degrees, the sines of which are to be read on *B*. For the decimal point it is to be noted that for 30 degrees on *S* we have 0.5. Again, the scale *T* gives the angles between 5 degrees 43 minutes and 45 degrees, the tangents of which are to be read on *C*. For the decimal point it is to be noted that for 30 degrees on *T* we have 0.577.

As the sines and tangents of small angles do not differ materially, tangents beyond the range given above may be determined from the sine scale *S*. For sines and tangents of angles smaller than the lowest division on this scale, namely 35 minutes, the "minute" or "second" gauge mark is set to the desired number or minutes or seconds on *A,* and the required value read on *A* over the index of the sine scale. In order to determine the decimal point of values so obtained, it should be remembered that sine 1″ is about 0.000005, and that sine 1′ is about 0.0003.

The common logarithm of any number may be read on the equal parts, or *L* scale, under the desired number on the *D* scale. The *L* scale indicates the numbers which would have been recorded on the *D* scale, had logarithms been recorded instead of the numbers to which they belong.

Given a quantity $u$ between zero and one on the *LL*0 scale, the logarithm of this quantity to the natural base $e$ is found on the scale *A*. The *LL*0 is here given a range from 0.97 to 0.05. It is so placed that $e^{-0.1}$ and $e^{-1.0}$ are in alignment with the end and middle indexes, respectively, on *A*. This pair of scales is equivalent to the first two columns of the table in Appendix E.

The particular feature of the slide-rule illustrated in the figure is the "log log" scale, divided into the three parts, *LL*1, *LL*2, and *LL*3. The raising of numbers to powers, greater or less than unity, and integral or fractional, is a simple operation in virtue of this scale in conjunction with *C*. Say we have the equation

$$y = a^n$$

wherein the exponent is any such number as mentioned. By taking the logarithm of this expression we obtain

$$\log y = n \log a$$

The quantity on the right is itself a product of two numbers. Let us take the logarithm of the entire expression once more:

$$\log \log y = \log n + \log \log a$$

If we now find $a$ on the log log scale, and add to it $n$ on the log scale *C,* under the index of the latter we will have, on the log log scale the value of $y$. The scales thus reduce the operation to one represented by the equation

$$y = n + a$$

The log log scale is divided on powers of $e$. Thus the separate parts have the following ranges:

| | | | | |
|---|---|---|---|---|
| *LL*1, | from | $e^{0.01}$ | to | $e^{0.10}$ |
| *LL*2, | from | $e^{0.10}$ | to | $e$ |
| *LL*3, | from | $e$ | to | $e^{10.0}$ |

This division permits us to read natural logarithms at once on $D$ without regard to the position of the slide, or to use them as factors when they occur in any formula.

It is important to observe that the values stamped on the log log scale are definite ones, and may not be taken as multiples or decimal fractions of the values stated, as is done with $A$, $B$, $C$, $D$, and their like. When the slide projects beyond the ends of the rule in taking powers or roots, the desired number can be read below the index at the other end of the slide, on the division of the scale adjoining, provided only that this number does not lie beyond the range of the entire scale. This reading is possible in virtue of the fact that the three parts are continuous.

The log log slide-rule may be obtained in ten- and twenty-inch lengths. For accuracy the latter is of course to be preferred.

## The Triangular Pattern for Wells

The situation for the location of wells in accordance with the triangular pattern is shown in Figure $H_1$. All degrees of interception—negative, zero, and positive—are of course possible. Here interception is shown to be 13.40 per cent, under which circumstances the drainage areas of the wells intersect at the center of the triangle. If we consider a single location, such as $L_1$, in the center of its drainage area, we see that its cone is intercepted by six neutral planes. The area is in fact a perfect hexagon for this and all greater values of interception. Whatever values we may determine with respect to the triangle $WL_1L_2$ we can apply to the hexagon without modification, for the portion $b$ is common to both areas, $a$ is exactly the same as $a'$, and $c$ is exactly the same as $c'$. The portions $a'$, $b$, and $c'$ form half the hexagon, and the other half consists of portions $a''$, $b''$, and $c''$, all alike.

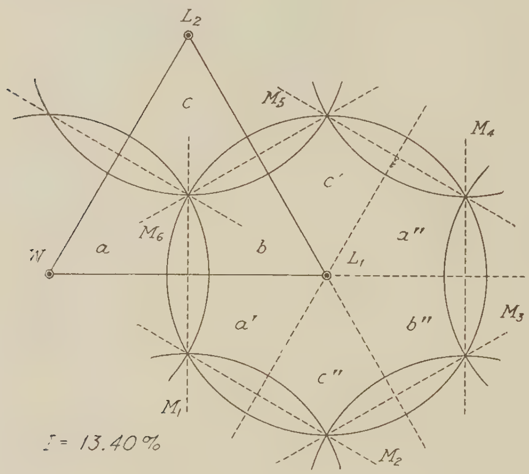



FIG. $H_1$

Corresponding to Figure 200 for the square pattern we now have Figure $H_2$. The curve $AB$ shows the percentage of the normal volume produced by each well for all positive degrees of interception, where the normal volume is understood to be that which is produced at zero interception. This curve is subject to the conditions outlined in section 184.

In place of Figure 202 we have Figure $H_3$ (p. 624). In view of the description given the former figure in section 186 we need only dwell upon differences in the present case. The triangle contains three sixths of volume cones which are identical. These are equivalent to one-half a complete cone. Of all the mobile fluid under the unit area only that from one-half a cone is recovered when interception is zero or negative. The volume of this half-cone is

$$\frac{1}{6}\,\pi R^2 h_{Vo}$$

where $R$ is the original drainage radius for the given value of potential pressure, and $h_{Vo}$ is the altitude of the cone, with its inclosing prism, in terms of volume of fluid.

The volume of the prism of mobile fluid under the triangle depends, of course, upon the length of the side of the triangle. Let us consider the case where this side

is of a length $2R$; that is, the case where interception is exactly zero. The volume of the prism is then

$$\frac{\sqrt{3}}{4}\ (2R)^2 h_{Vo}$$

The ratio in percentage between the two volumes is the percentage of recovery for zero interception. This is 30.23 per cent. The recovery is of such value with

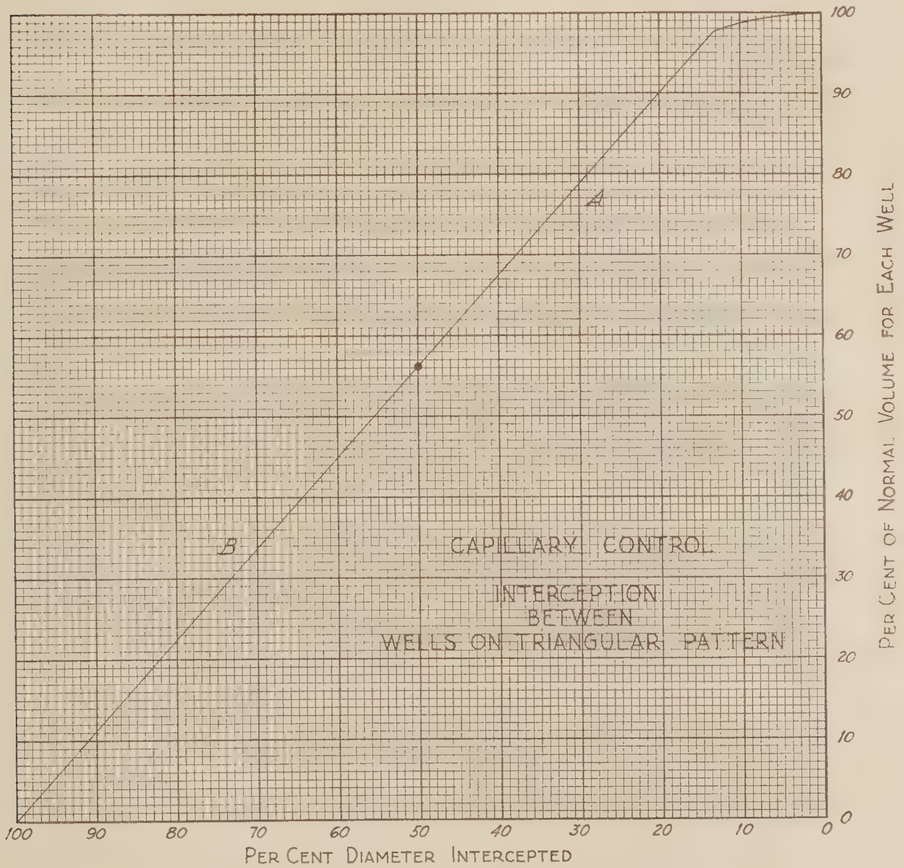


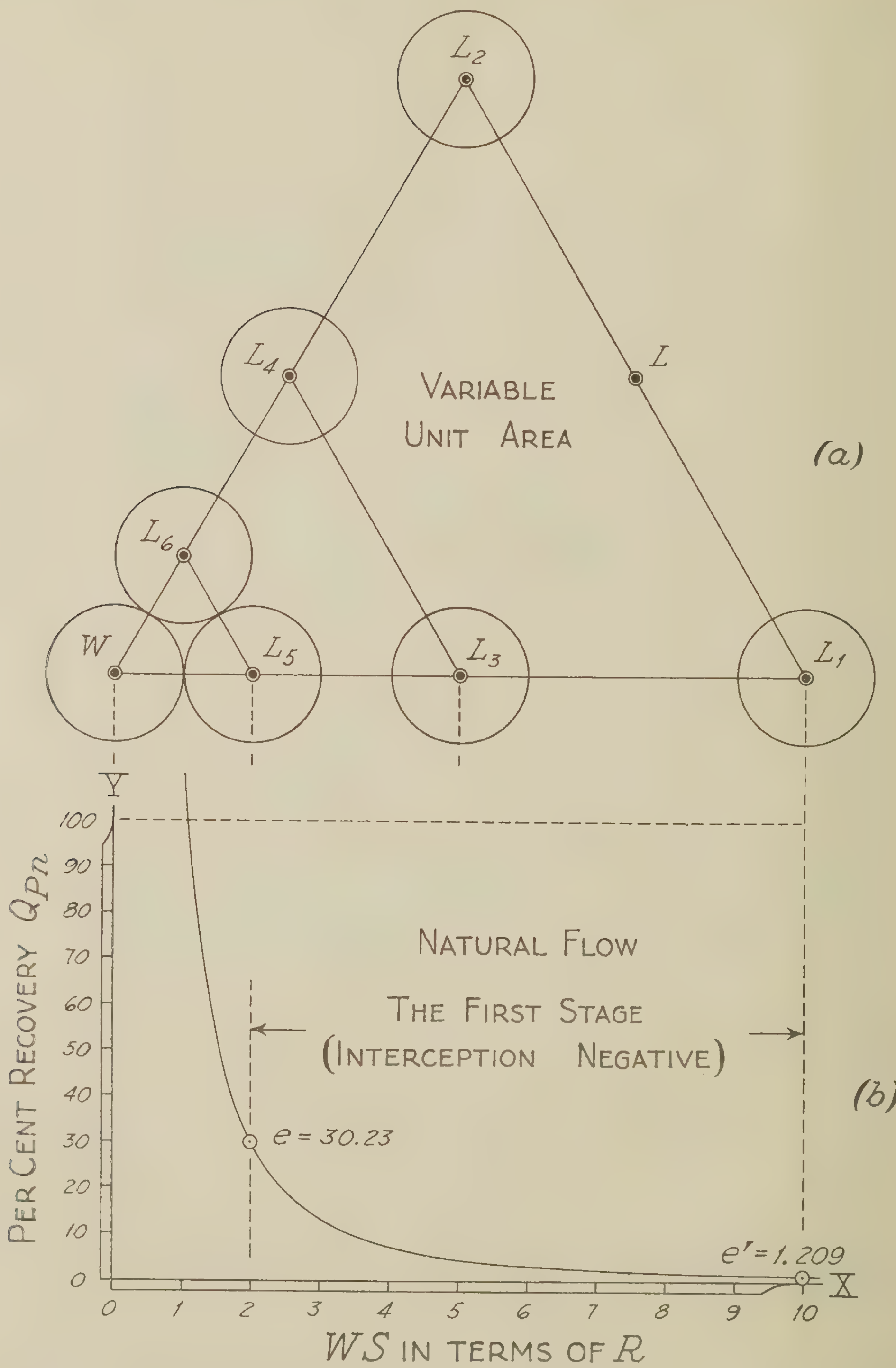

FIG. $H_2$

respect to the triangle, the hexagon, and the field at large, provided the field is homogeneously drilled throughout its area. Where recovery for zero interception in accordance with the square pattern is 26.18 per cent, it is greater than this amount in accordance with the triangular pattern.

We might take triangles of any other length of side. If we say that $WS$ is the distance between wells, as expressed in terms of $R$, then all values of recovery will be found to satisfy the equation

$$(WS)^2 Q_{Pn} = 120.9$$

where $Q_{Pn}$ is the value of such recovery. This equation corresponds to Equation 607 (p. 557). Like its predecessor it is the equation of a quadratic hyperbola. If



$L_2$
$L_4$
$L$
VARIABLE
UNIT AREA
(a)
$L_6$
$W$
$L_5$
$L_3$
$L_1$
Y
100
90
80
70
60
50
40
30
20
10
0
PER CENT RECOVERY $Q_{Pn}$
NATURAL FLOW
THE FIRST STAGE
(INTERCEPTION NEGATIVE)
(b)
e = 30.23
e′ = 1.209
X
0 1 2 3 4 5 6 7 8 9 10
WS IN TERMS OF R


FIG. $H_8$

we change its form to that of a variation we obtain the same result as before, for the differing constants are caused to disappear by doing this. *Recovery in percentage of the mobile volume of fluid varies inversely as the square of the distance between wells, regardless of the pattern of locations.*

Whereas the constant previously had the value 104.7, it now has, as we see, the value 120.9. This is an increase of 15.47 per cent, if the first value is taken as of the basis of comparison. *Recovery at given values of interception is greater by 15.47 per cent when wells are located on triangles than when located on squares.* This percentage of increase only holds for such values of interception for which the equations themselves hold. As before, the present equation breaks down in cases where $WS$ is less than $2R$. Figure $H_3$ therefore shows the recovery law for the first stage, wherein interception is negative or zero.

For positive interception we have Figure $H_4$ corresponding to Figure 203. This

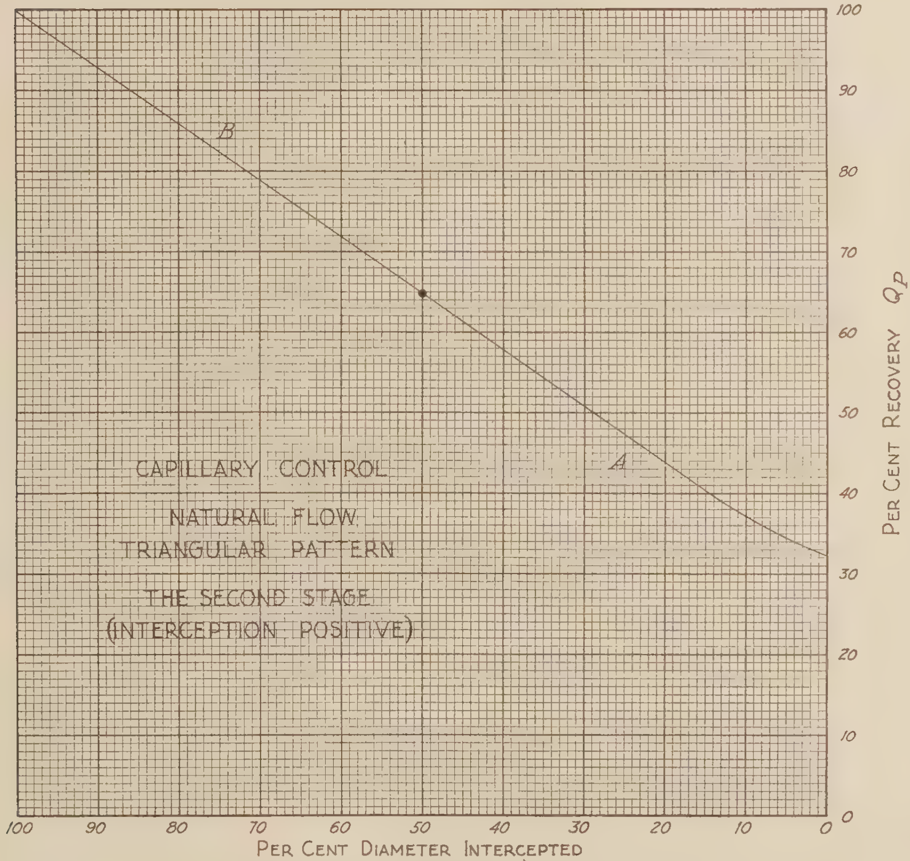



FIG. $H_4$

figure is computed in the same manner as the earlier one, and it is subject to the same conditions, as explained in section 186. For given values of interception the increase in recovery by the triangular pattern may be determined by comparing corresponding ordinates in the two figures. This increase begins at 15.47 per cent

for zero interception and diminishes as we approach one hundred per cent interception. At one hundred per cent interception the increase is exactly zero per cent.

To cover a given field a greater number of wells are required when the locations are in accord with triangles than when they are in accord with squares, interception being of the same degree in both cases. Let us see what the increase in the number is. We shall consider the square and the hexagon of sides $2R$ as a special case, and thereafter generalize our comparative results.

For the square:

The area allotted to one well is

$$A = 4R^2$$

The recovery is

$$Q_{Pn} = 26.18\%$$

The number of wells per area $R^2$ is

$$N = 0.25$$

For the triangle, in accordance with Figure $H_5$:

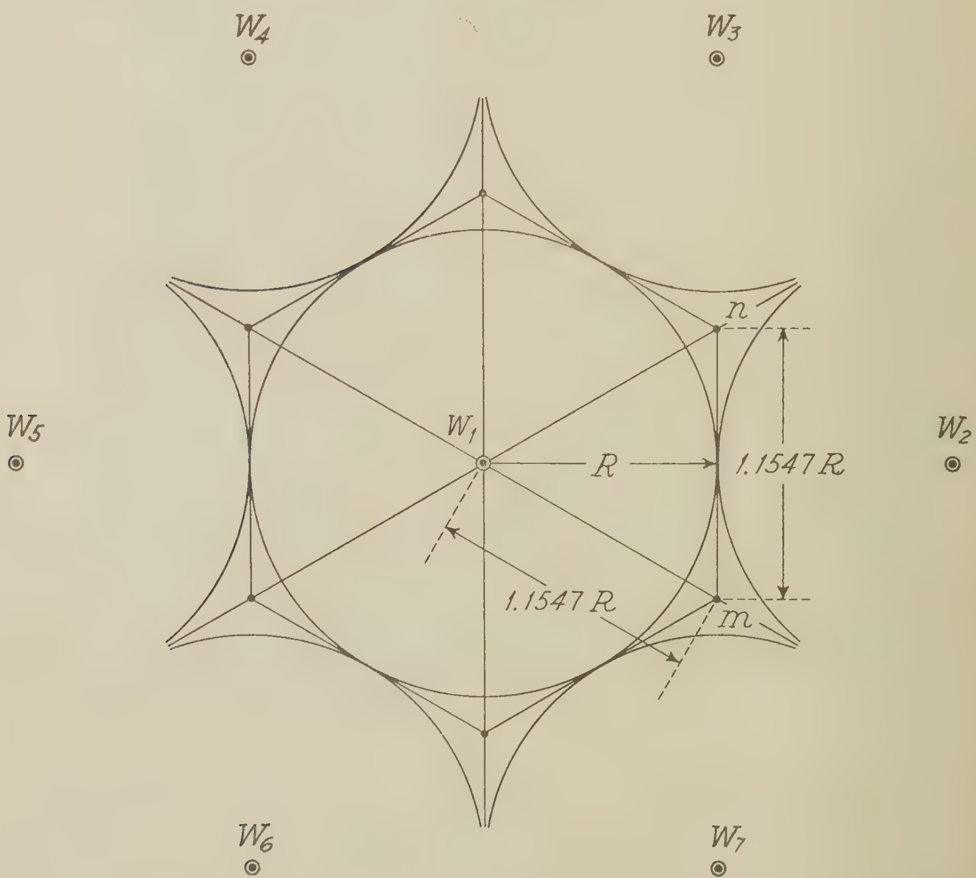



FIG. $H_5$

The area allotted to one well is

$$A' = 6 \times \tfrac{1}{2} \times R \times 1.1547\,R$$
$$= 3.4641\,R^2$$

The recovery is

$$Q'_{Pn} = 30.23\%$$

The number of wells per area $R^2$ is

$$N' = 0.28866$$

The ratio in percentage between $Q_{Pn}$ and $Q'_{Pn}$ is

$$\frac{30.23}{26.18} \times 100\% = 115.47\%$$

The ratio in percentage between $N$ and $N'$ is

$$\frac{0.28866}{0.25000} \times 100\% = 115.47\%$$

Both ratios are the same; both show an increase of 15.47 per cent as between the two patterns. *It is therefore evident that the increase in recovery is due simply to the increase in the number of wells required to cover the field.*

Now the ratio between $N$ and $N'$ is purely a geometrical one. It holds between all squares and hexagons having sides of the same length. It follows, then, that the increase in the number of wells is 15.47 per cent regardless of the values of $R$ and of interception. On the other hand, the increase in recovery is 15.47 per cent only in case interception is negative or zero. For positive interception this increase diminishes as we pass from zero to one hundred per cent, being precisely a zero increase for the latter value. We may therefore come to the following conclusion: *While the ratio between the number of wells remains fixed, the advantages to be gained in recovery diminish as we pass from zero to one hundred per cent interception.*

## APPENDIX I

### The Zoar Storage Field[1]

The Zoar Storage Field is a part of the natural gas plant which furnishes gas to the city of Buffalo, New York. Previously exhausted wells are filled during each summer from distant sources, and this gas is held in storage for distribution in the winter months. Observations on pressure and volume are tabulated throughout operations. The actual curves for the process of filling during the summers of 1917 to 1922, inclusive, are accurately reproduced in Figure $I_1$.

These curves illustrate in a very beautiful manner the relations between pressure and volume in natural reservoirs of Capillary Control. They represent a practical example of the principles discussed in section 151. It is clear that they are

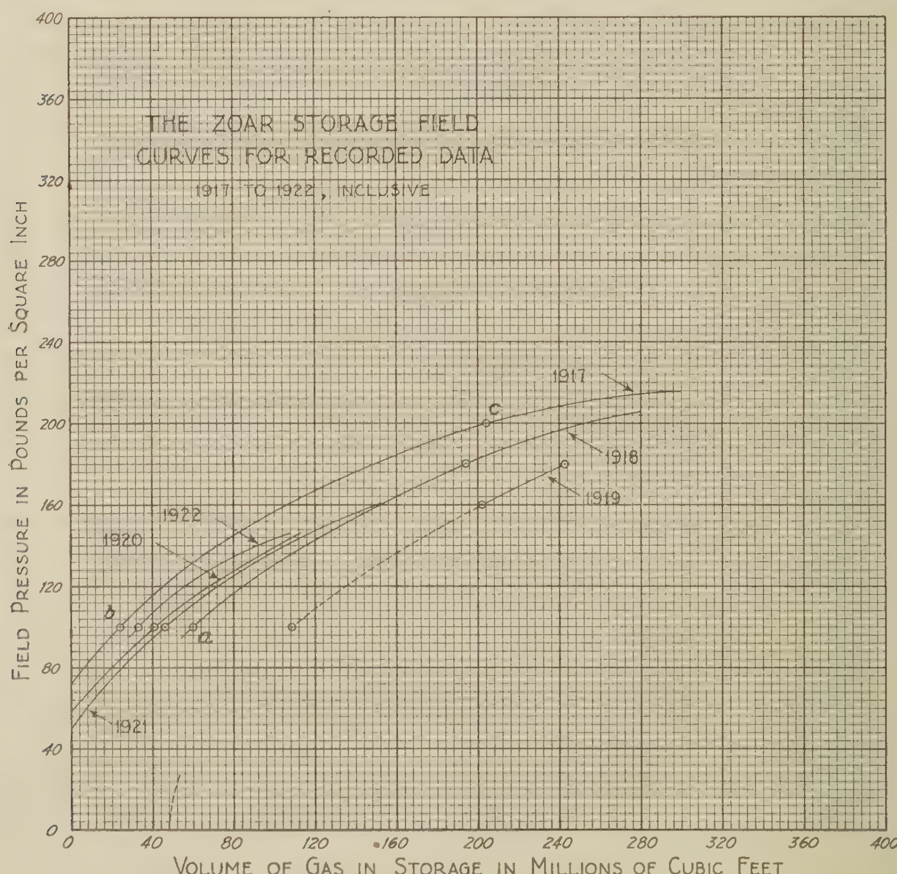



Fig. $I_1$

[1] The data for operations in this field were furnished by Thomas R. Weymouth, President, Iroquois Gas Corporation, Buffalo, New York, and by Ralph E. Davis, Engineer, Pittsburgh, Pennsylvania.

of consistent curvature, being portions of true parabolas in accordance with the equation

$$P = K Vo^{1/2}$$

Let us examine them for the purpose of seeing how closely they conform to this equation in spite of their horizontal positions on the plat.

For the point *a* on the curve of 1918

$$P = 100 \text{ pounds per square inch}$$

and

$$Vo = 60{,}000{,}000 \text{ cubic feet of gas}$$

By substituting these values into the foregoing equation we find that the constant

$$K = 12.9086$$

or

$$K = 12.91, \text{ approximately}$$

The specific equation is therefore

$$P = 12.91\ Vo^{1/2}$$

Now if we assume values for volume, say from 60,000,000 to 280,000,000, the corresponding values for the pressure may be computed by means of this equation. These pressures may then be compared with values as shown by the curve. Thus we have the following table:

| Assumed Volume | Computed Pressure | Observed Pressure | Error | Per Cent Error |
|---|---|---|---|---|
| 60* | 100.0* | 100.0* | 0.0* | 0.00* |
| 80 | 115.5 | 116.0 | +0.5 | 0.44 |
| 100 | 129.1 | 130.0 | +0.9 | 0.70 |
| 120 | 141.5 | 143.0 | +2.5 | 1.77 |
| 140 | 152.7 | 154.0 | +1.3 | 0.85 |
| 160 | 163.3 | 164.5 | +1.2 | 0.73 |
| 180 | 173.2 | 173.5 | +0.3 | 0.17 |
| 200 | 182.5 | 182.0 | −0.5 | 0.27 |
| 220 | 191.4 | 190.0 | −1.4 | 0.73 |
| 240 | 200.0 | 196.5 | −3.5 | 1.75 |
| 260 | 208.0 | 202.0 | −6.0 | 2.88 |
| 280 | 216.0 | 206.0 | −10.0 | 4.63 |

* These values are assumed to be correct in accordance with the specific equation.

The column to the right shows the close adherence of the actual curve to the computed one for that year. The last two observations indicate an excessive error. Perhaps this is due to the fact that the pressure gauges did not record accurately above 200 pounds.

The curve for 1917 appears to be quite similar to the one for 1918, except for the fact that its position is shifted to the left. We can say at once that its position is incorrect, for at the extreme left we find a recorded pressure of 72 pounds per square inch corresponding to zero volume. This is absurd, for if the gas can exert this pressure when a well is closed, it must be able to escape when the well is opened.

Let us assume that we do not know the proper position for the 1917 curve on the plat. Inasmuch as pressure is intensive in its nature, it may be measured at single observations. We shall assume that the recorded values of this function are correct. Volume is extensive in its nature. Two observations are required for its measurement. We shall assume that the recorded values of this function are incorrect, while differences between any two of its values are correct. We shall see presently whether we are fully justified in making these assumptions.

Now for a selected observation $b$,

$P_1 =$ 100 pounds per square inch,

$Vo' =$ 24,000,000 cubic feet, as indicated by the curve, and

$Vo_1 =$ the true volume for the point $b$ to be determined

And for another selected observation $c$,

$P_2 =$ 200 pounds per square inch,

$Vo'' =$ 204,000,000 cubic feet, as likewise indicated by the curve, and

$Vo_2 =$ the true volume for the point $c$, also to be determined.

It is clear that $P_1/P_2 = 50$ per cent; that is, when on consumption the pressure declines from 200 to 100 pounds, it has 50 per cent of its first value remaining. From the relative curve in Figure 92 we learn that a decline to 50 per cent pressure remaining corresponds to a decline to 25 per cent volume remaining. If $Vo_2$ represents 100 per cent initial conditions, to correspond with $P_2$ as such, $Vo_1$ then represents 25 per cent final conditions, to correspond with $P_1$ as representing 50 per cent final conditions. Evidently 75 per cent of the volume is withdrawn from the reservoir in the decline of pressure from 200 to 100 pounds. Then

$Vo_2 - Vo_1 = Vo'' - Vo' = 75$ per cent total volume initially present.

From the data of observations we have

$Vo'' - Vo' =$ 180,000,000 cubic feet

This amount is 75 per cent of the initial volume. Thus

$Vo_2 =$ 100 per cent = 240,000,000 cubic feet

and

$Vo_1 =$ 25 per cent = 60,000,000 cubic feet

Now the values

$P_1 =$ 100 pounds per square inch

and

$Vo_1 =$ 60,000,000 cubic feet of gas

show the correct position for the point $b$ on the plat. This point obviously falls on $a$, our point on the 1918 curve. The 1918 curve happened to have its correct

position on the plat, while that for 1917 is shifted to the left by 36,000,000 cubic feet of volume. If we move this curve to its proper location, we shall find the section between the points *b* and *c* to agree with the computed curve above even more accurately than does the 1918 curve. For pressures above 200 pounds the curve is again quite in error.

The curve for 1919 is shifted to the right by 48,000,000 cubic feet. The upper section conforms accurately with the data, but the lower section, being obviously incorrect in the extreme, is replaced by computed values in accordance with the equation, without disturbing the position of the curve on the plat. If this reconstructed section is extended to the edge of the plat, we see the absurdity of 48,000,000 cubic feet of gas in storage, ready to be produced, but at zero pressure.

It is important for us to interpret correctly the quantities which give title to ordinates and abscissas in this figure. Pressures are those above the pressure of the atmosphere, as recorded directly by pressure gauges. Volume of gas in storage is that which, measured at atmospheric pressure, will be produced as against the pressure of the atmosphere. These are "potential" functions according to our definition. The potential axis here coincides with the atmospheric axis, a circumstance which might be caused to exist with any reservoir, artificial or natural.

The fact that the curves are shifted from their proper positions in all except the case for 1918 is due solely to the loss of the count in volume during the winter months of consumption.

It is a very simple matter to calibrate the reservoir at any time, in order to check the record of stored gas. Mathematically the process is exactly in accord with our computation concerning the curve for 1917 above. The accuracy of this reservoir as a producer is greater than that of any mechanical measuring device we may attach to it.

The curve between pressure and volume is a single path year after year. This is shown in Figure $I_2$ (p. 632). It is based on the unique assumption that the data for the points *a*, *b*, and *c*, in the preceding figure, are consistently correct. The points for the present curve are as follows:

| Pressure | Volume |
|---|---|
| 0.0 | 0.0 |
| 20.0 | 2.4 |
| 40.0 | 9.6 |
| 60.0 | 21.6 |
| 80.0 | 38.4 |
| 100.0 | 60.0 |
| 120.0 | 86.4 |
| 140.0 | 117.6 |
| 160.0 | 153.6 |
| 180.0 | 194.4 |
| 200.0 | 240.0 |
| 220.0 | 290.4 |
| 240.0 | 345.6 |
| 260.0 | 405.6 |
| .... | .... |

These when plotted give the section *AB*. On extension we have the section *BC*, as shown in the figure.

For production against a constant back pressure greater than the atmosphere we can simply shift the vertical scale for pressures downward through a distance corresponding to the excess of this back pressure over that of the atmosphere. For example, suppose the reservoir were to produce consistently against a line pressure

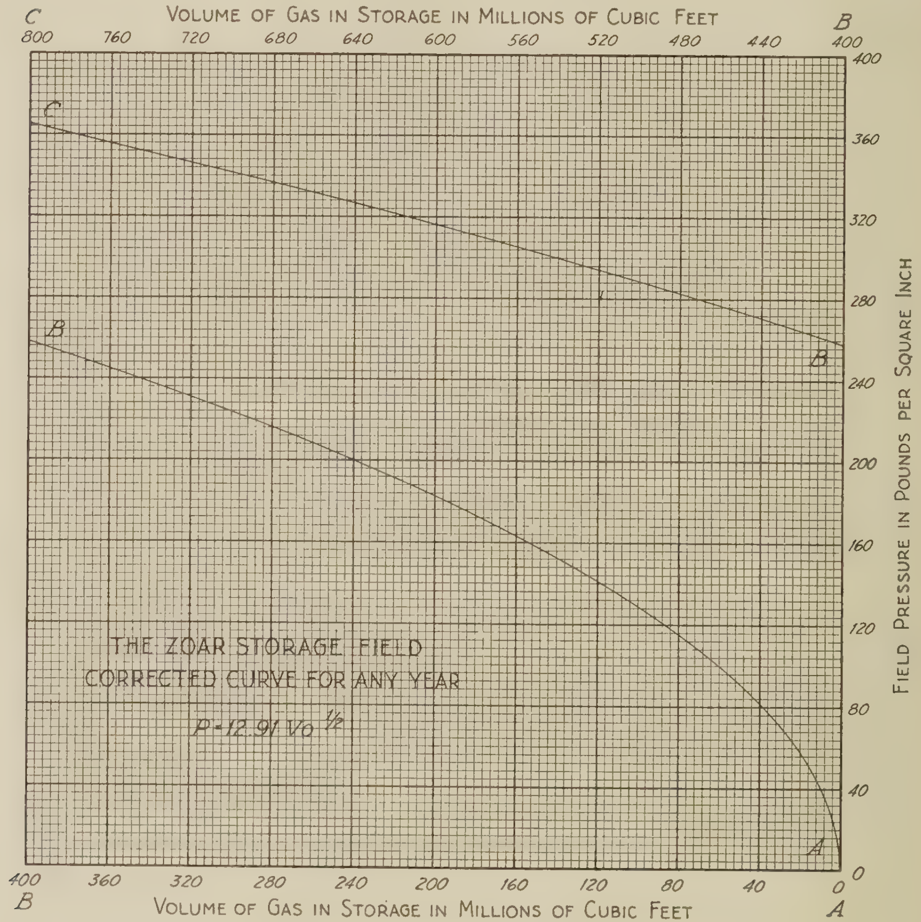



Fig. I₃

of 50 pounds above atmosphere. We shift the scale downward 50 pounds, and thereupon we have the volume of gas to be produced from the reservoir, as against this line pressure. For a vacuum applied to the reservoir the scale must be shifted correspondingly upward.

APPENDIX J

## A Natural Forced Drive

Given a natural reservoir in a more or less consolidated, yet porous and permeable formation, such reservoir containing both liquid and gas in globules and bubbles, we must admit that Jamin action exists within it. If this reservoir is performing in either Hydraulic or Volumetric Control, we can say that Nature is operating a forced drive on her own account, in so far as she is driving with the water which constitutes a column of liquid bearing its weight upon, say, a pool of oil in the vicinity of a well or group of wells. Aside from this general situation, however, there is a special one that is worthy of particular attention. It is the situation wherein all wells producing from the formation would possess potential reservoirs in Capillary Control, except for the fact that there is a small, yet appreciable inflow of water from the surface, either at the outcrop of the formation or along a fault or fault zone.

Figure $J_1$ represents in profile a vertical section along the dip of a productive formation. This formation extends over an undefined distance perpendicularly in front of and behind the plane of the drawing. For the sake of simplicity the surface is imagined to be plane at the elevation of $I$. The formation outcrops at $a$, and a well $W$ penetrates it at $b$. A pump at the well removes the liquid at $b$ as rapidly as it arrives there; consequently the lines $J$ and $N$ coincide in the manner shown.

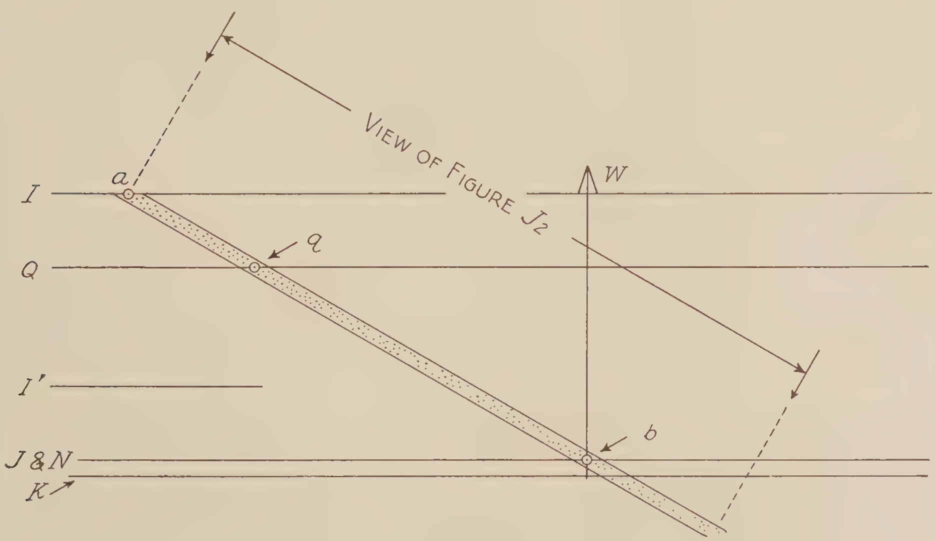



FIG. $J_1$

The Jamin action within the formation sets up a line $Q$ in the same manner as in Figures 111 and 112. The distance between $Q$ and $N$ therefore represents the potential pressure of the reservoir at $b$. We already know that $W$ performs in accordance with Hydraulic Control if the free surface of the column of liquid is maintained

at $a$ or at any position below this point within a certain range that is about to be defined. We also know that $W$ performs in accordance with Volumetric Control if the free surface of the liquid lowers in accordance with production from $W$. In this case $I$ and $Q$ lower, maintaining a constant distance between them, until $Q$ coincides with $J$ and $N$, $I$ taking the position $I'$. When this happens $W$'s reservoir is converted from Volumetric to Capillary Control. In fact $I'$ defines the lowest position for the maintenance of the free surface for production from $W$ in accordance with Hydraulic Control.

We shall imagine the free surface of the liquid to be maintained at any position, say at $a$, above $I'$. This requires an inflow of water at a rate equal to the rate of production from $W$. What can be determined concerning the events and conditions within the productive formation at large under these circumstances? Let us refer to Figure $J_2$, a perpendicular view of the formation, as indicated in Figure $J_1$.

We have again the line $Q$ and the point $b$ where the well $W$ penetrates the formation. The potential pressure $P$ pertaining to $b$ has, in virtue of Jamin action

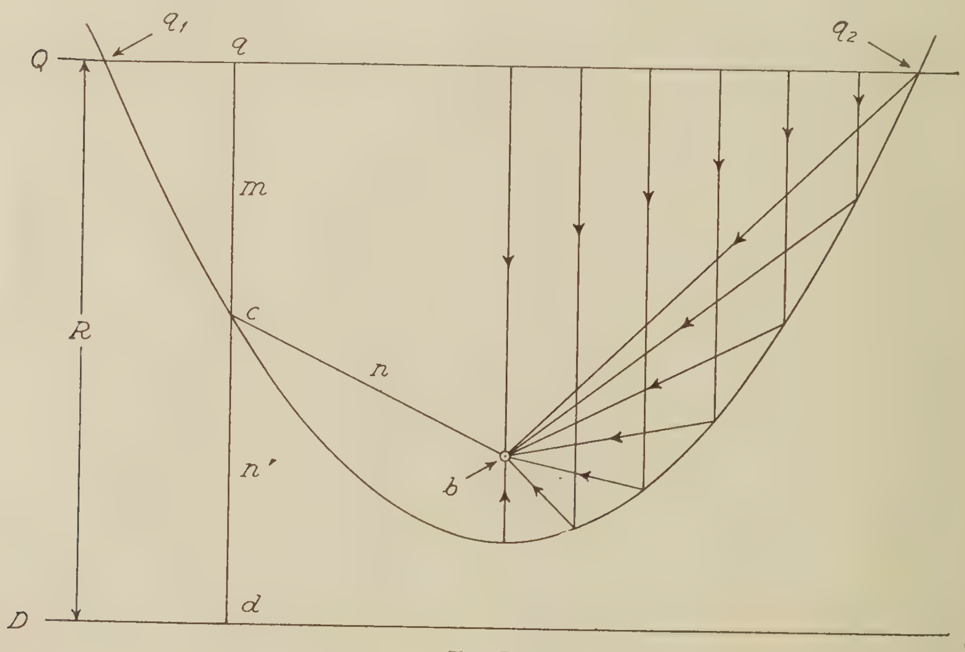



FIG. $J_2$

within the formation, a radius of action $R$. Since $Q$ is assumed to be a straight horizontal line, the radius $R$ defines a parallel line $D$. $D$ lies below $b$; otherwise $W$ would of necessity be producing in accordance with Capillary Control, a condition contrary to the stipulation that the free surface of the liquid is maintained at a position above $I'$. The strip of the formation between $Q$ and $D$ is under stress due to the pressure $P$. None of the fluid within this strip can move except that which lies within some radial distance $R$ with $b$ as a focus. The situation is analogous to that encountered in the forced drive. In the present instance, however, the pressure is applied along a straight line, as at $Q$, instead of being applied at another single point designating the location of a second well. The ellipse of the forced

drive no longer holds; nevertheless we encounter some sort of a geometrical figure defined by a pressure acting at all points along a straight line, a pressure having a radius of action $R$. And this geometrical figure has a single focus at $b$. What is this figure?

Assuming our figure to be drawn to scale, we may place a pin at $b$ and fasten to it a thread of length $R$ on the same scale. Now we can say that the movement of the fluid within the formation is confined to all possible positions in which this thread may be imagined to extend between the line $Q$ and $b$. $P$ can always move fluid over a path of length $R$. *It of course can move fluid over a shorter path, but certainly it cannot move fluid over a longer one.* If we take a pencil, and draw an area with it while it is held in the loop of the thread, always maintaining a portion of this thread perpendicular to $Q$, we circumscribe the area to be swept out by $P$. The point $c$ represents one position for the pencil; $m$, is perpendicular to $Q$, and $n$ extends from the pencil to the pin, so that

$$m + n = R$$

But it is clear that

$$m + n' = R$$

Consequently

$$n = n'$$

The curve circumscribed by the pencil is therefore a parabola. The point $b$ is its focus, and $D$ is its directrix. Under these circumstances Nature is driving fluid to $W$. *Not one molecule of the driving fluid can pass the circumference of the parabola, and not one molecule of the fluid beyond the circumference of the parabola can be driven to the well.* The free surface of the column of liquid in Figure $J_1$ may be altered in its position, either by a change in the rate of inflow or by flow from $W$ in accordance with Volumetric Control; therefore the value of $P$, and consequently the value of $R$, is altered. But if these happen, we merely repeat the statement with respect to molecules of driving and driven fluids as applied to a parabola of larger or smaller dimensions.

During the driving process there is focal radial flow within the parabola, as indicated on the right in Figure $J_2$. This situation is likewise analogous to that of the forced drive.

$W$ has a unique potential reservoir. It is taking care of all water furnished by Nature within the formation structurally above it. Although we have considered $W$ to be a single well, we see that the same principle holds with respect to a group of wells capable of acting as a multiple orifice.

If there is another well or group of wells at some locality down the regional dip of the formation, and below the present location of $D$, performance will there take place in accordance with Capillary Control. This is necessarily the case, for it is clear that if performance were to be in accordance with either Hydraulic or Volumetric Control, the well or group of wells would not be located below $D$, but above it, acting with $W$ in the way of a multiple orifice.

We see in this problem the possibility that a given productive formation might readily have wells in Hydraulic or Volumetric Control in its fields near the outcrop, and at the same time have wells in Capillary Control in its fields farther down the regional dip.

Nothing is to be construed here as a contradiction to statements made in the text. We retain our two aspects of a reservoir, that of a potential reservoir capable of producing fluid under the given conditions of production, and that of a reservoir as a physical container. While the elliptical area of the forced drive described in chapter xxxiii is in either Hydraulic or Volumetric Control, any exterior wells producing by natural flow are in Capillary Control. The same physical container possesses potential reservoirs in different controls. Obviously the same situation holds with regard to the present parabolic drive.

If we will agree to say that the elliptical forced drive is an artificial one, we may further agree that the parabolic forced drive is a natural one.

*For any given porous and permeable formation, containing oil and gas in association, and capable of producing the same, Capillary Control is the normal control. Hydraulic and Volumetric controls are merely special variations of this normal control.* In this we revert to the beginning of our investigation where we distinguished between Hydraulic and Volumetric controls, on the one hand, and Capillary Control, on the other. (See section 171, the first paragraph.) In the variations the directrix may be within the formation, in which case this directrix is real; or it may be beyond the limits of the formation, in which case it is imaginary. Both situations undoubtedly exist in our known productive formations.

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